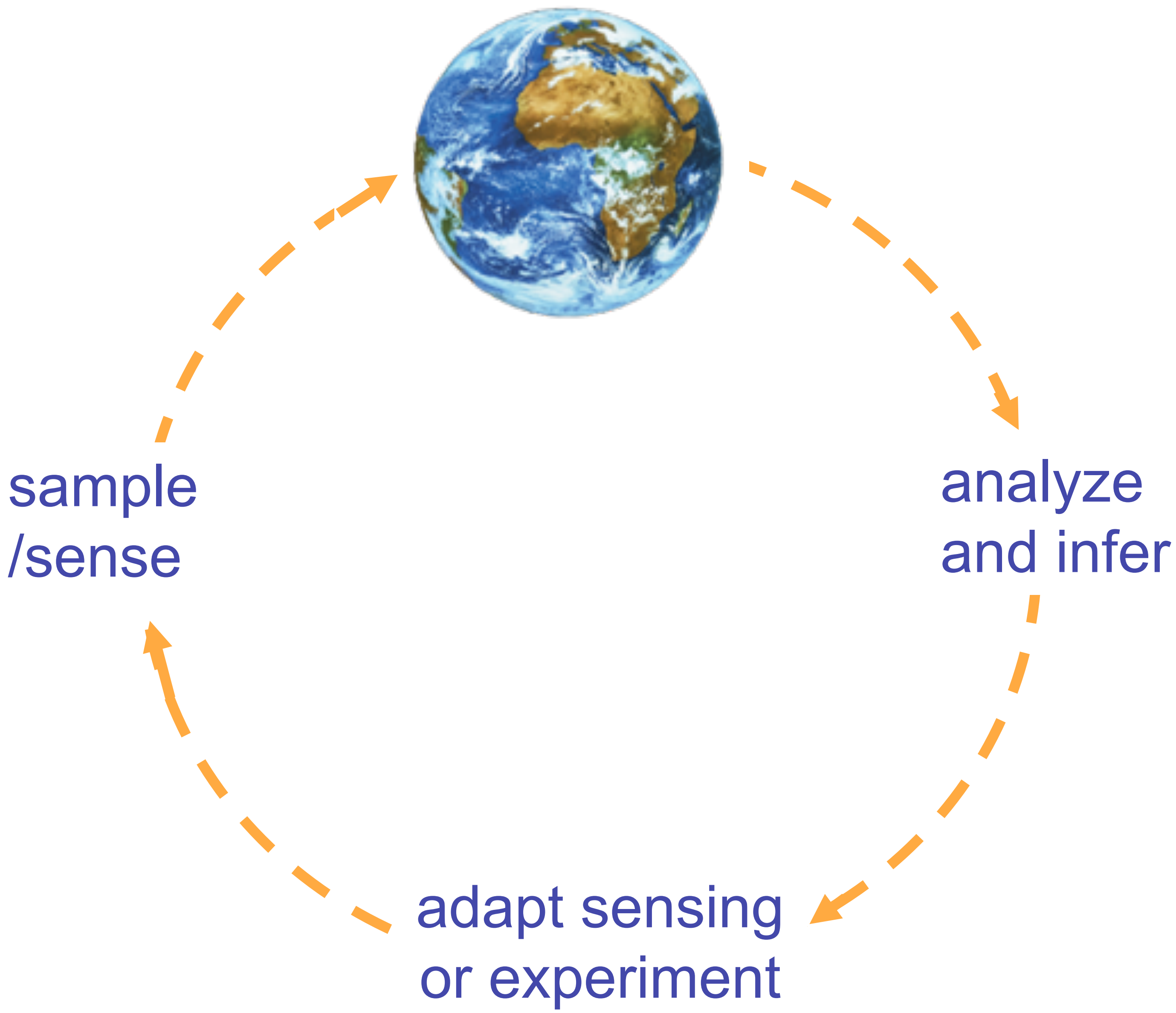
Active Learning



Machine Learning

Training examples come in pairs, feature X and label Y.

Goal: Design a rule for predicting Y given X

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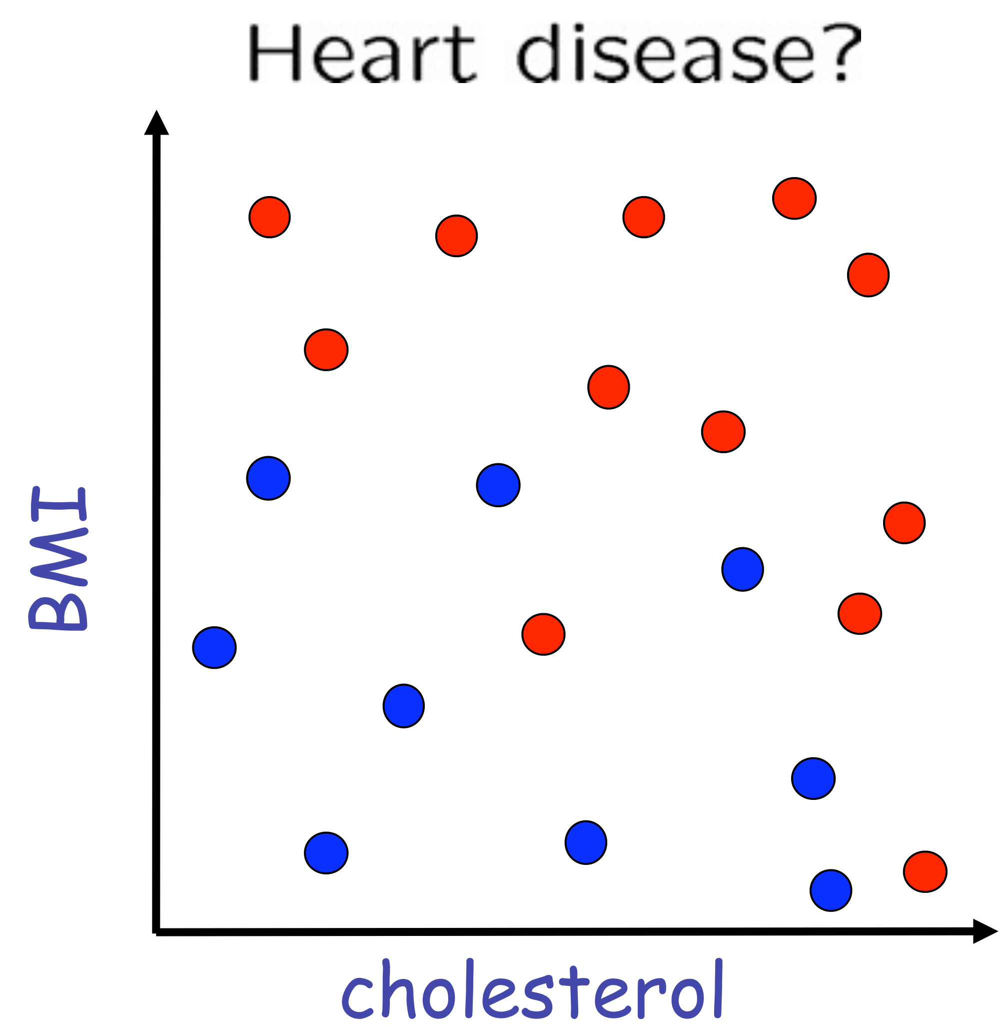

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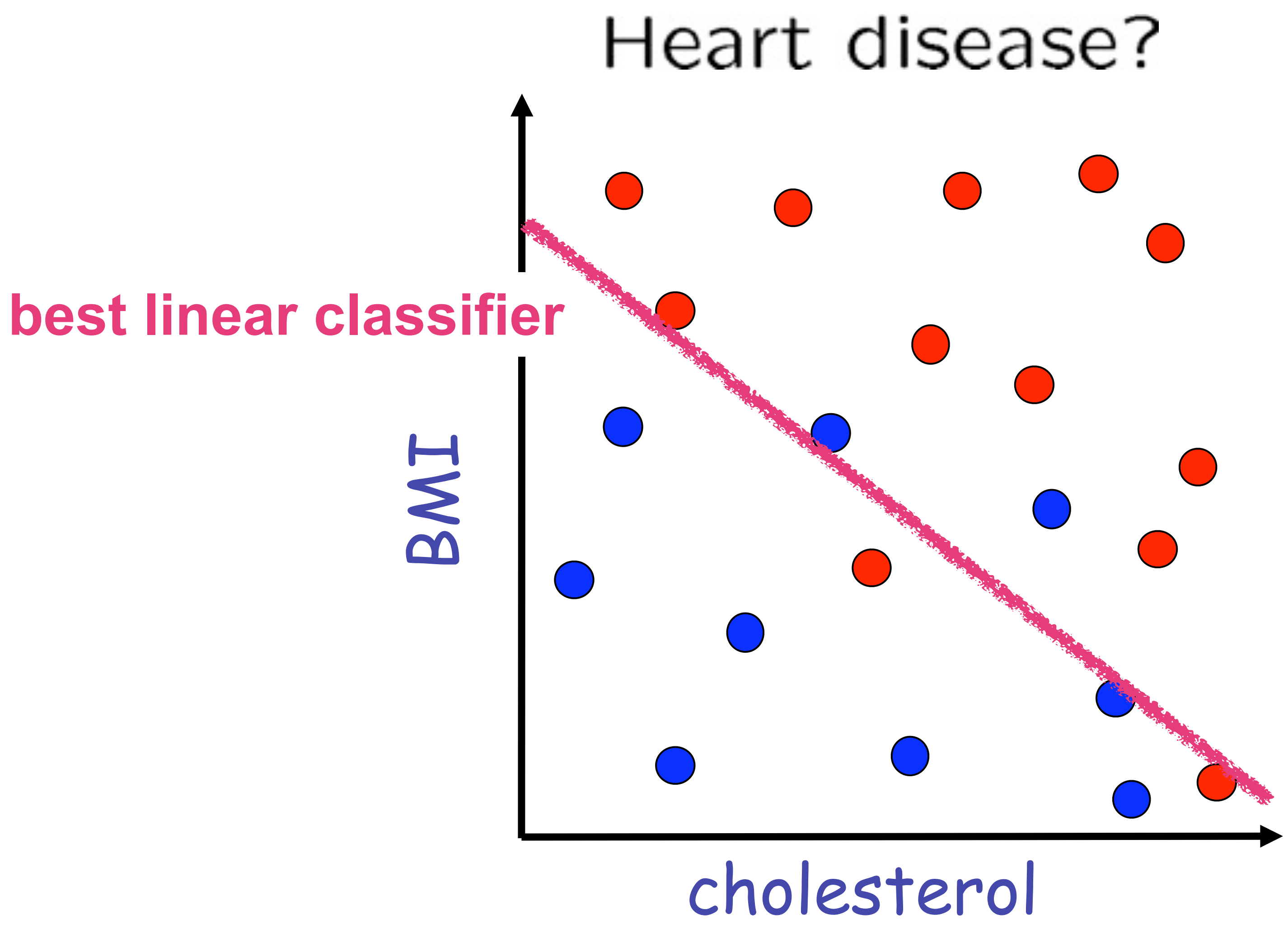

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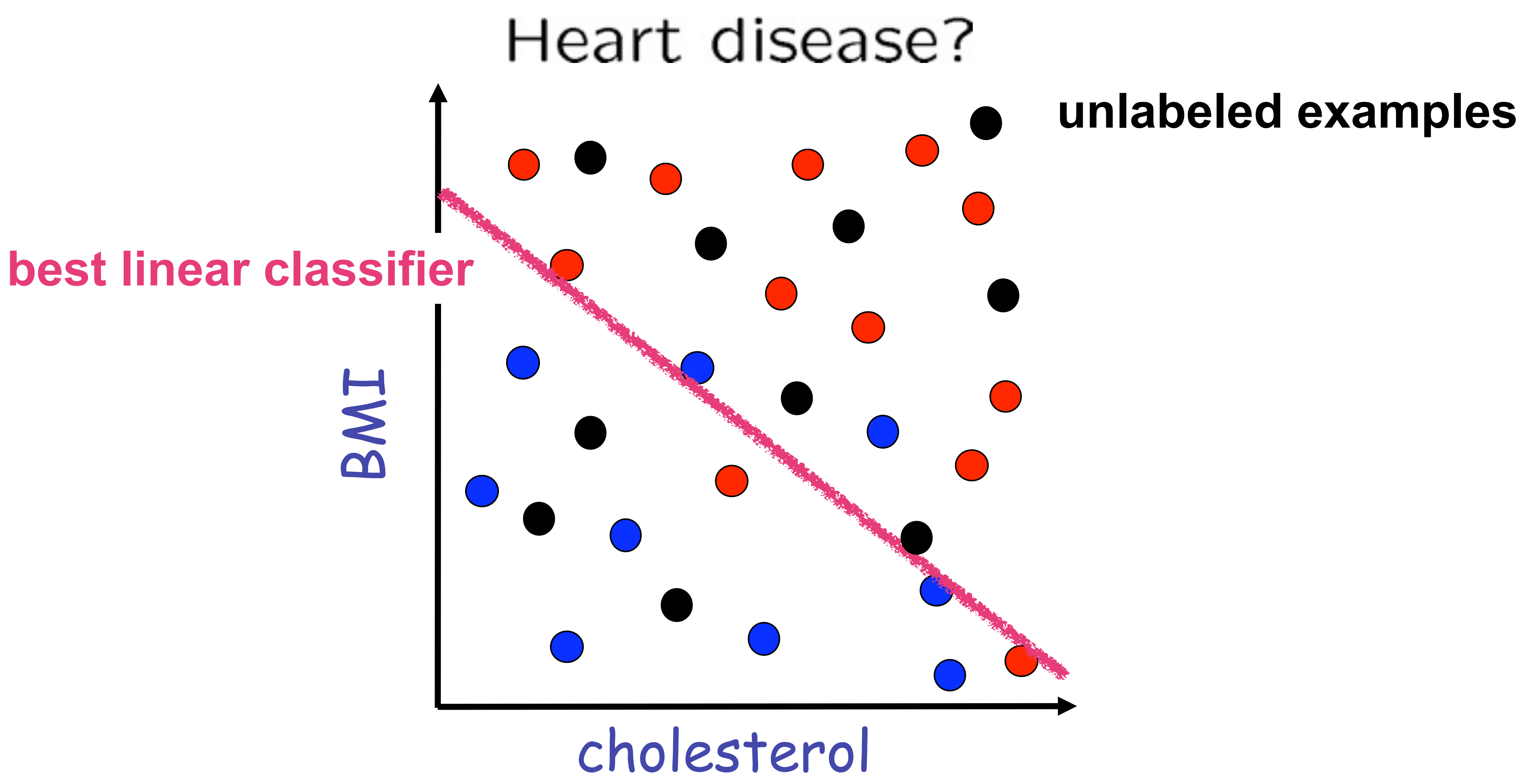

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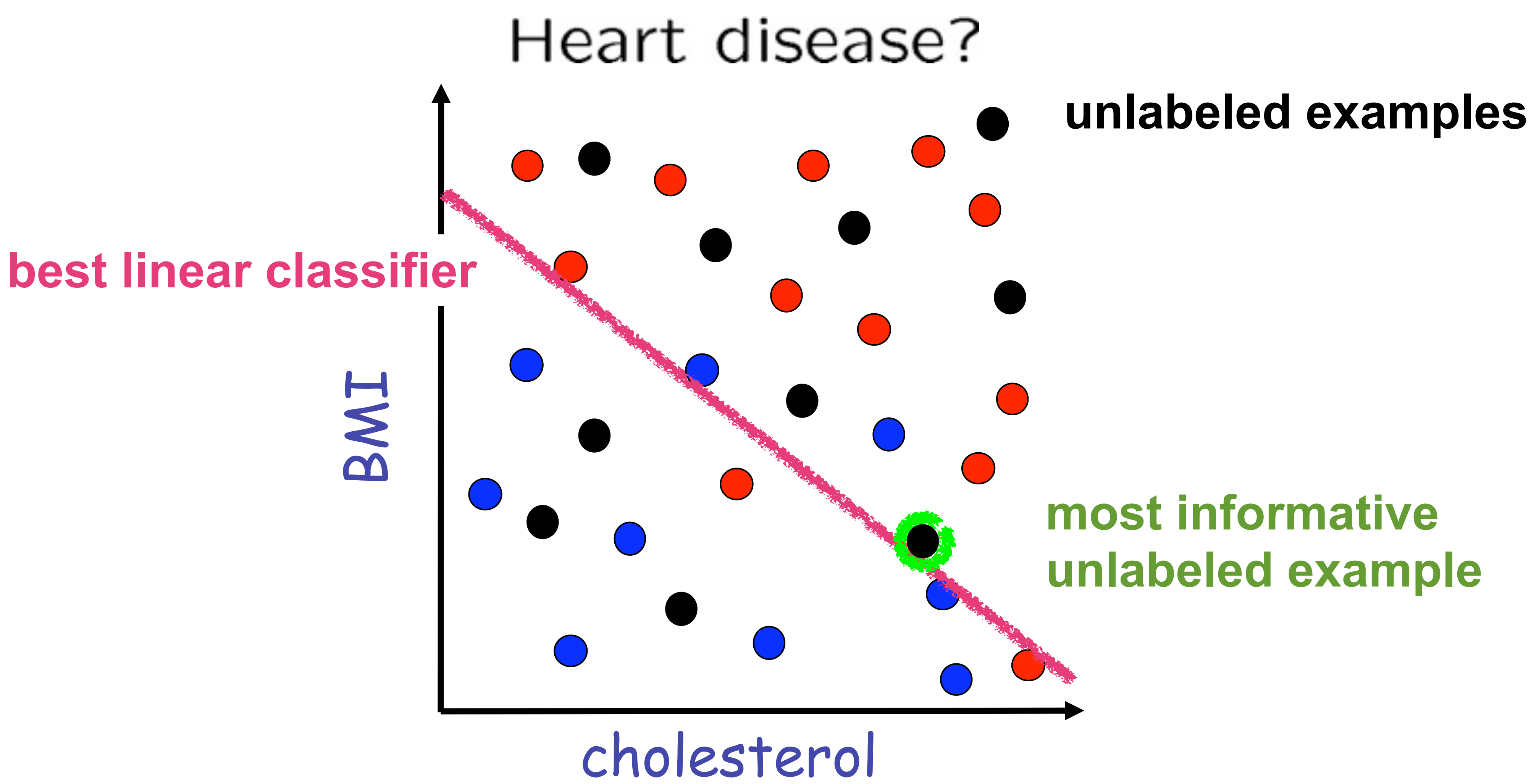
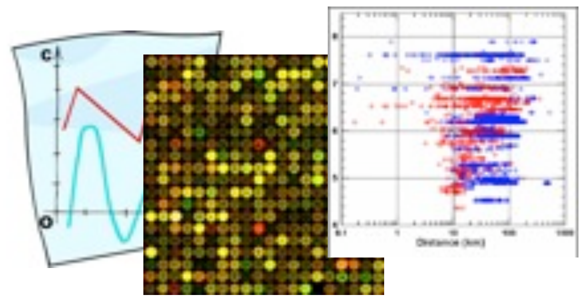


Machine Learning (Passive)

Raw unlabeled data



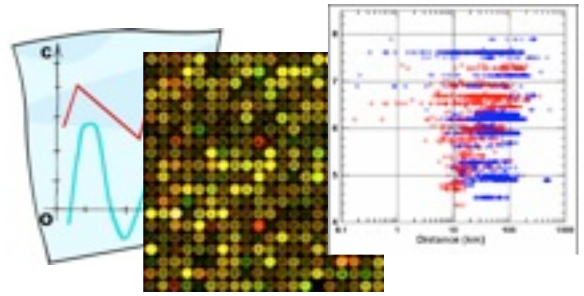
$$X_1, X_2, X_3, \dots$$

passive learner

expert/oracle
analyzes/experiments
to determine labels

Machine Learning (Passive)

Raw unlabeled data



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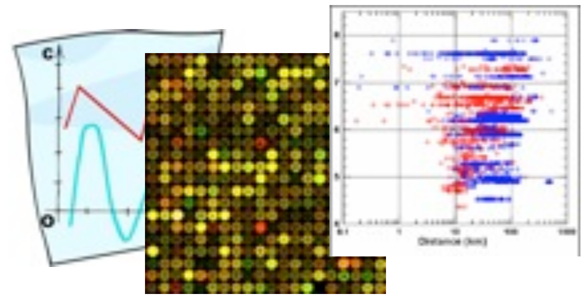


$X_1, X_2, X_3, \ldots$

$(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), \ldots$

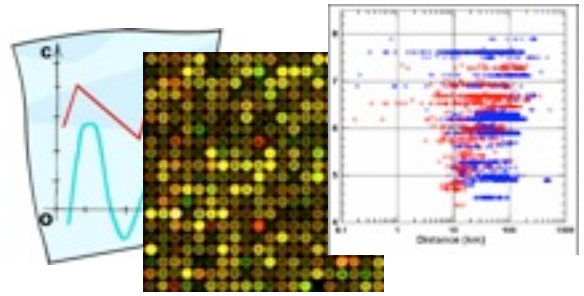
Labeled data

passive learner

expert/oracle
analyzes/experiments
to determine labels

Machine Learning (Passive)

Raw unlabeled data



$$X_1, X_2, X_3, \ldots$$

$$(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), \ldots$$

Labeled data

passive learner

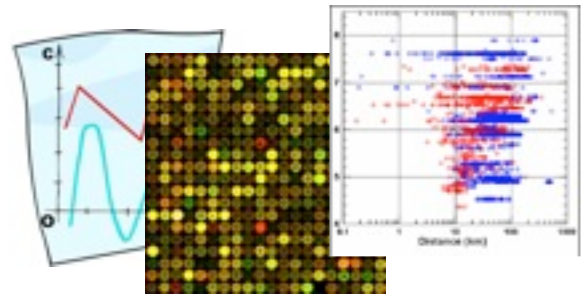
automatic classifier

expert/oracle

analyzes/experiments to determine labels

Active Learning

Raw unlabeled data



$$X_1, X_2, X_3, \ldots$$

active learner

expert/oracle
analyzes/experiments
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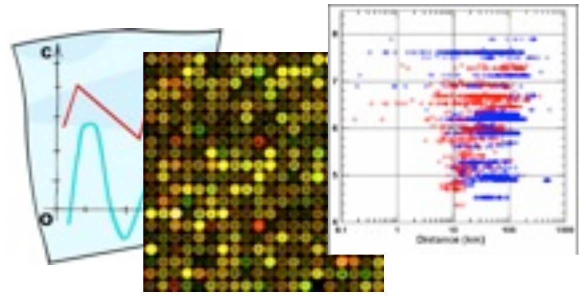
Active Learning

Raw unlabeled data



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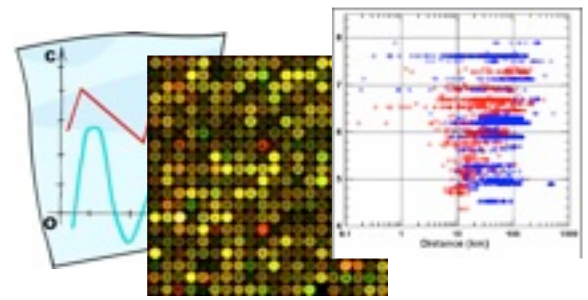


$X_1, X_2, X_3, \ldots$

Learner requests labels
for **selected** data

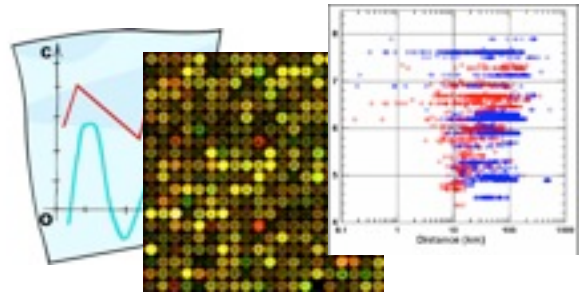
$(X_1, ?)$

active learner

expert/oracle
analyzes/experiments
to determine labels

Active Learning

Raw unlabeled data



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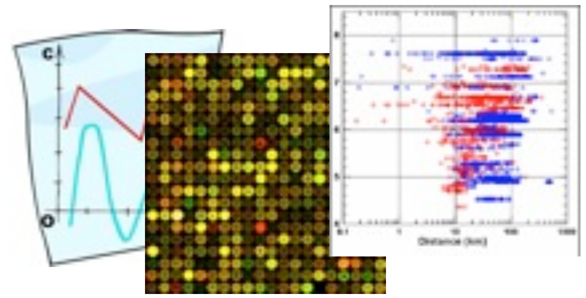

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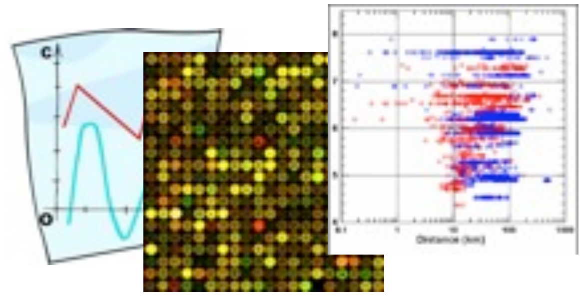

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$(X_3, ?)$

(X_3, Y_3)

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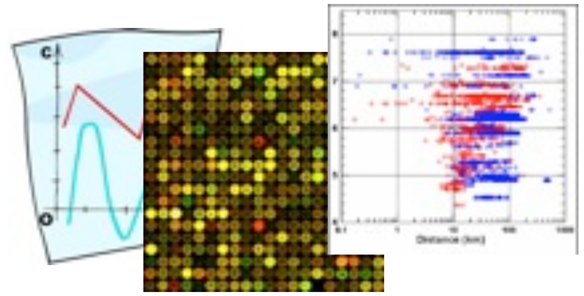

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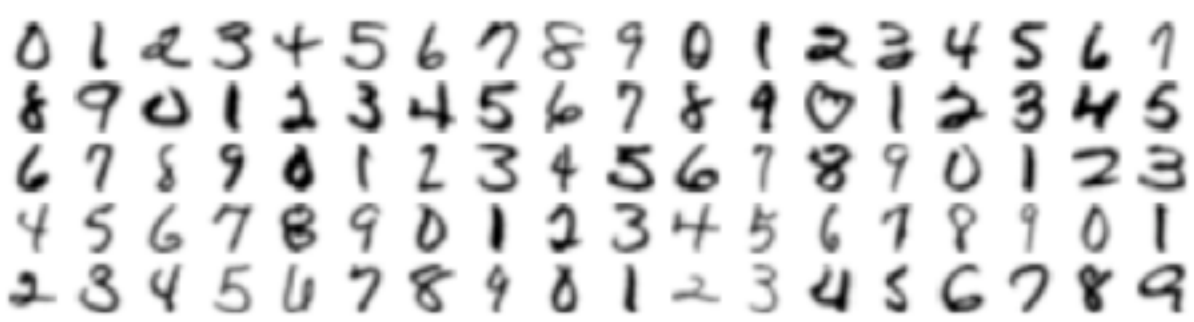
automatic classifier

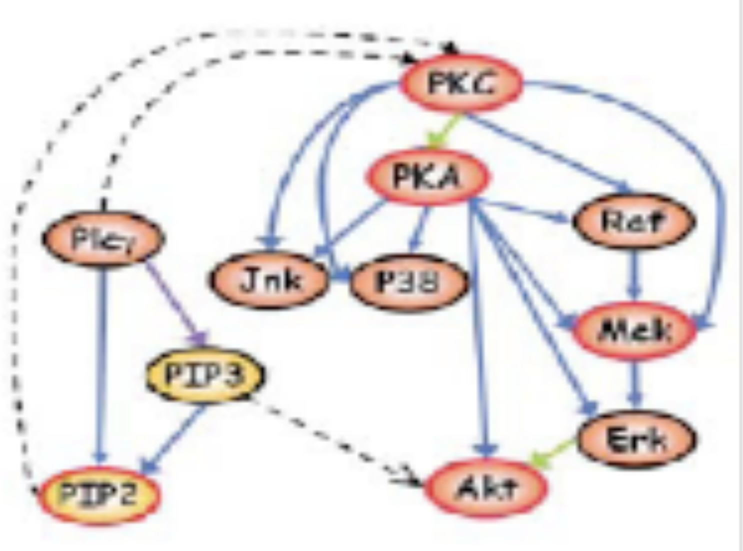
Applications of Active Learning

Hand-written character recognition



Document classification

Systems biology



Sensor networks

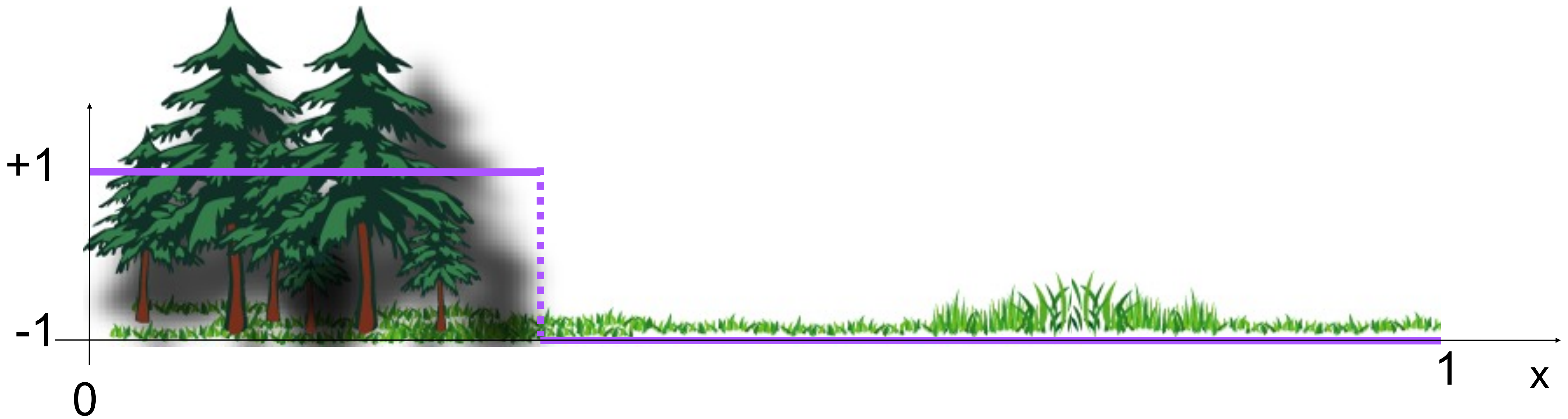
In many applications, obtaining labels or running experiments is costly !

A Stylized Environmental Sensing Task

x

Where is it shady vs. sunny ?

A Stylized Environmental Sensing Task



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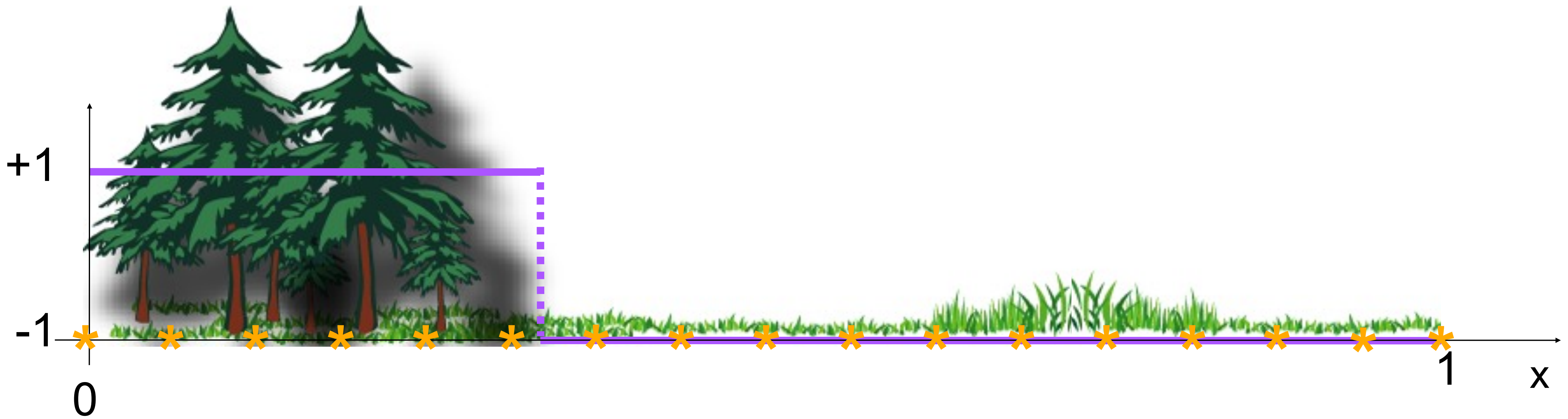
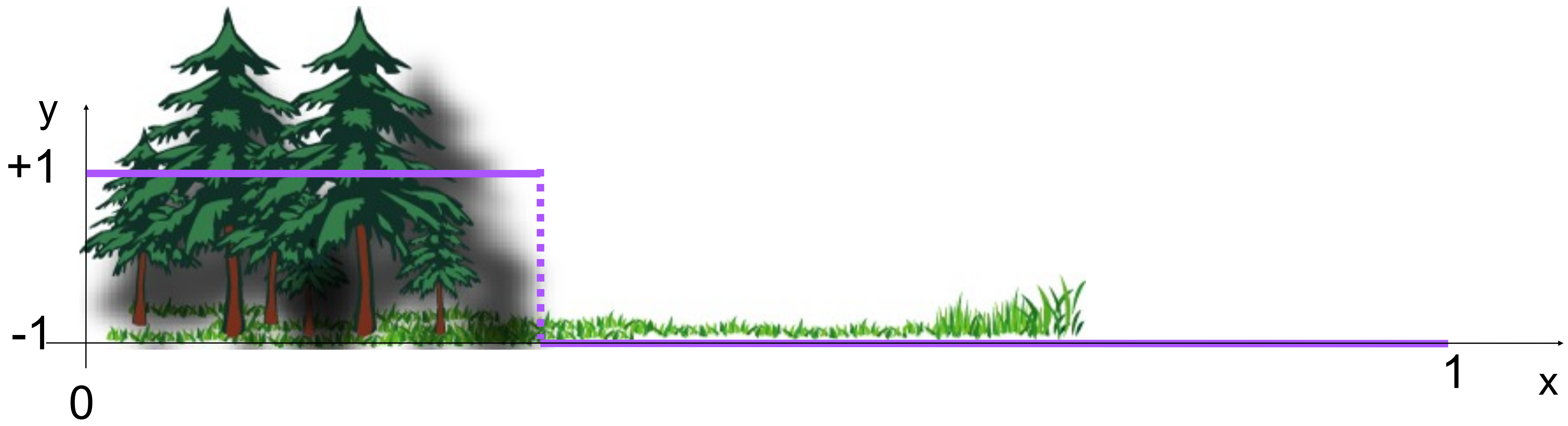
A Stylized Environmental Sensing Task



Where is it shady vs. sunny ?

Suppose we have N wireless sensors. Do we need to query them all?

Classic Binary Search



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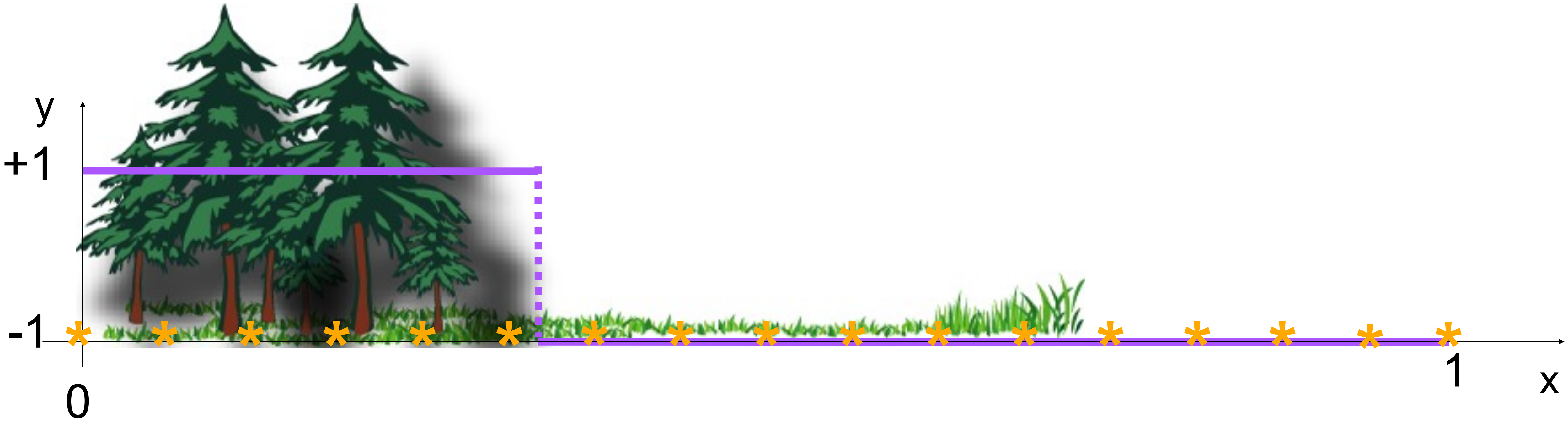
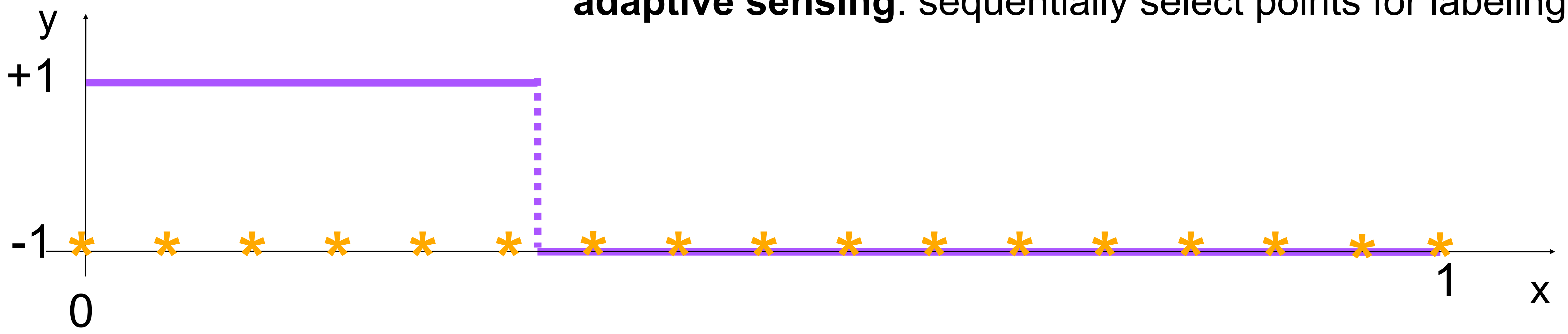


Where is it shady vs. sunny ?

Classic Binary Search

adaptive sensing: sequentially select points for labeling



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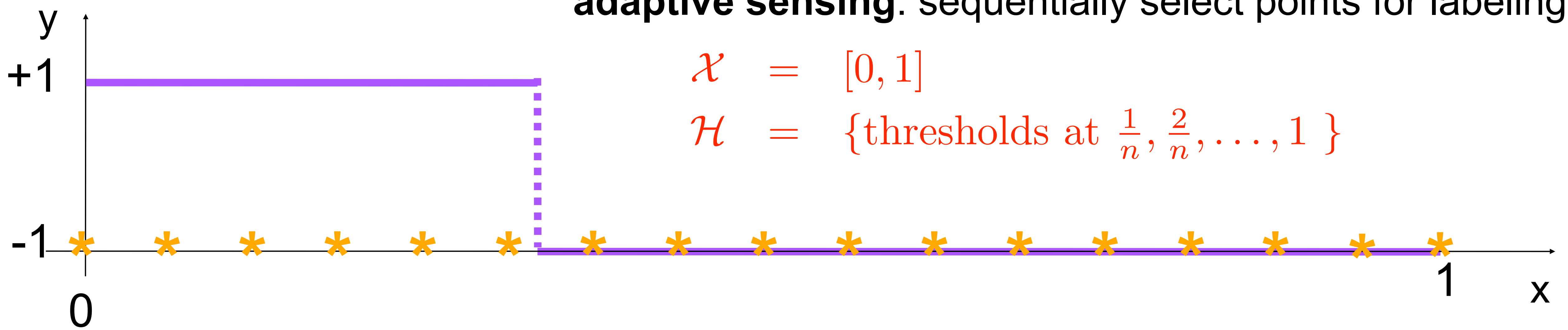

adaptive sensing: sequentially select points for labeling

$$\mathcal{X} = [0,1]$$
$$\mathcal{H} = \{\text{thresholds at } \tfrac{1}{n}, \tfrac{2}{n}, \dots, 1 \}$$

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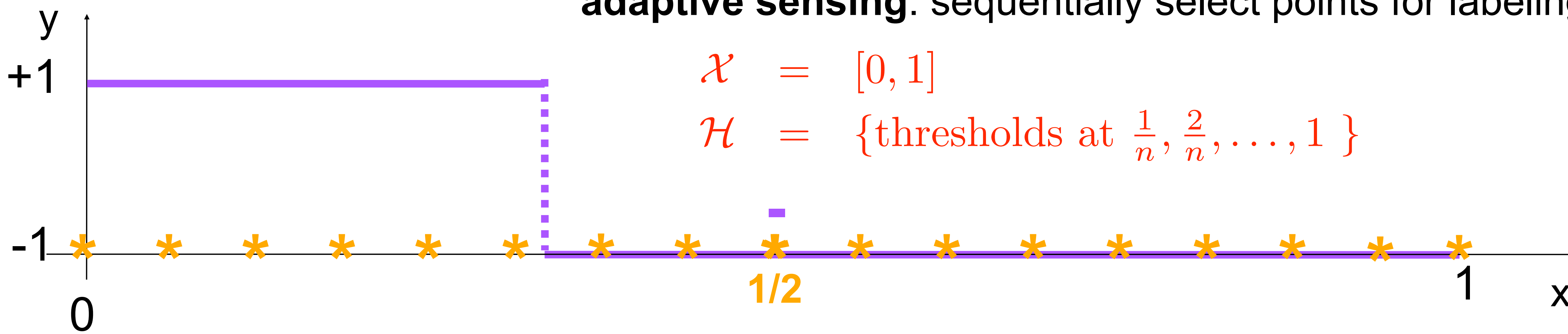

1/3 = 0…

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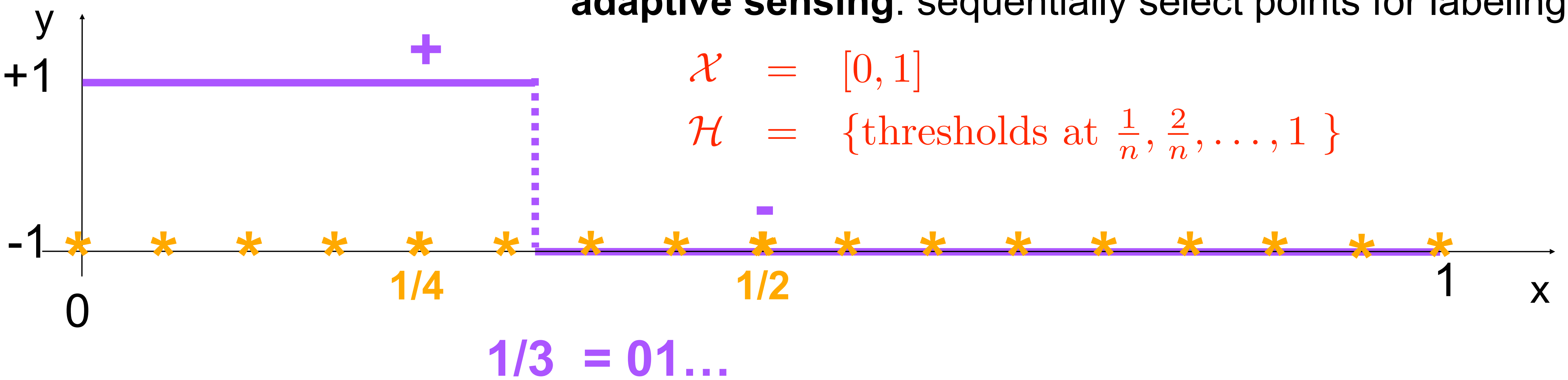

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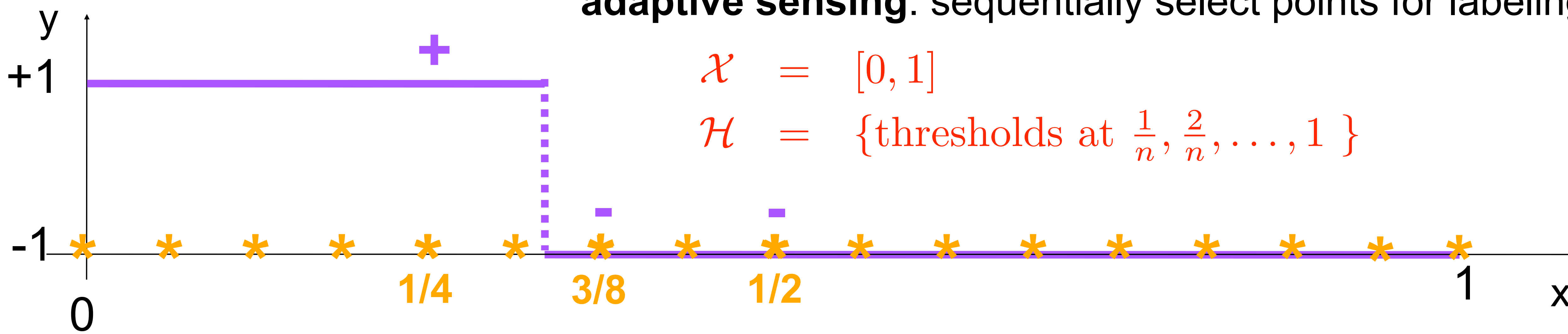

1/3 = 010…

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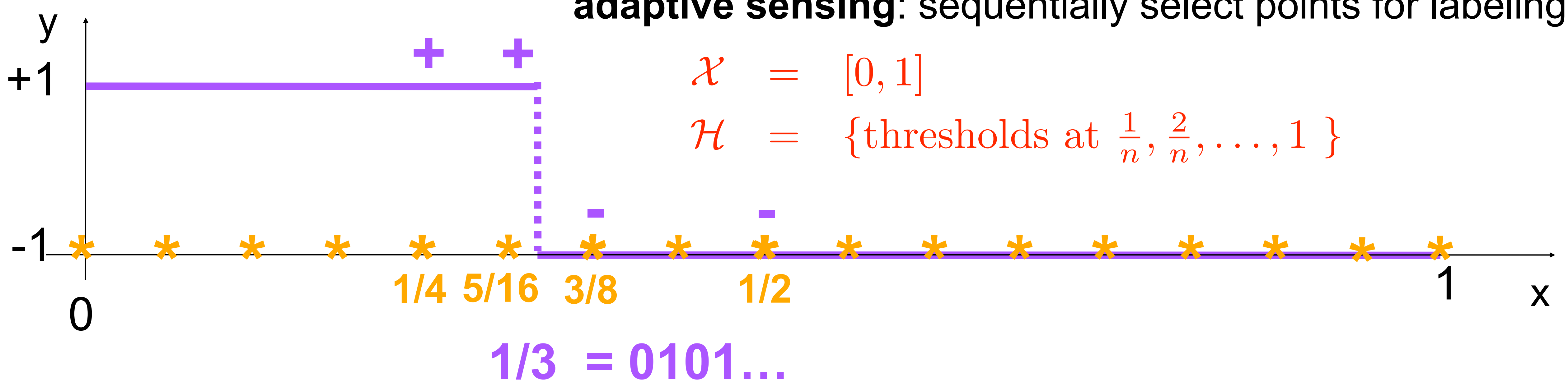

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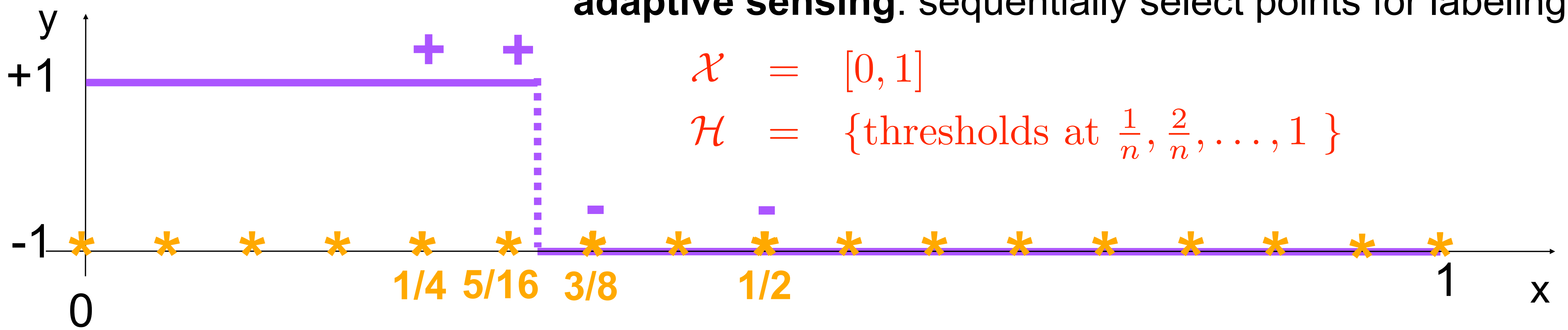

1/3 = 0101…

requires $\log_2 N$ queries

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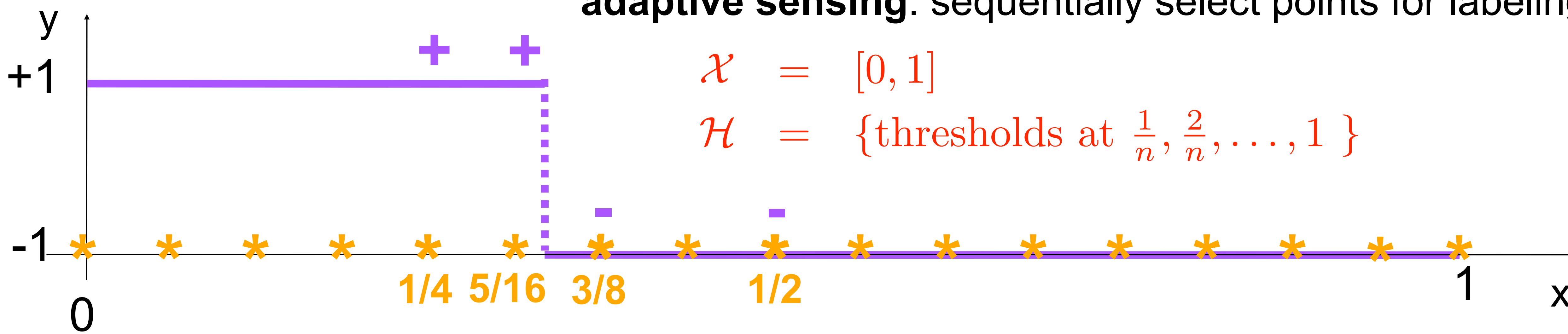

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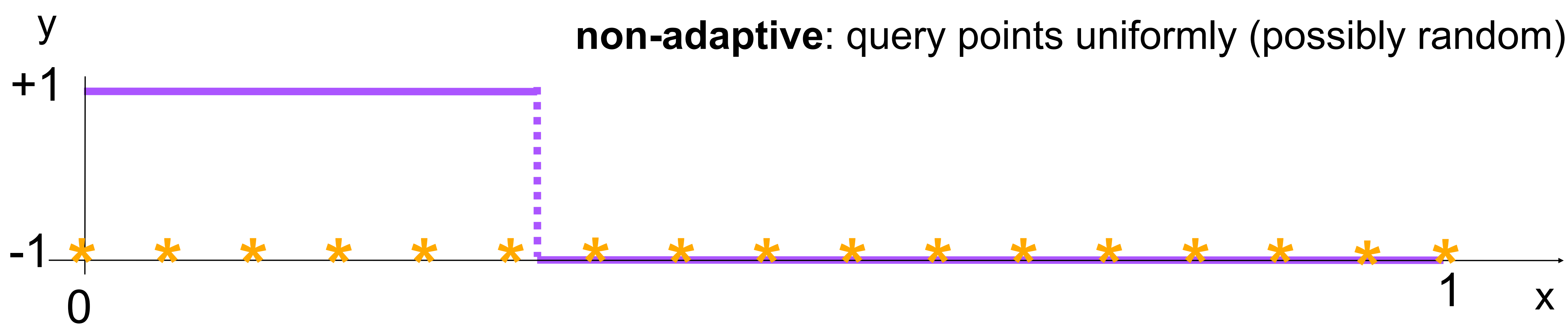
non-adaptive: query points uniformly (possibly random)



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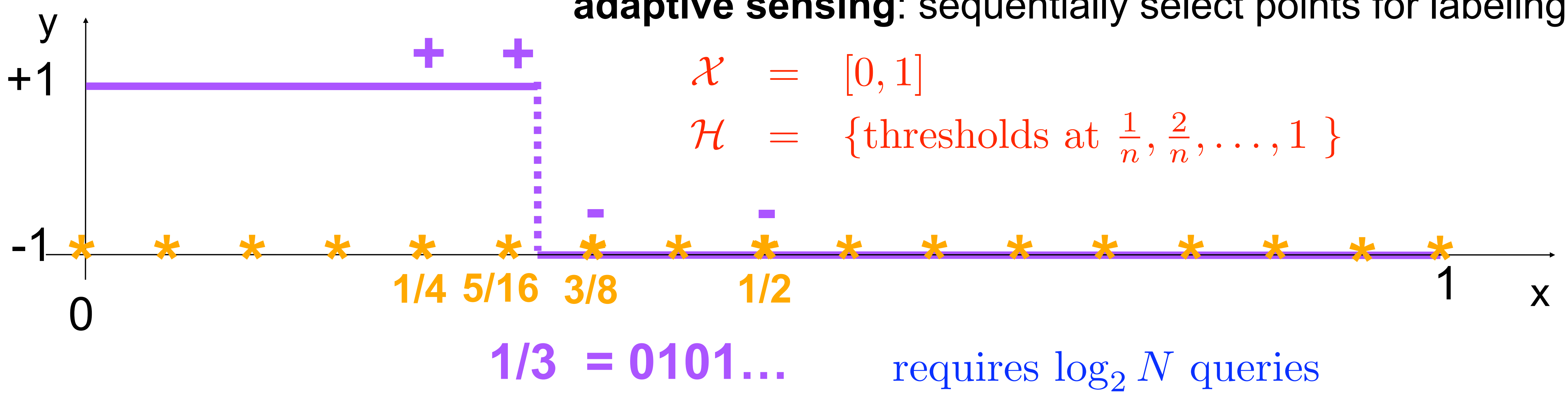

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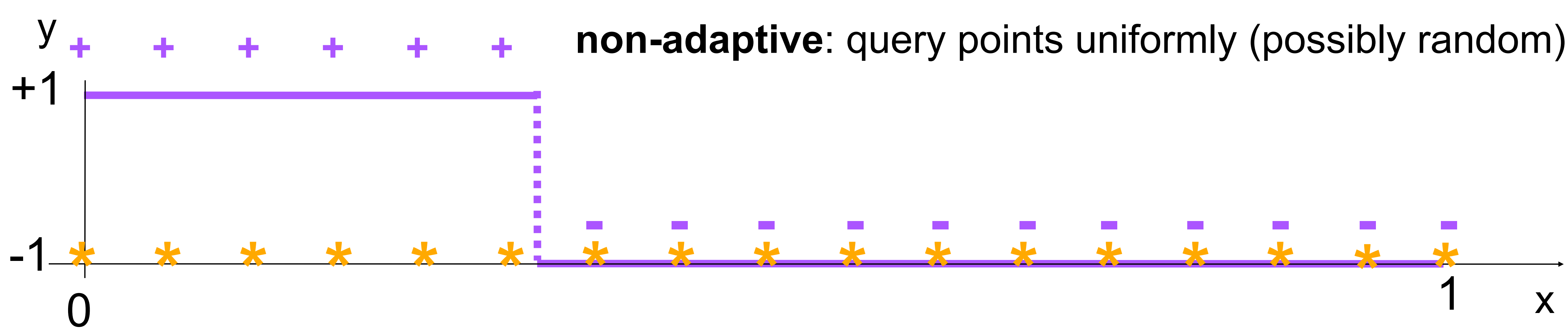

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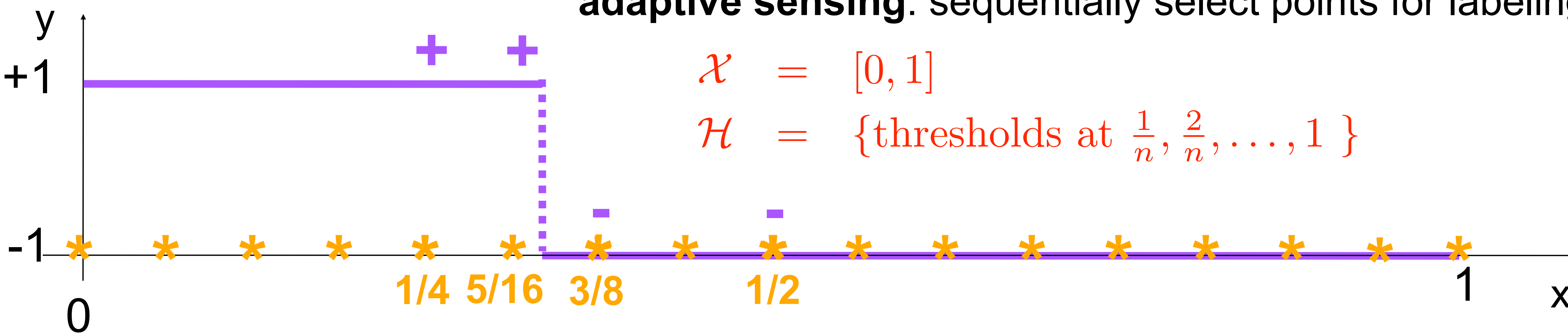

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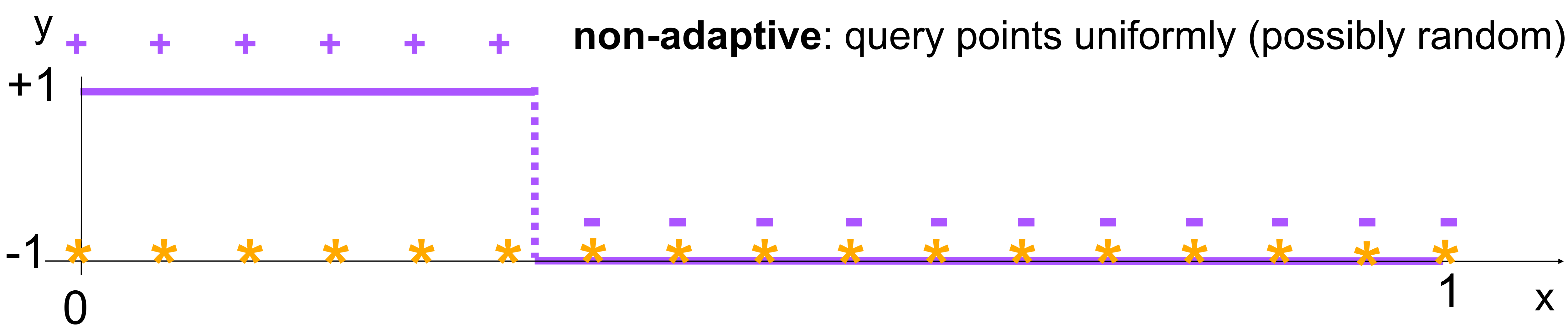

requires $O(N)$ queries

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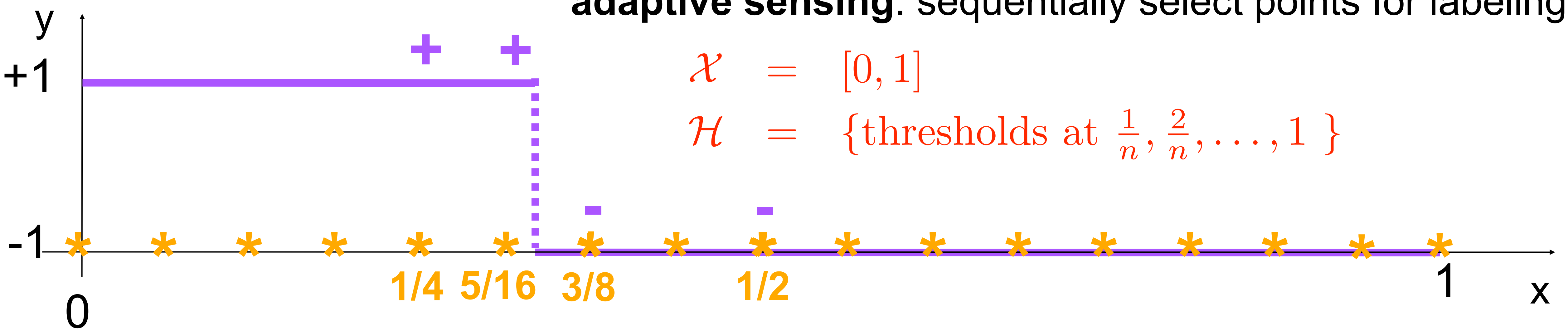

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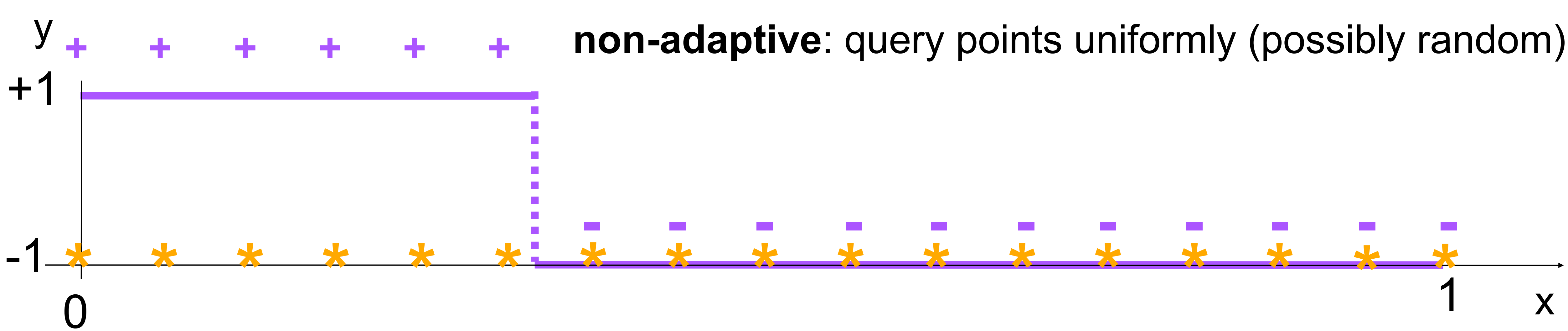


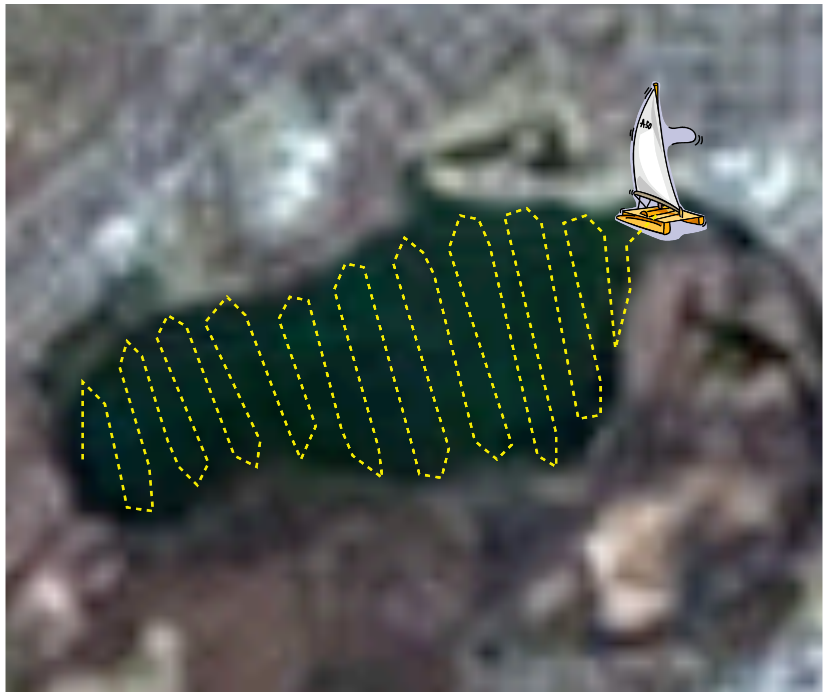
requires $O(N)$ queries

adaptive sensing is dramatically more efficient

Environmental Sensing

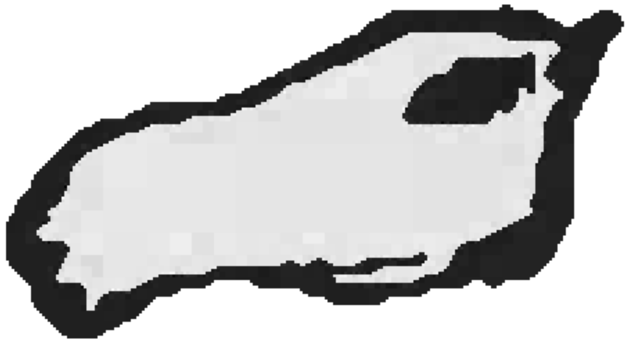
Chin Wu, Civil & Environmental Engr.
http://limnology.wisc.edu/

Lake Wingra, Madison WI



acoustic doppler sensing of
water current in Lake Wingra

water current velocity map
(darker = high velocity)



classification into high-
and low-velocity regions

Non-adaptive Survey: 48 hrs
Adaptive Survey: 14 hrs

A. Singh, R. Nowak and P. Ramanathan. **Active Learning for Adaptive Mobile Sensing Networks**. *ACM/IEEE Interntional Conference on Information Processing in Sensor Networks, IPSN 2006*.

Outline of Part 3

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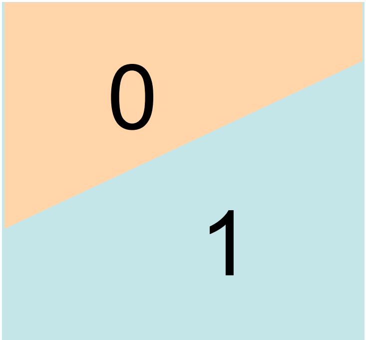

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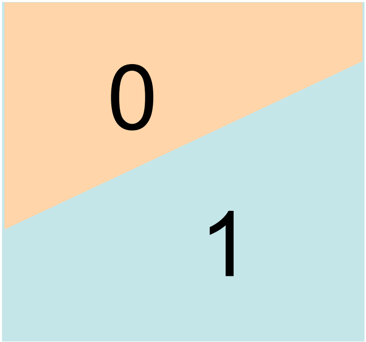
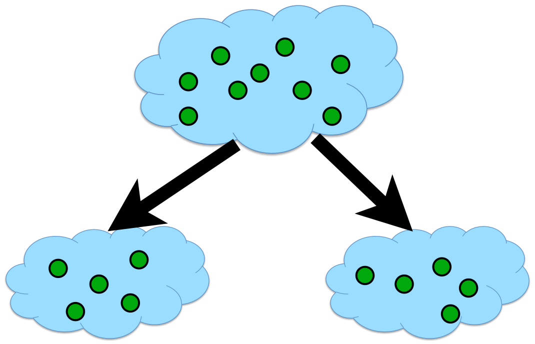
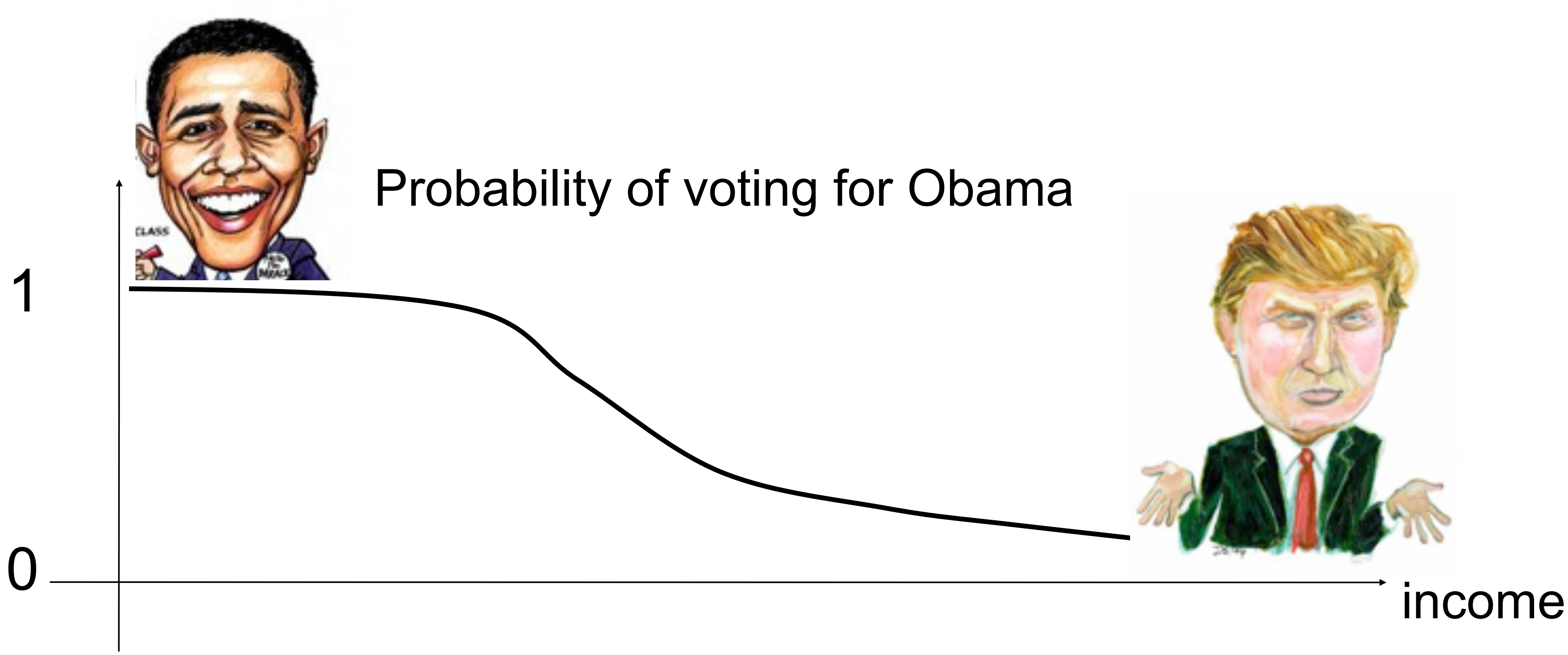


Unsupervised Active Learning: Can active learning help in unsupervised learning problems such as clustering ?



Binary Search and Noise

At what income level is a person more likely to be Republican vs. Democrat ?



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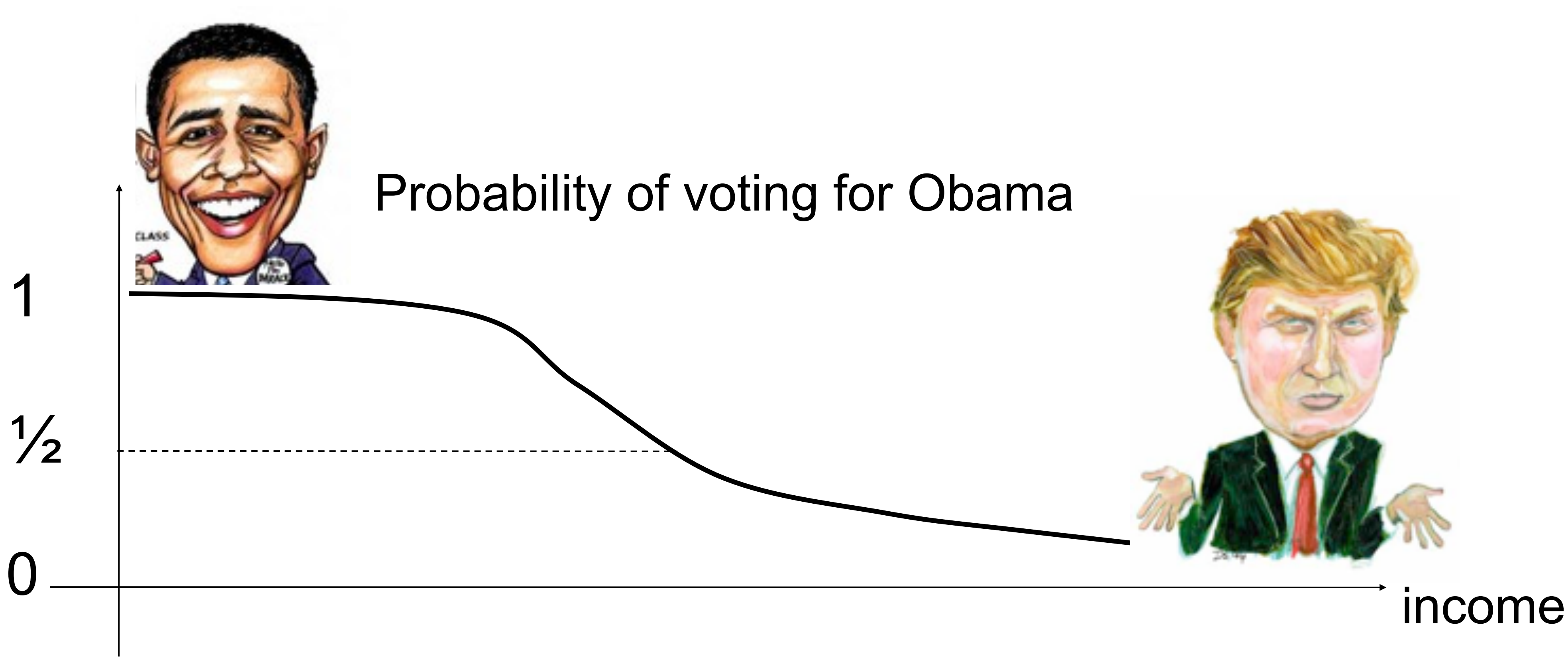

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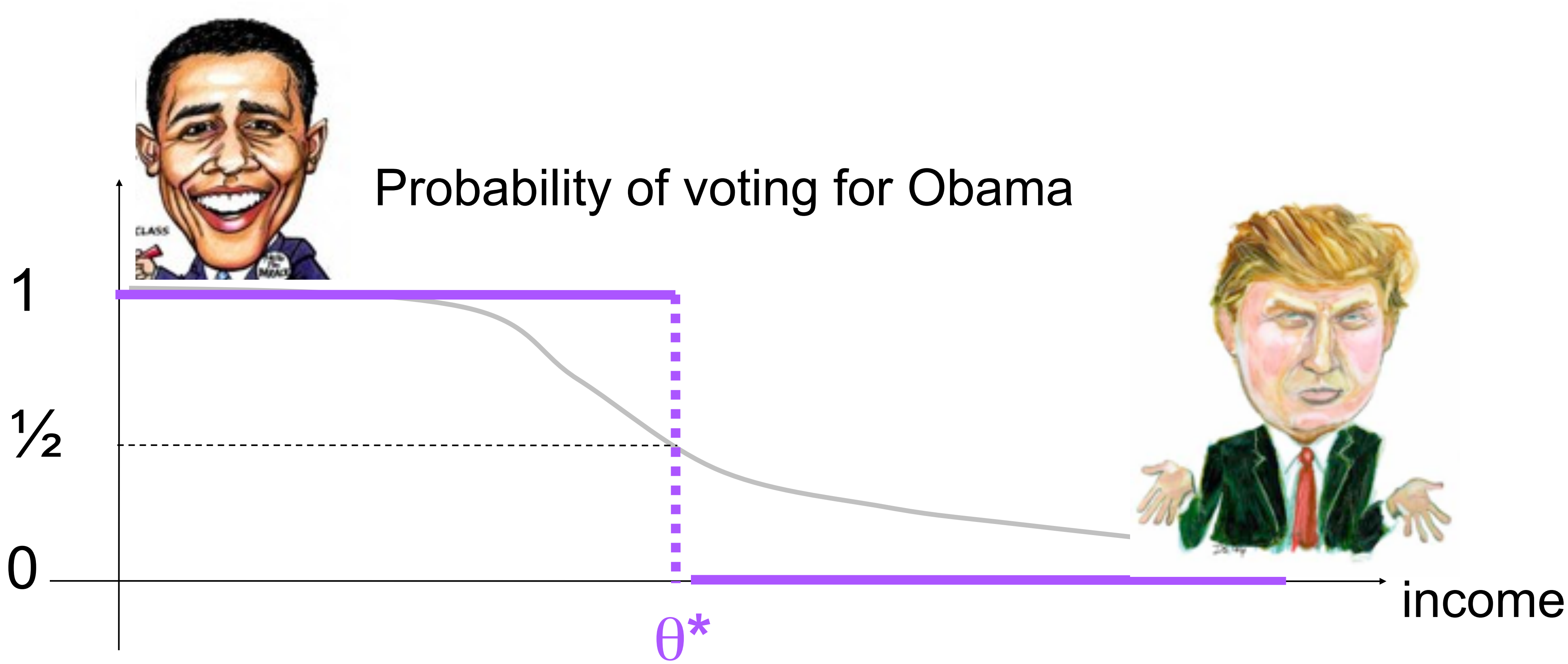

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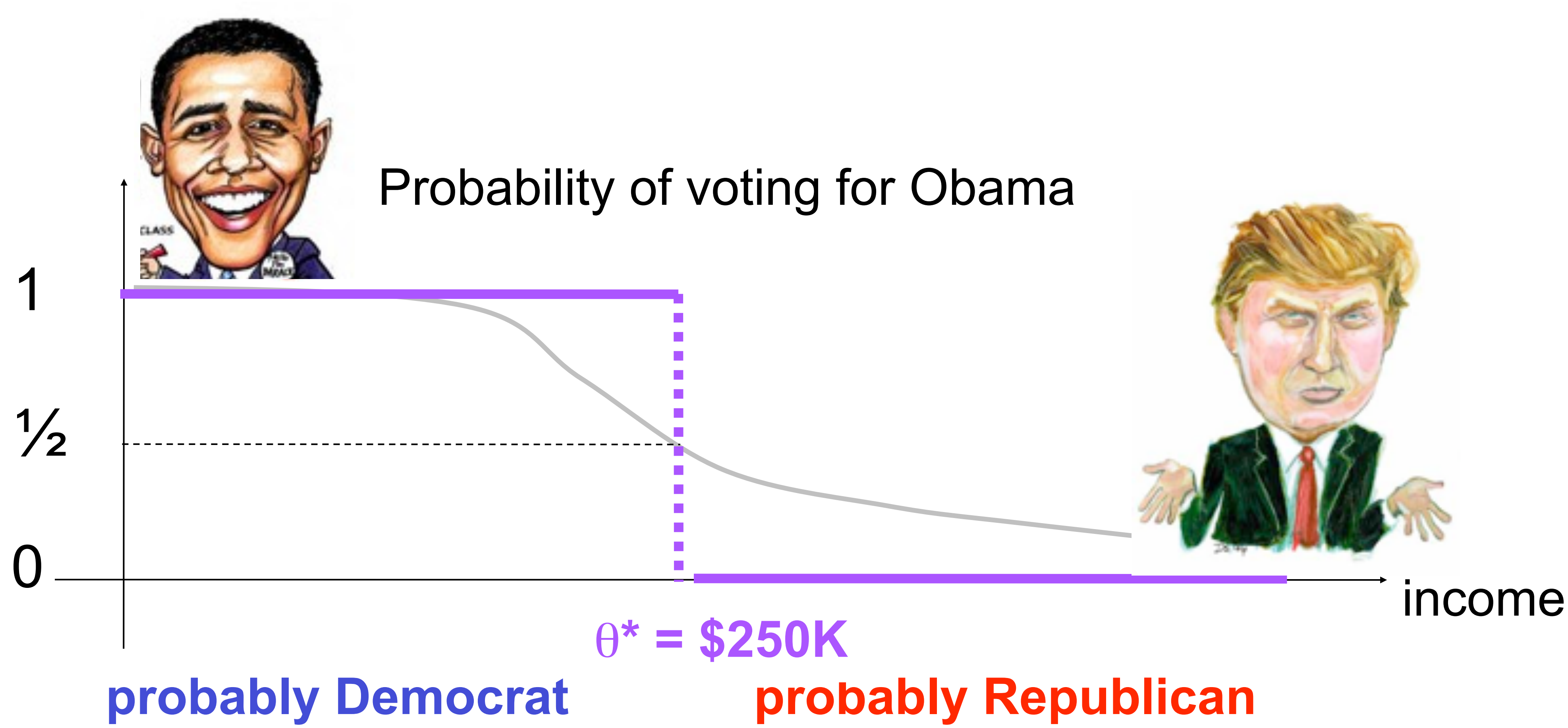
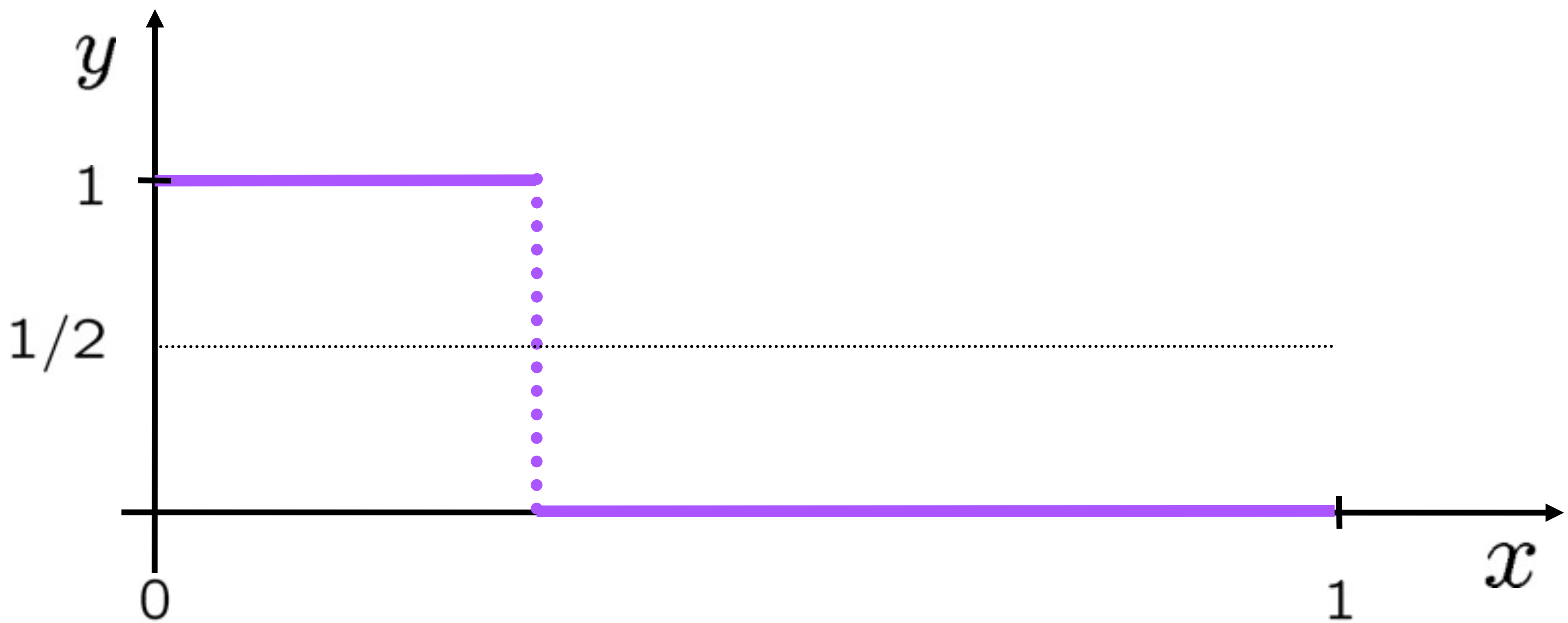


Bounded and Unbounded Noise



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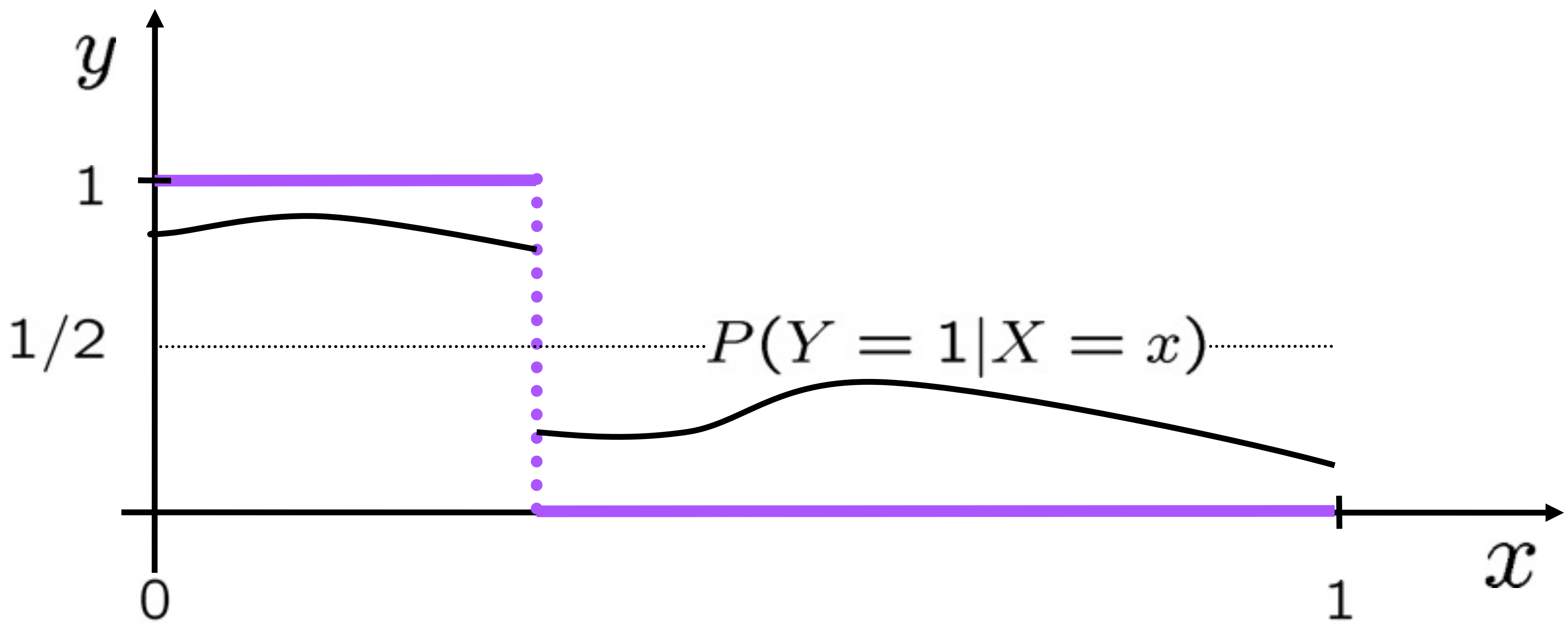

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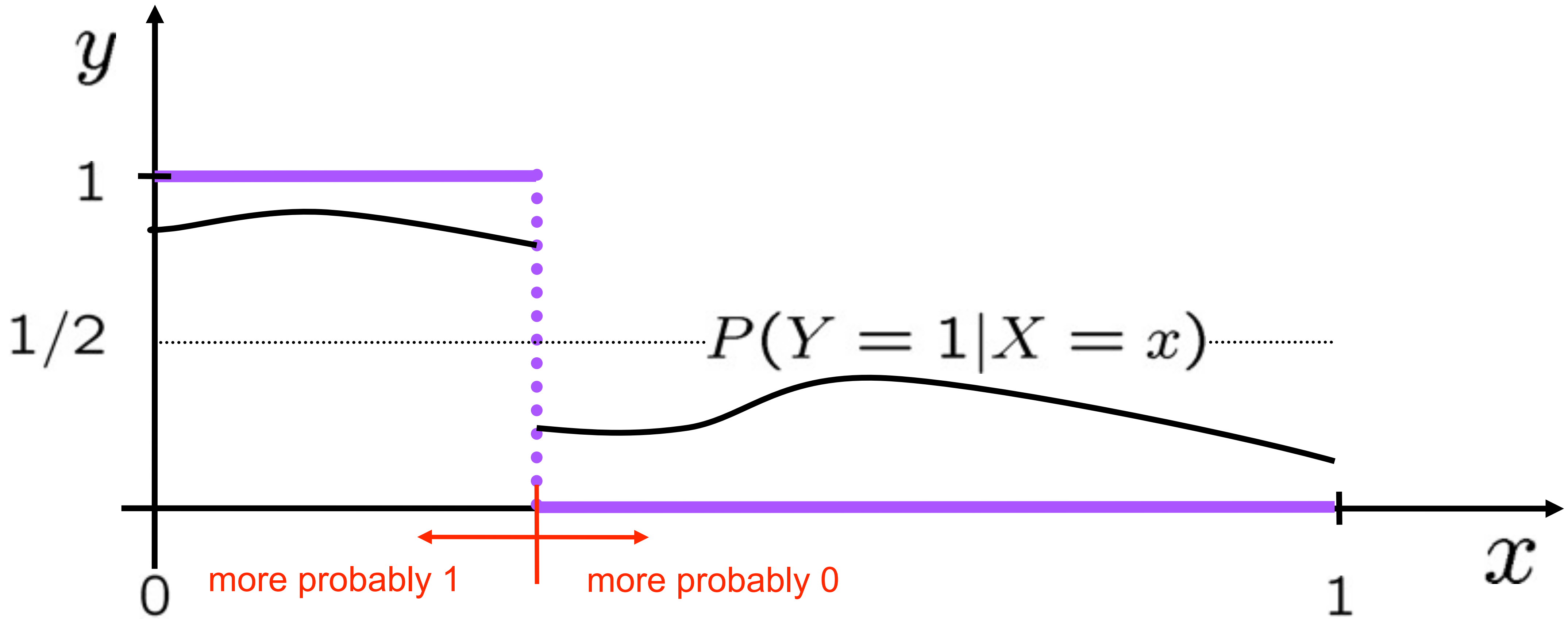

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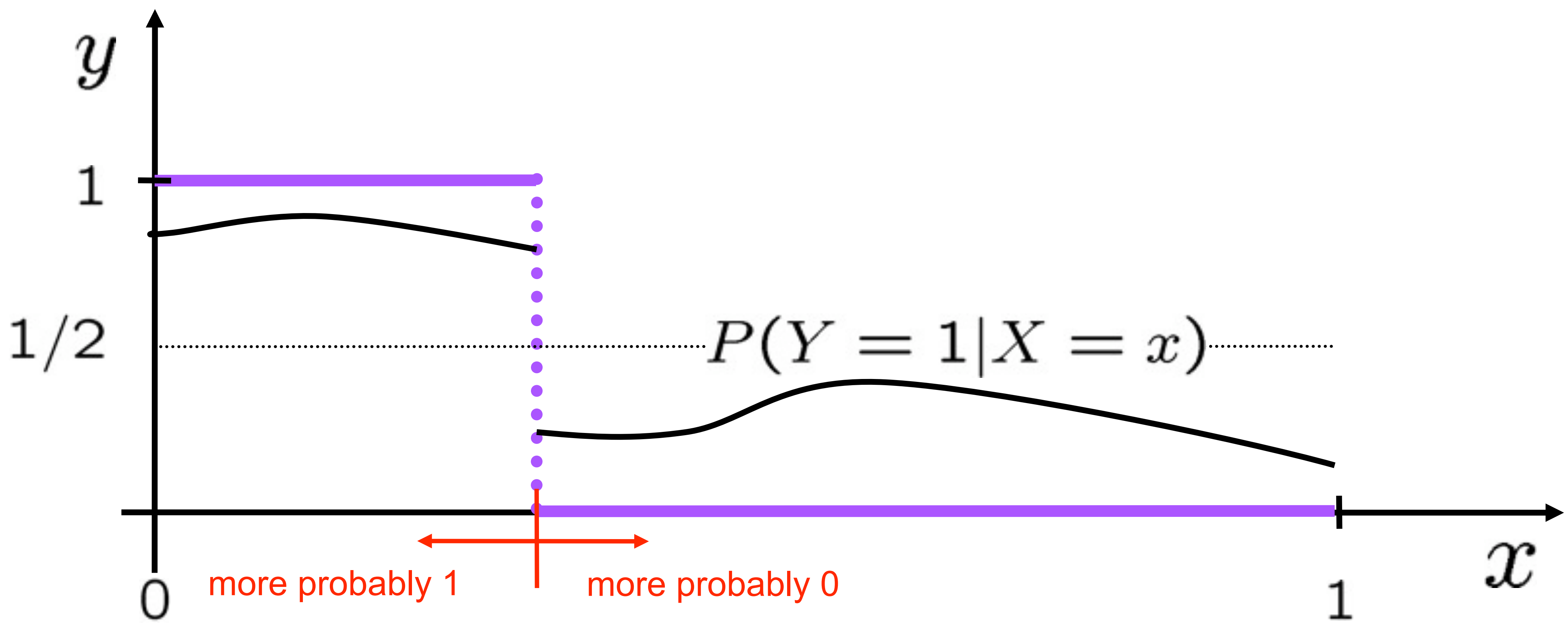
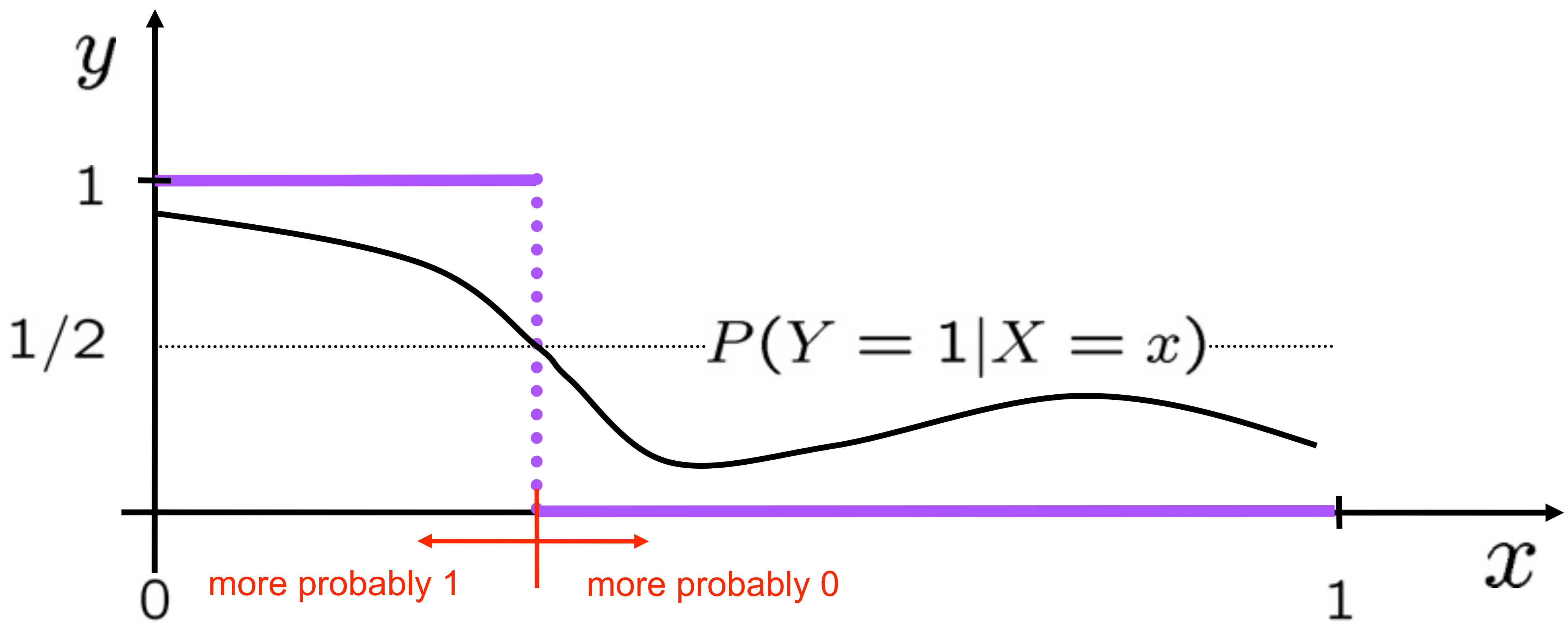


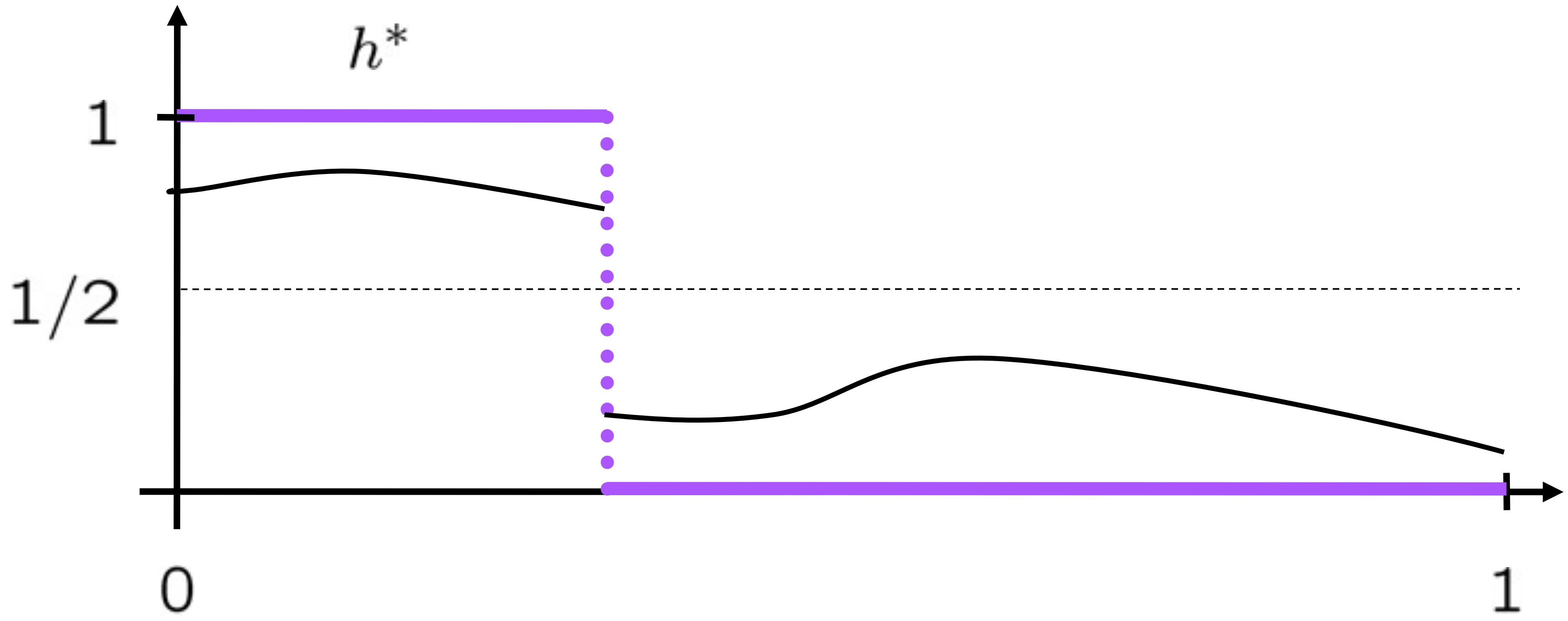
“bounded noise” : strictly more/less probably 1 at all locations

Bounded and Unbounded Noise



"unbounded noise" : like the toss of a fair coin at threshold

Horstein’s Multiplicative Weighting Method



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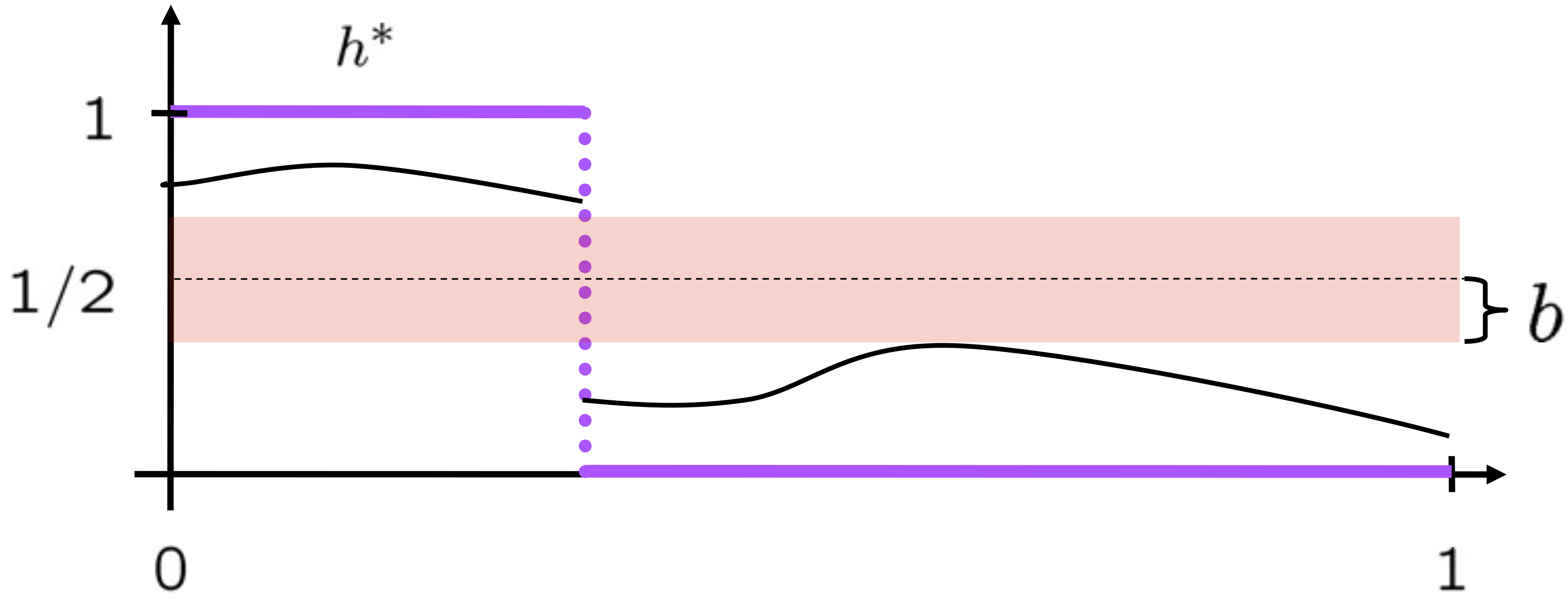
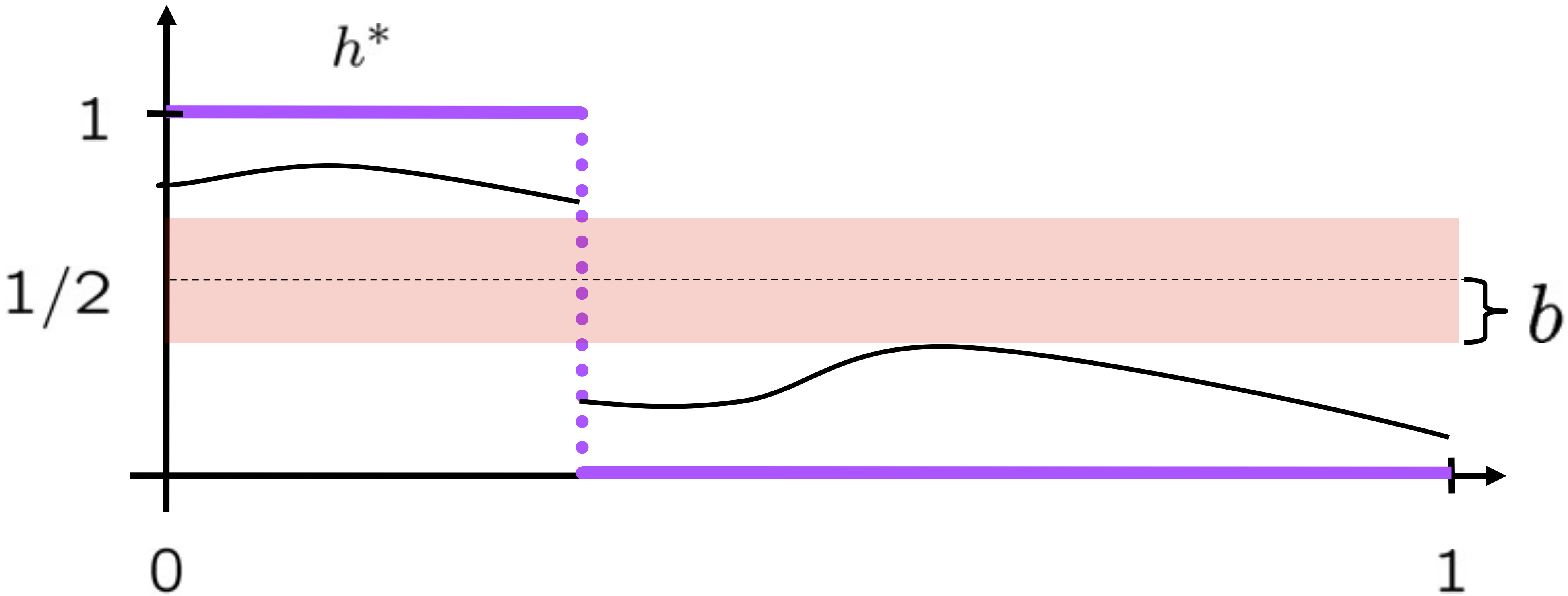
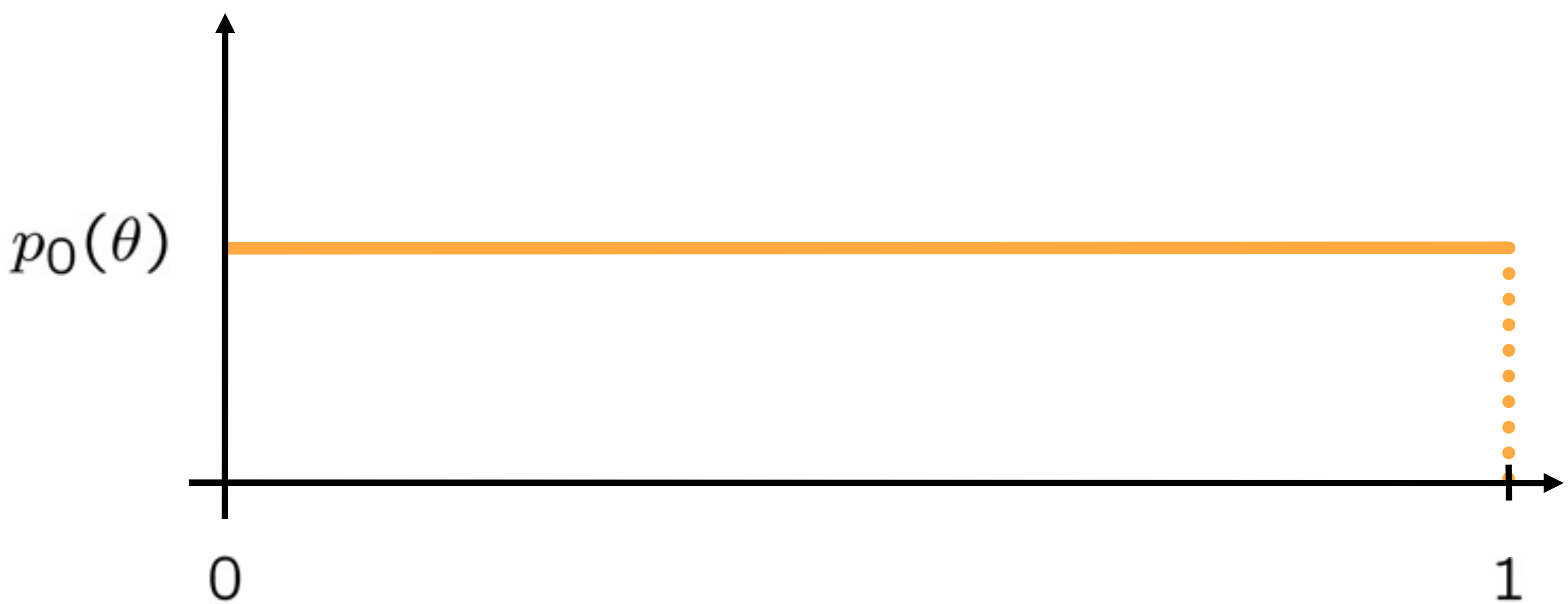


Horstein's Multiplicative Weighting Method





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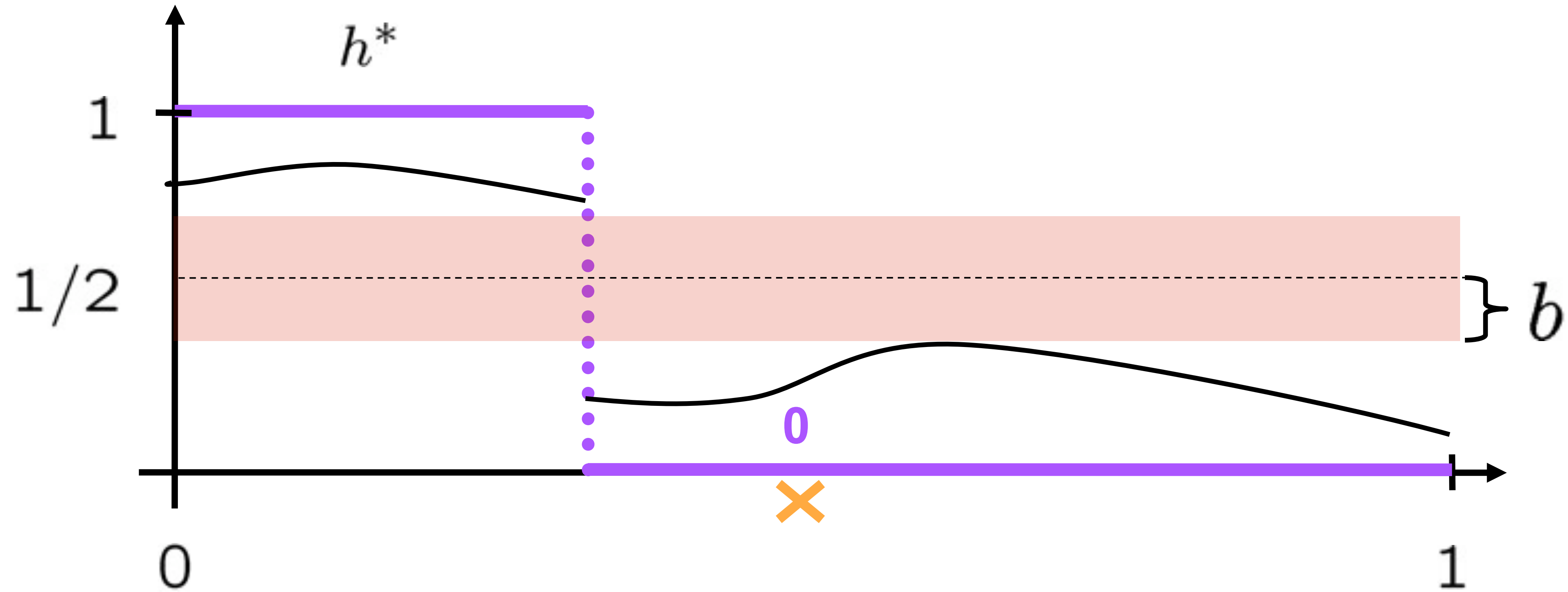

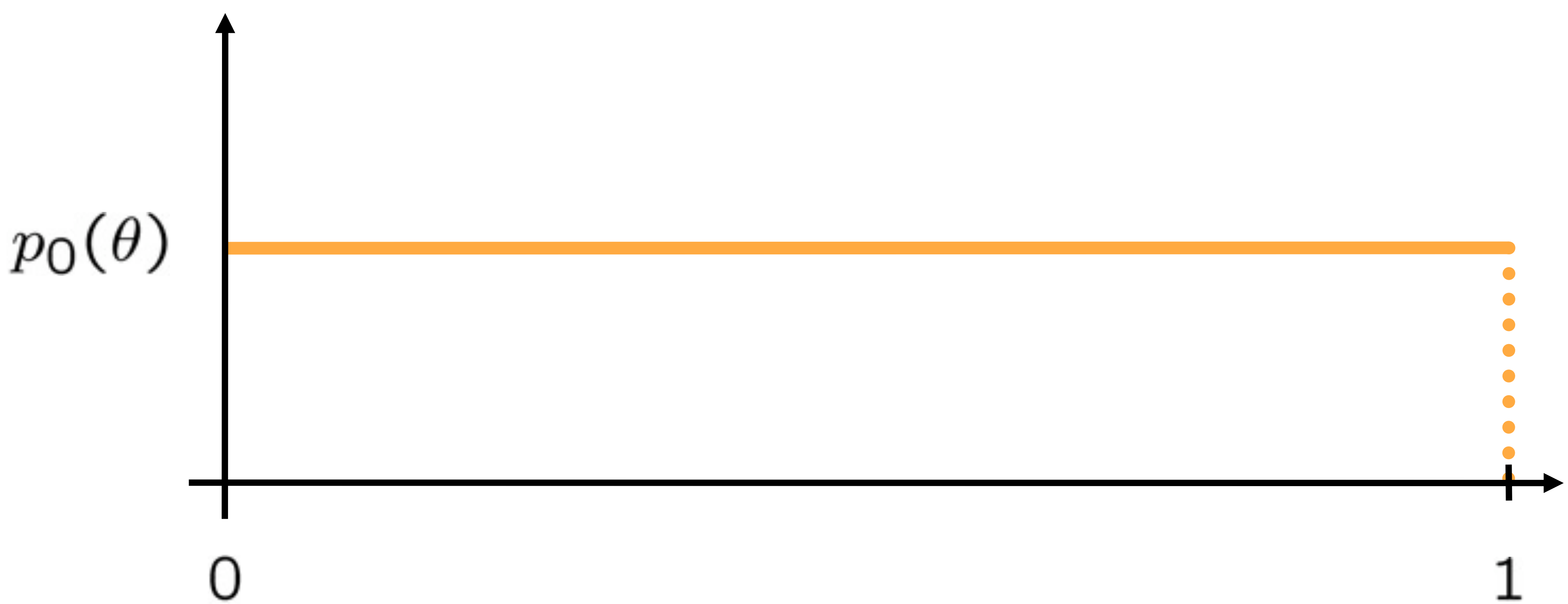

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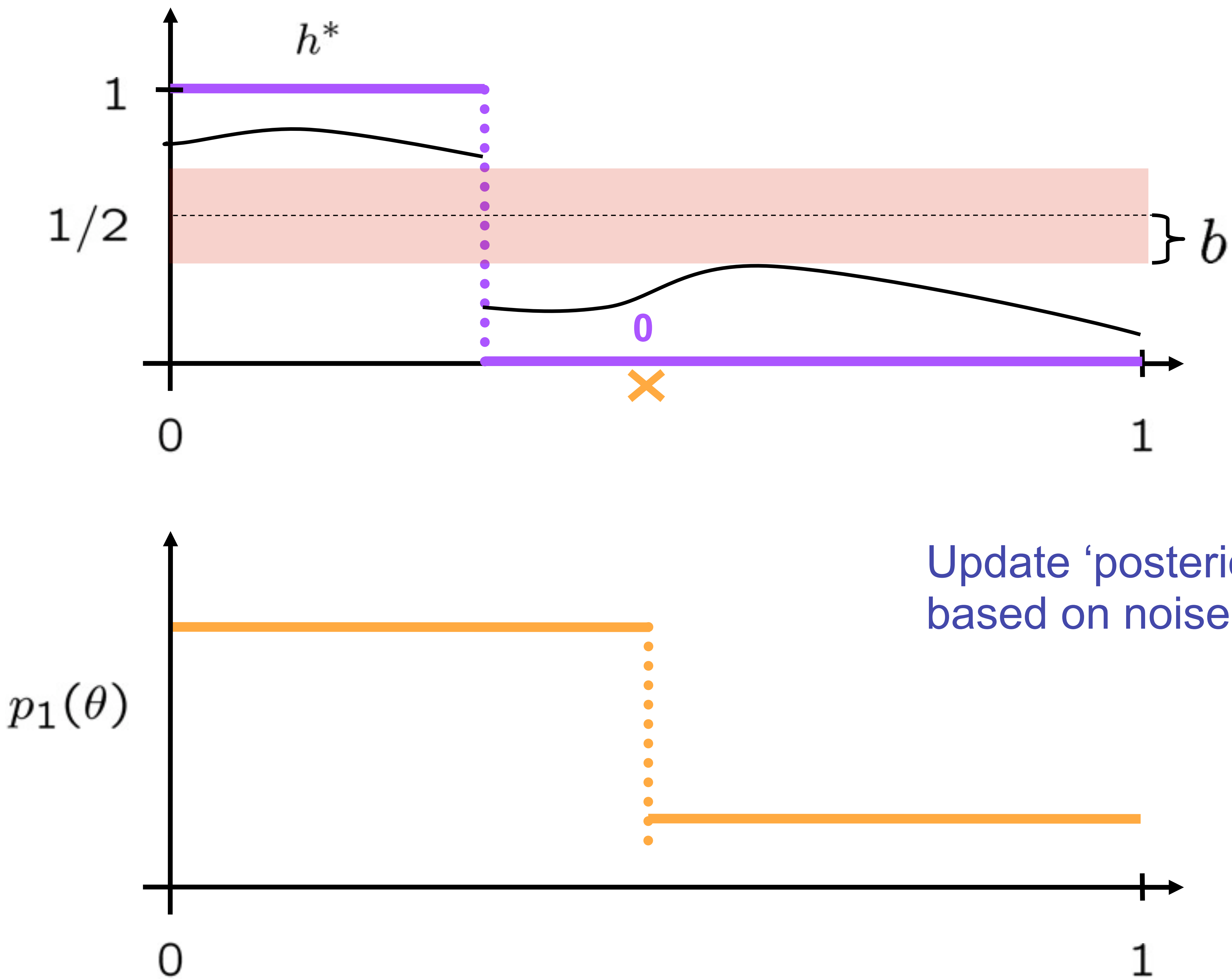
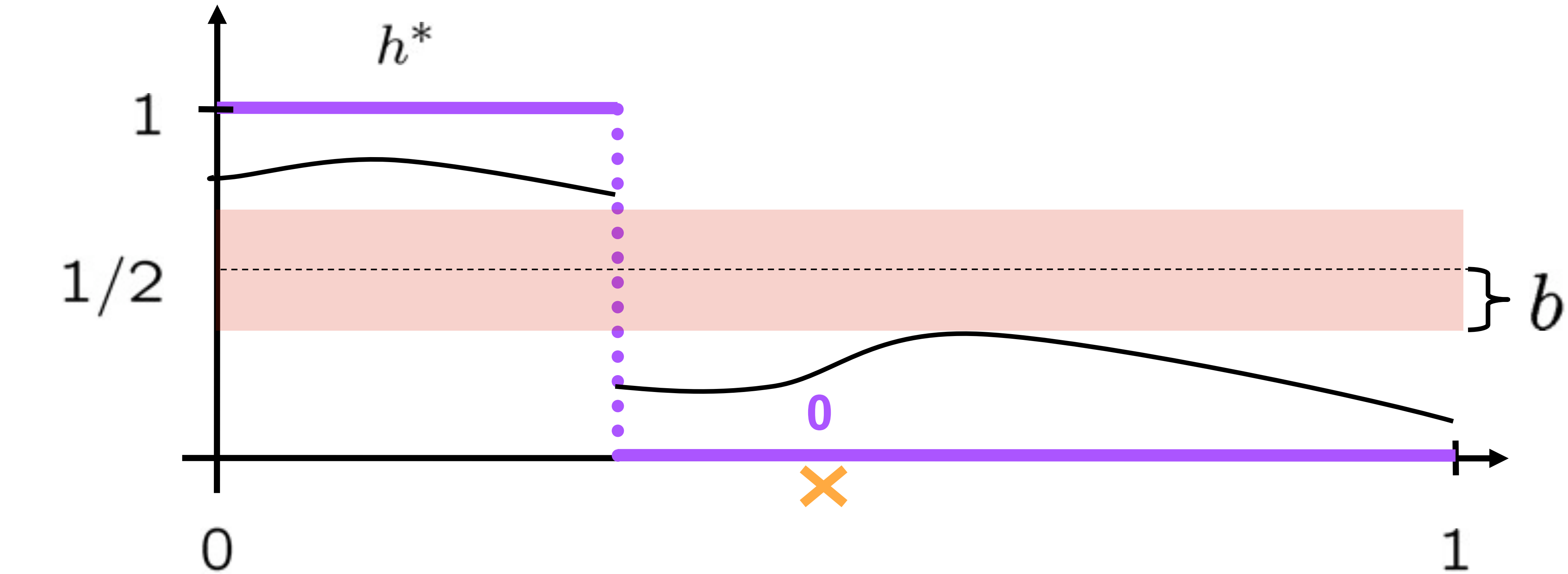
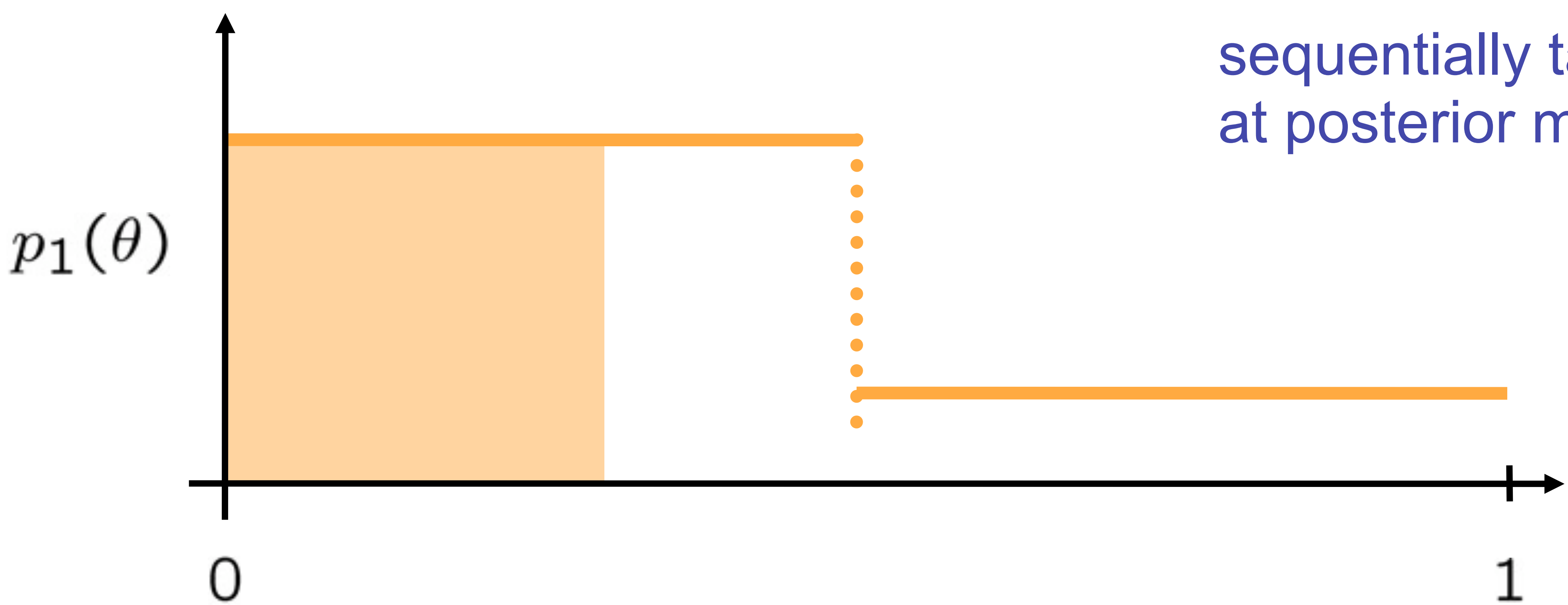


Update 'posterior' density based on noise bound b

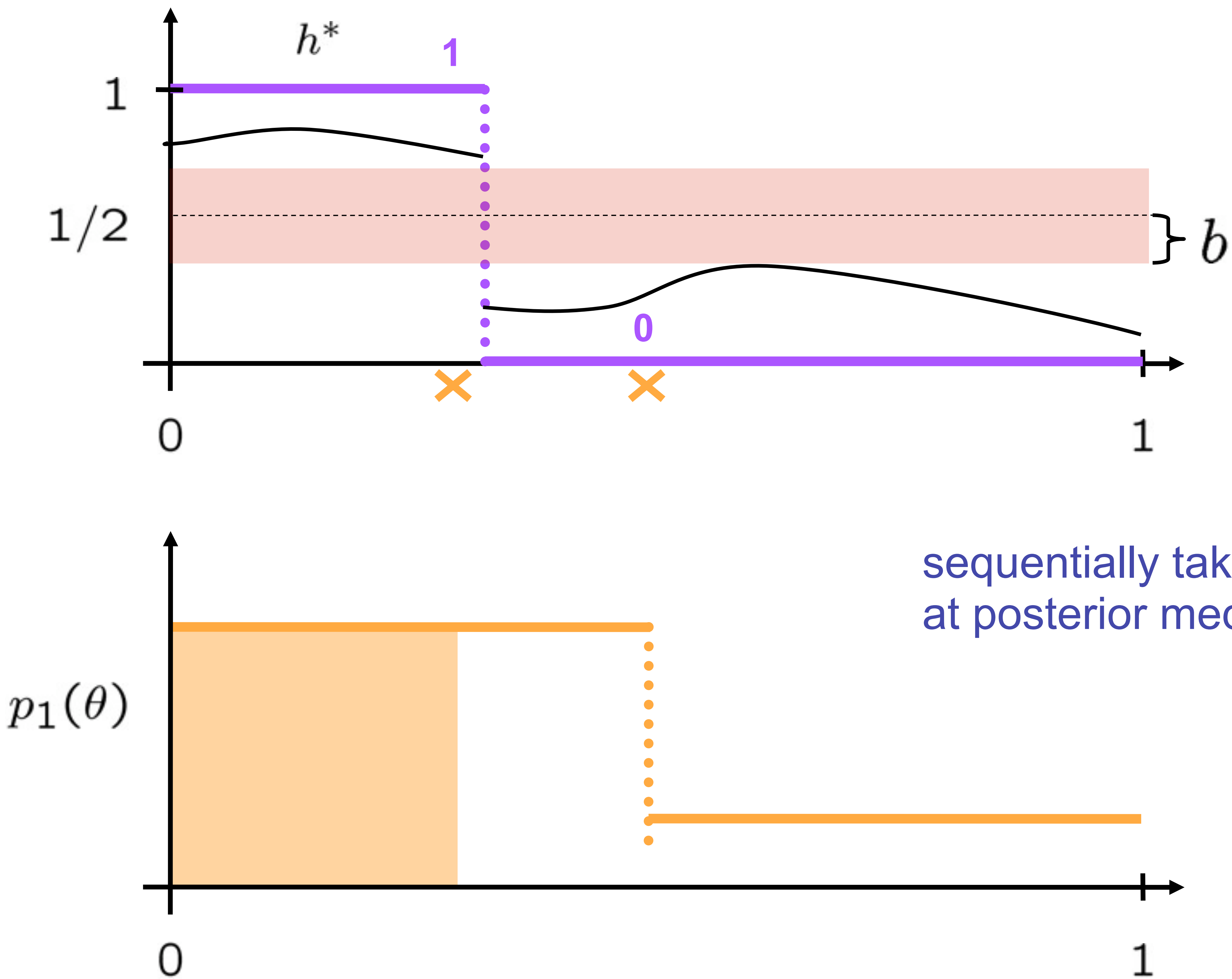
Horstein's Multiplicative Weighting Method





sequentially take samples at posterior median

Horstein's Multiplicative Weighting Method



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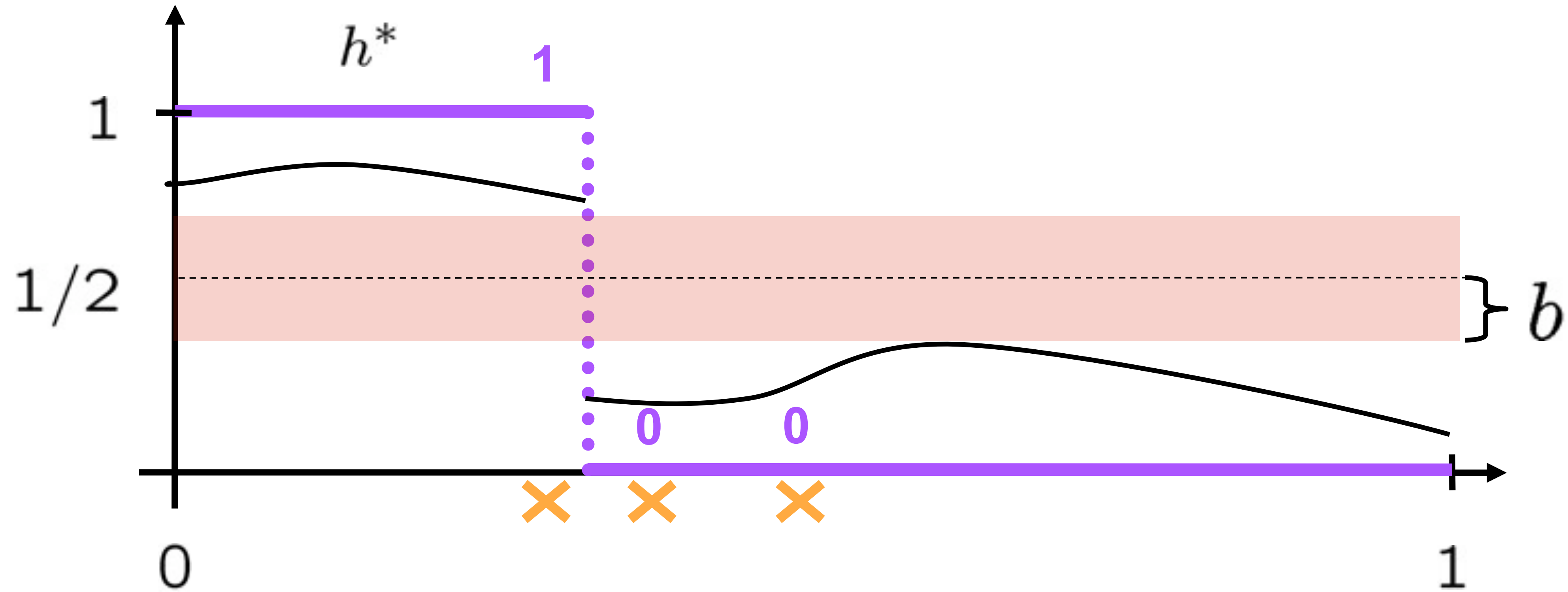
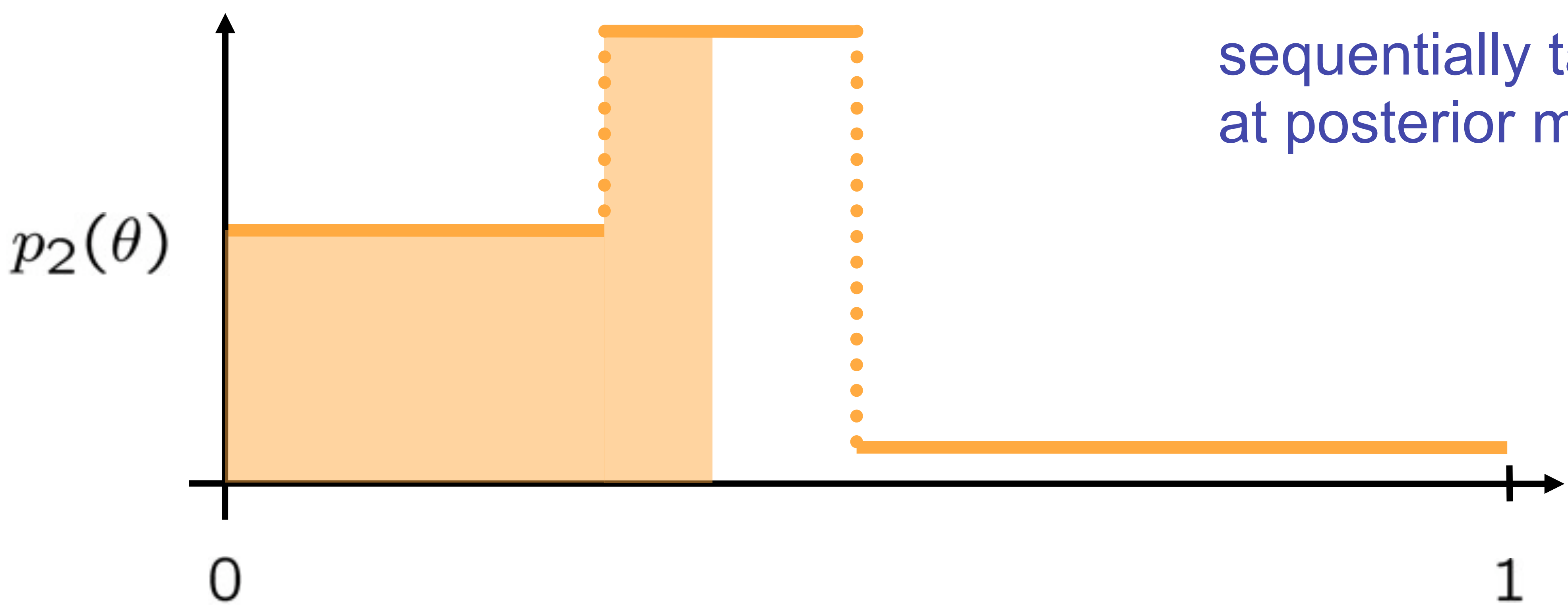
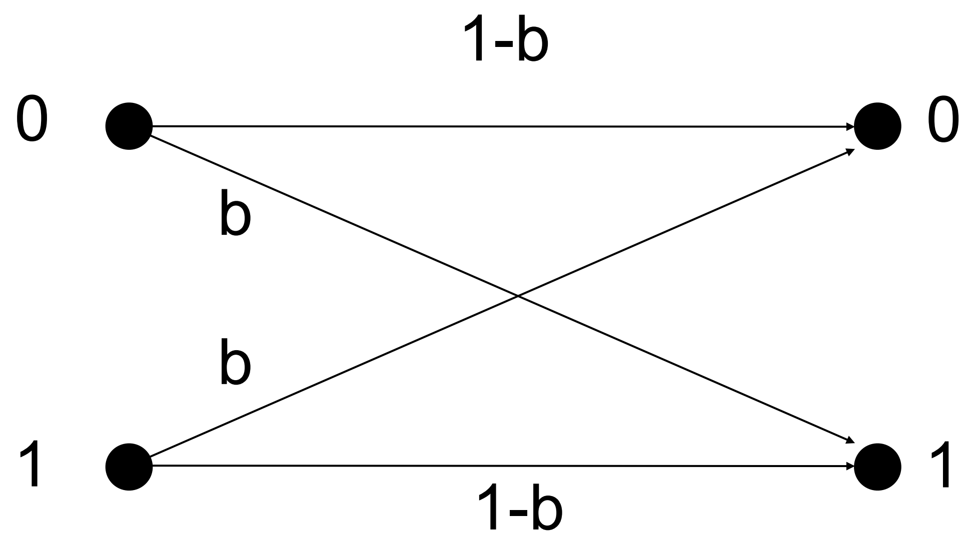




sequentially take samples at posterior median

Channel Coding with Noiseless Feedback

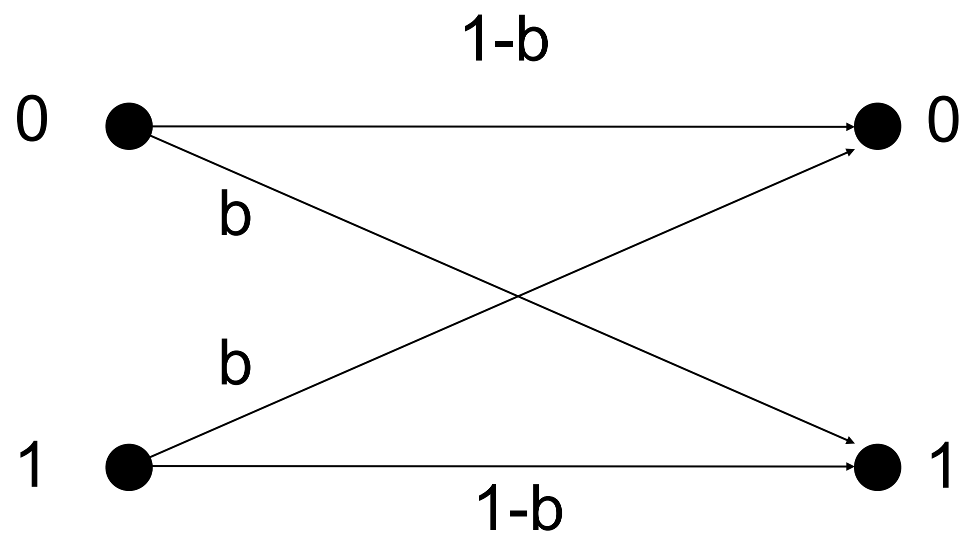
sender



receiver

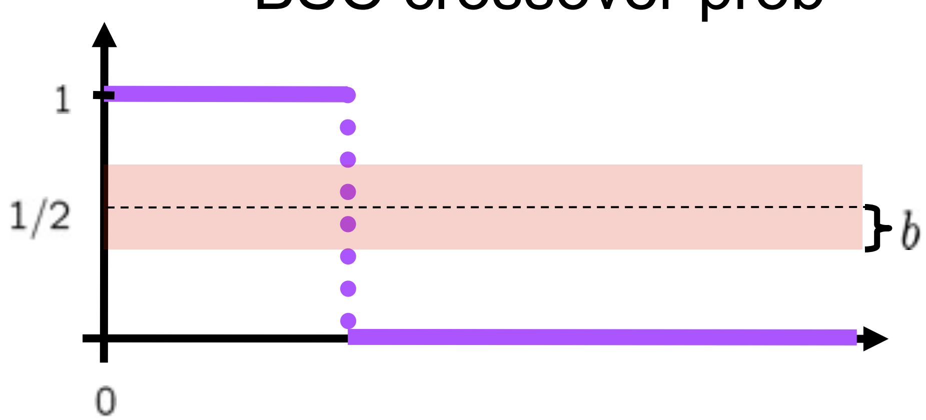
Channel Coding with Noiseless Feedback

sender



receiver

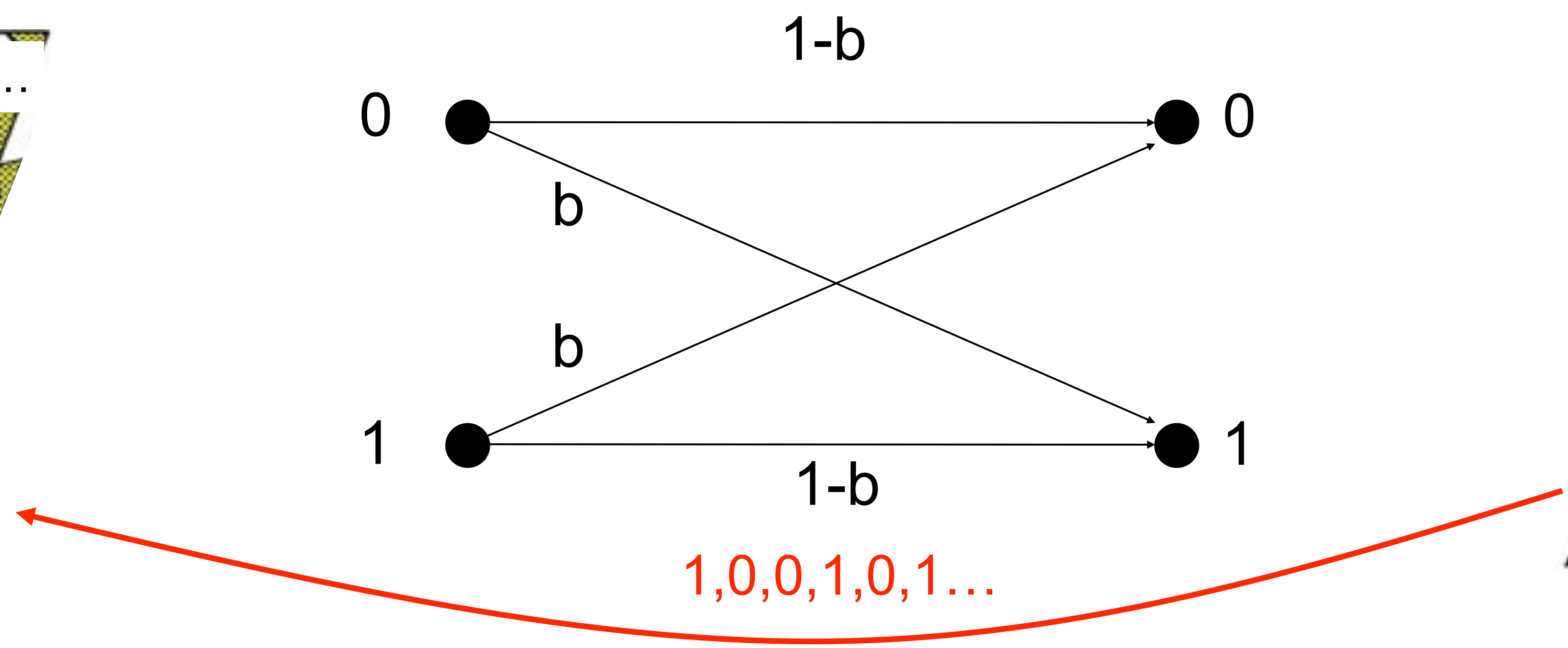
noise bound
= BSC crossover prob



threshold location
= n bit message

Channel Coding with Noiseless Feedback

sender



receiver

noiseless feedback

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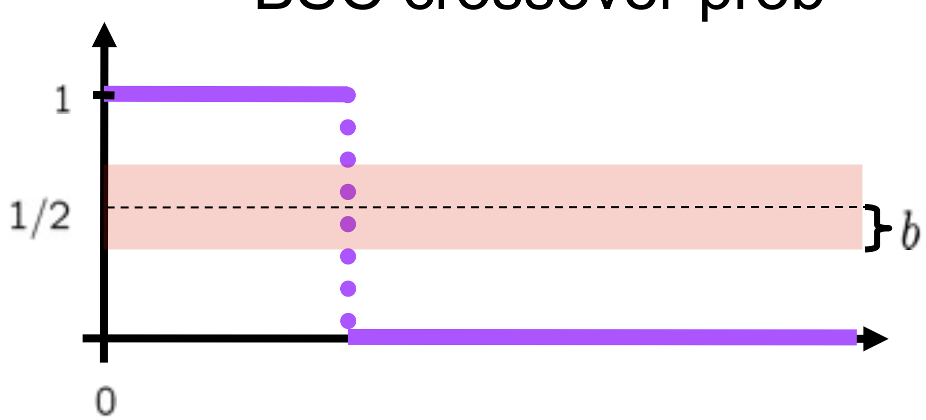

threshold location
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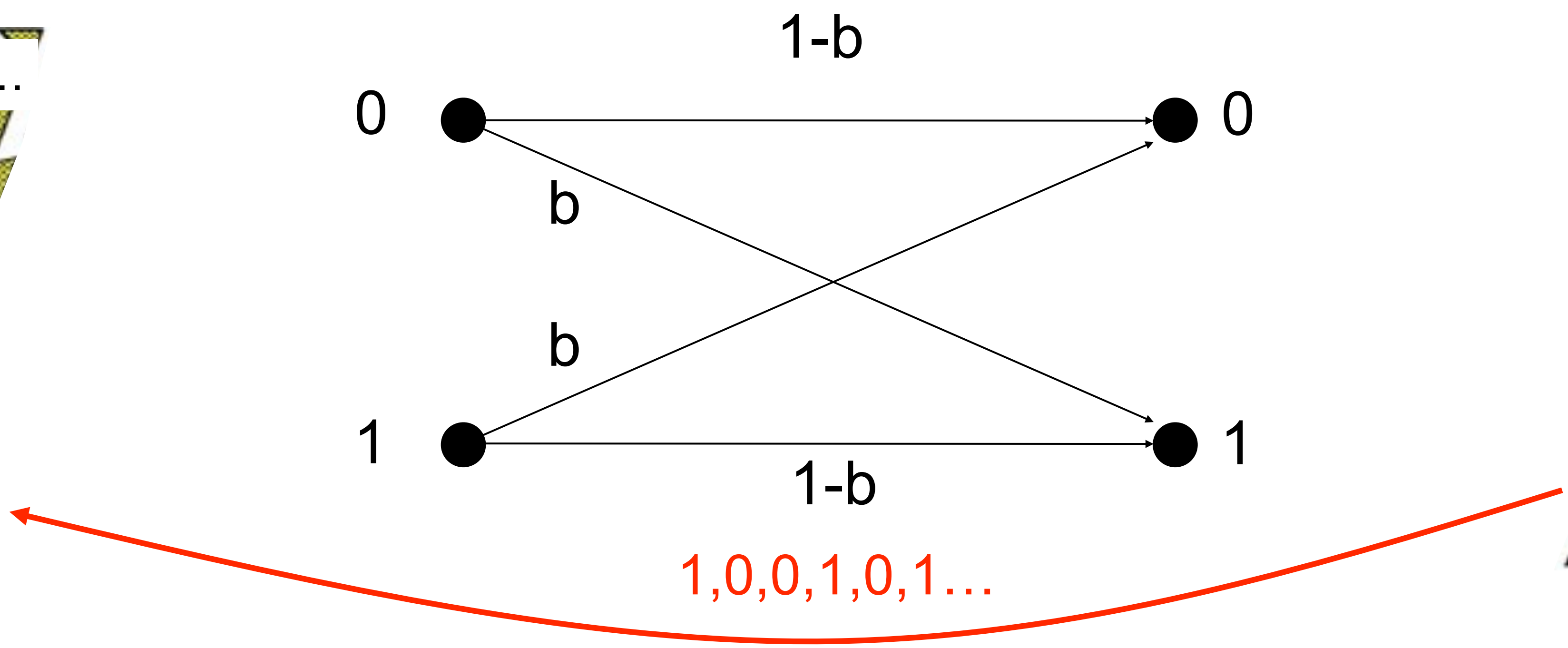
Channel Coding with Noiseless Feedback

sender



noiseless feedback

receiver

noise bound
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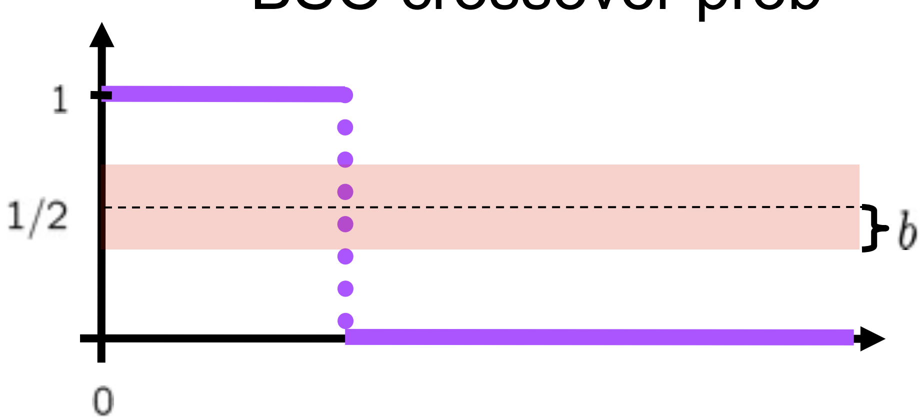


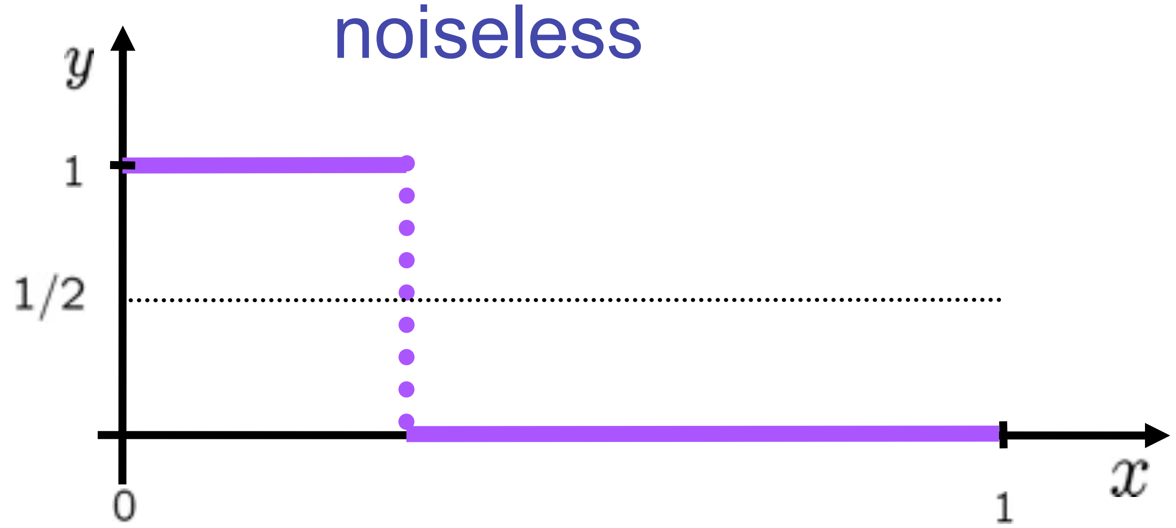
threshold location
= n bit message

Both sender and receiver implement Horstein’s algorithm

Sender deduces which binary symbol to send next in order to yield the greatest possible reduction in the receiver's uncertainty about n-bit message

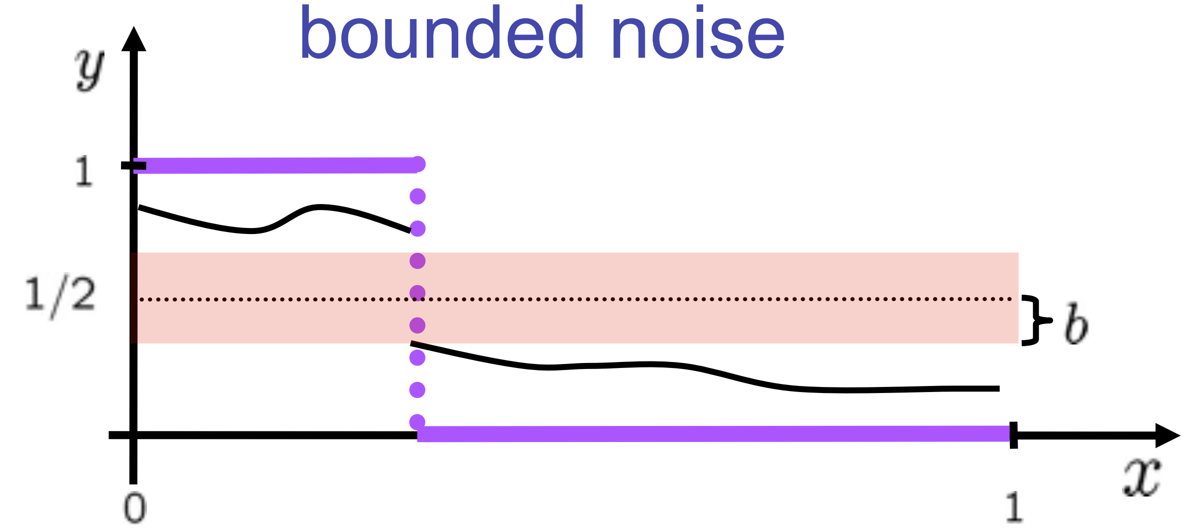
Active Learning in Unbounded Noise

noiseless



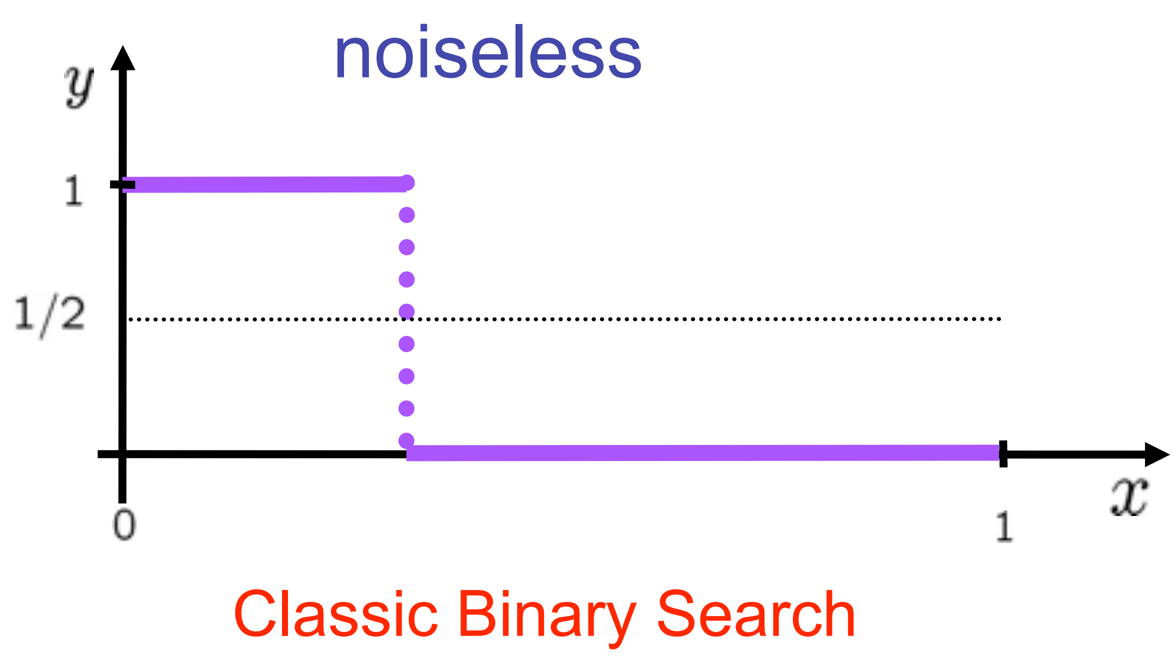
Classic Binary Search

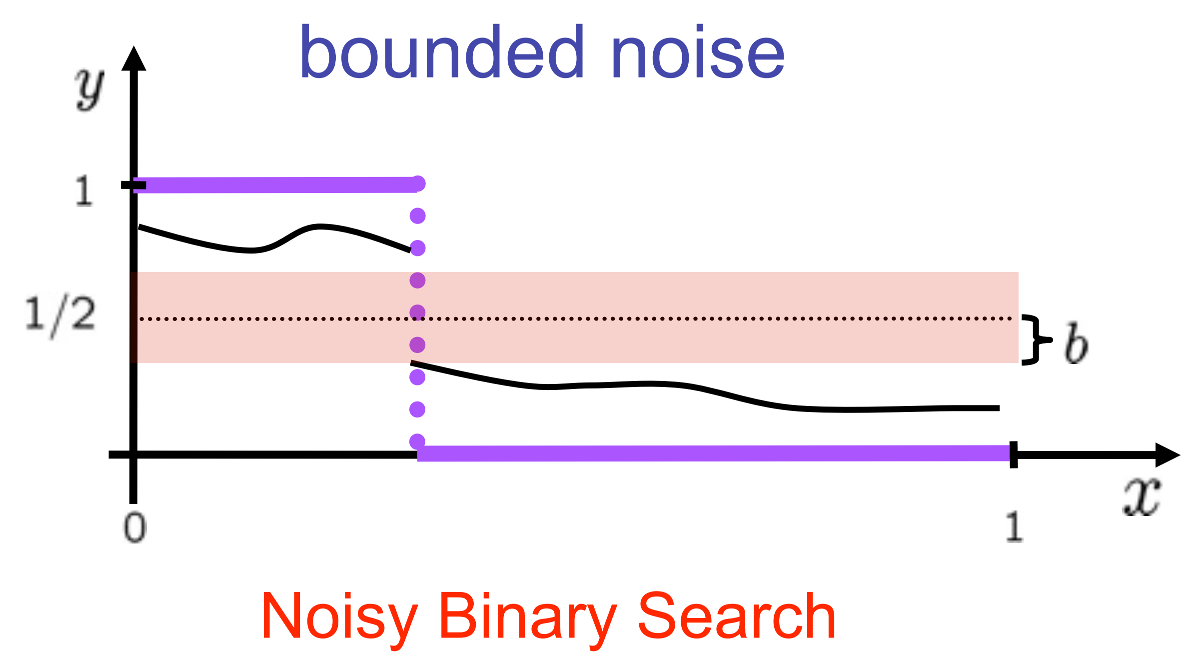
bounded noise

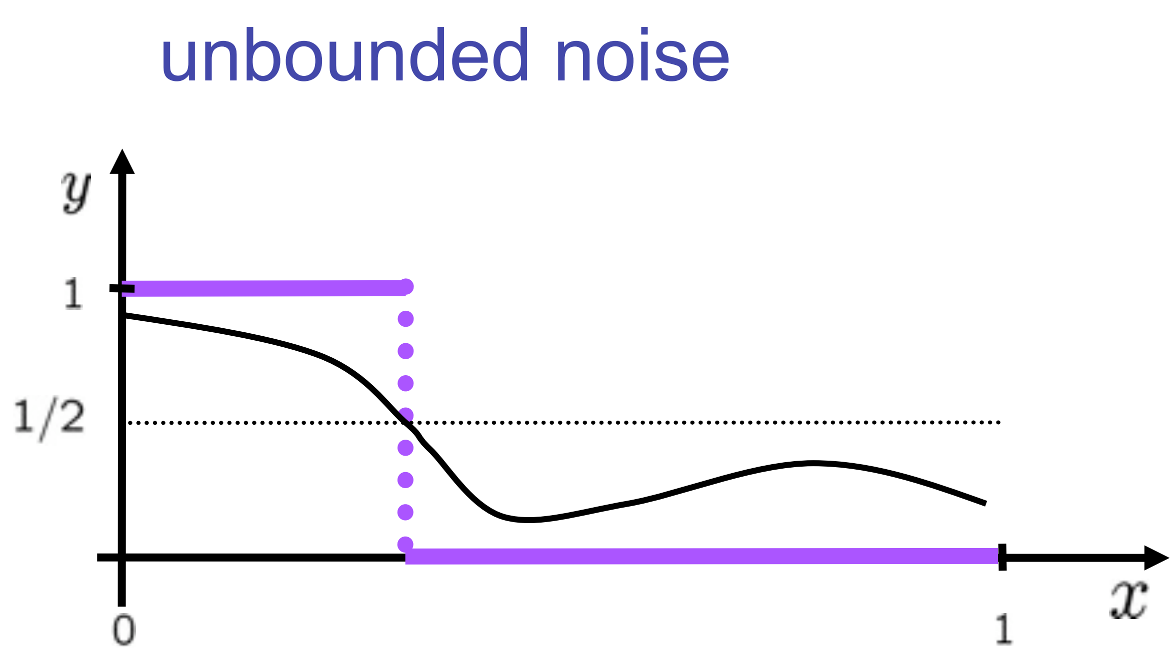


Noisy Binary Search

Active Learning in Unbounded Noise

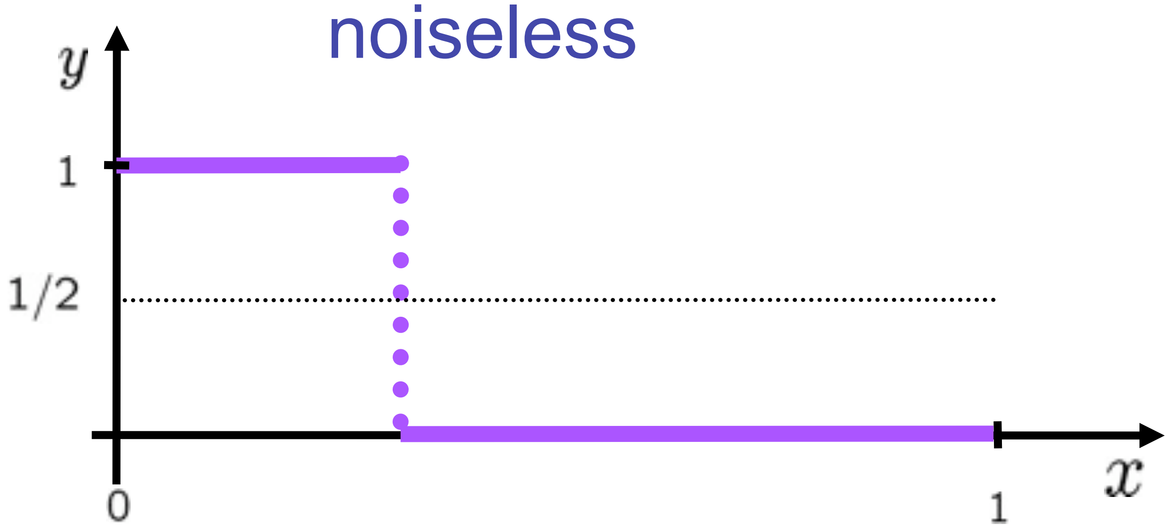






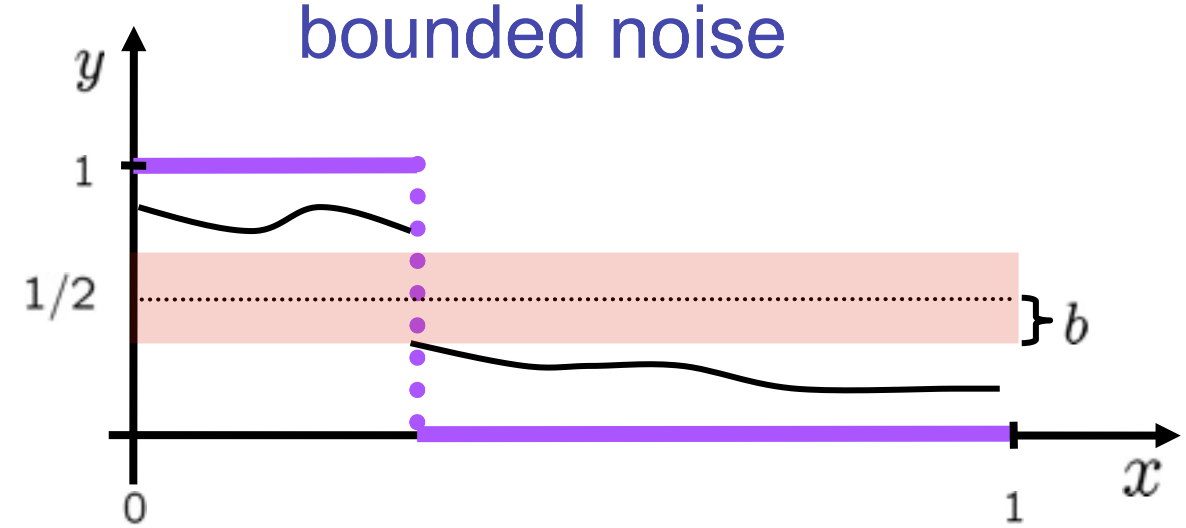
Active Learning in Unbounded Noise

noiseless



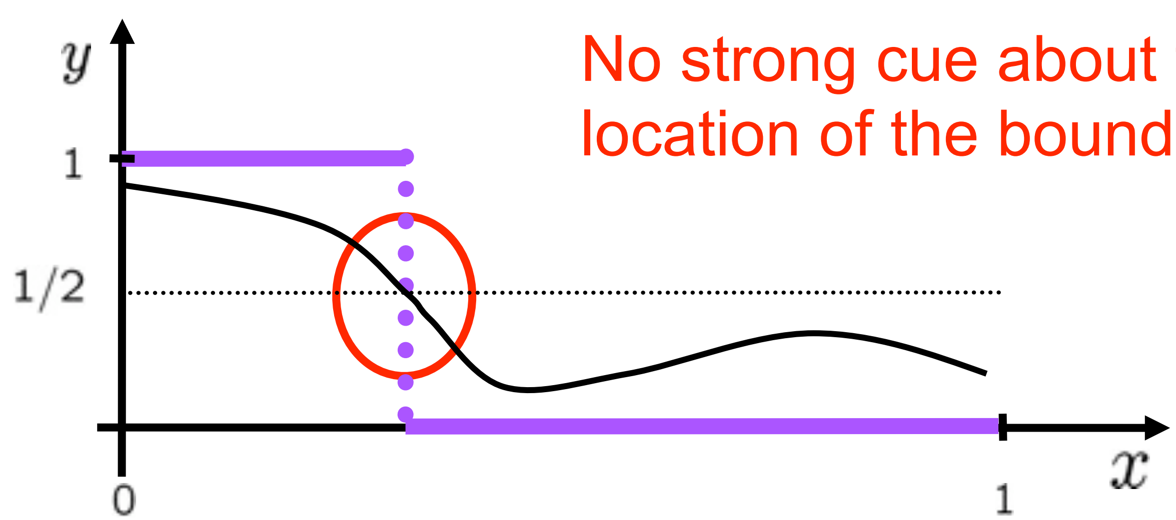
Classic Binary Search

bounded noise



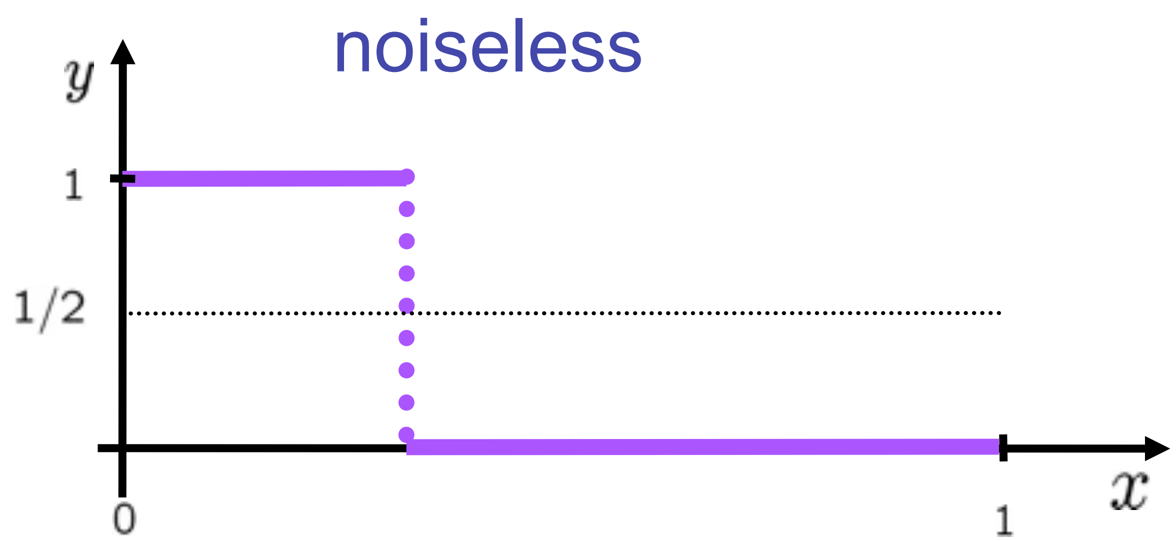
Noisy Binary Search

unbounded noise

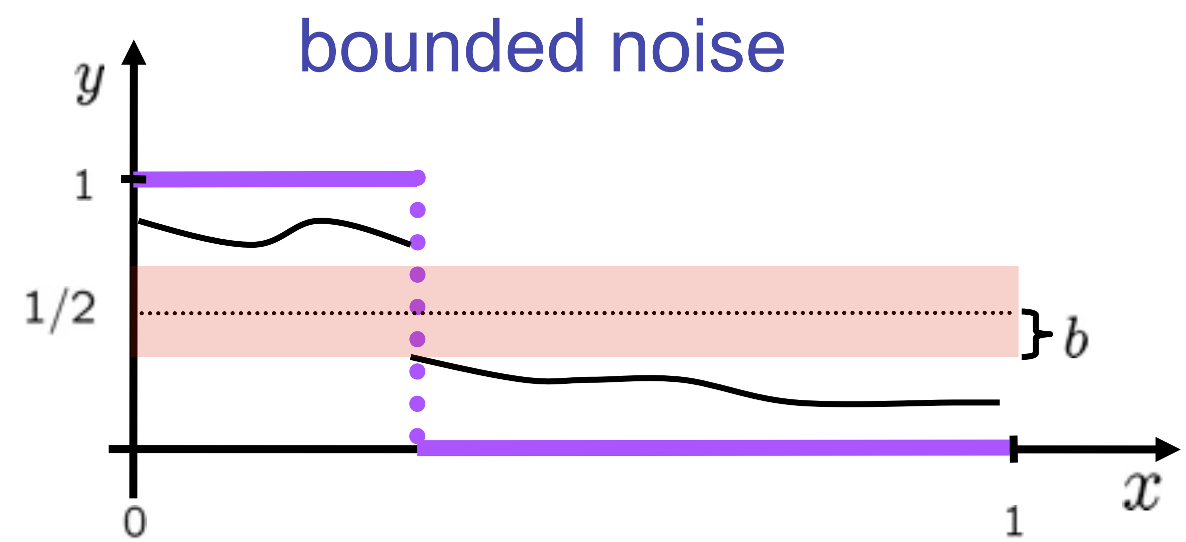


No strong cue about the location of the boundary

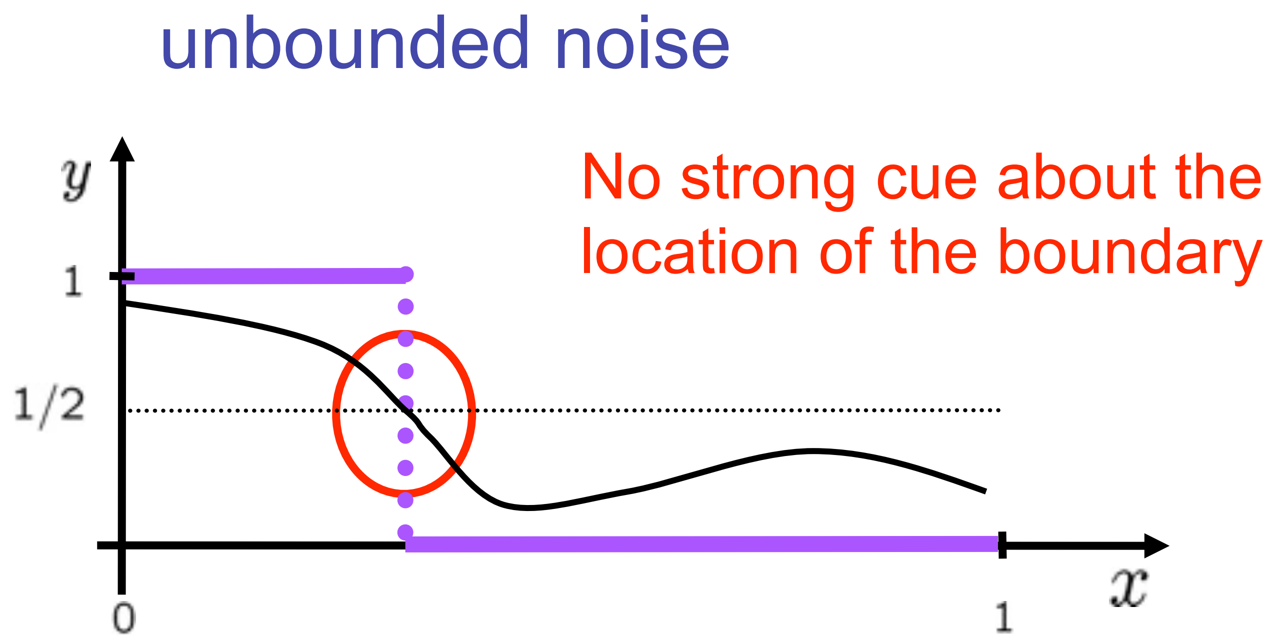
Active Learning in Unbounded Noise



Classic Binary Search



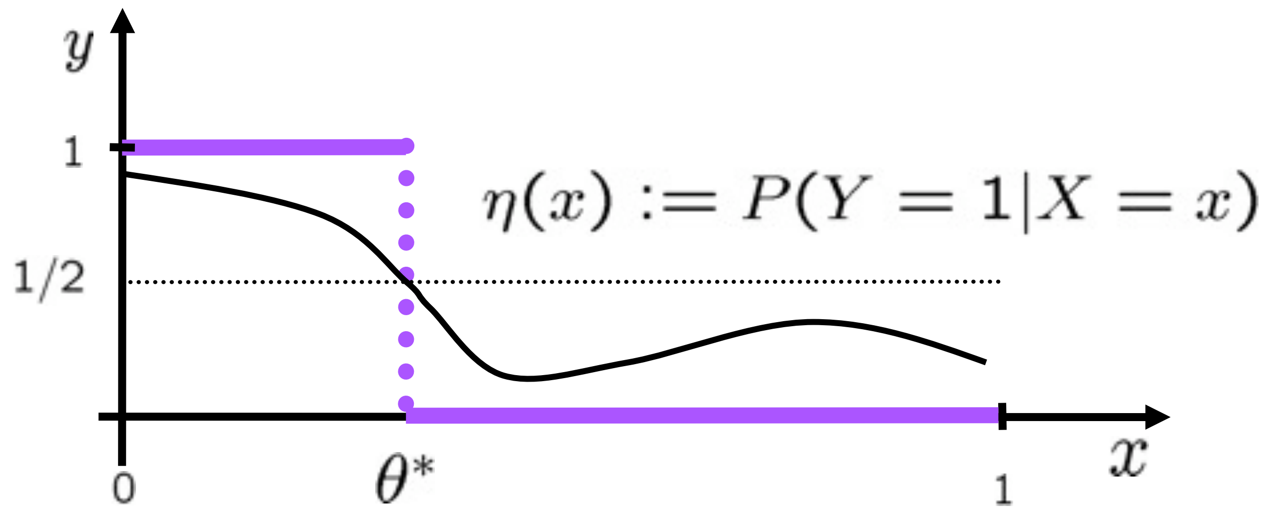
Noisy Binary Search



Rui Castro (Columbia): “How much does active learning help in this case ?”

Unbounded Noise Effects

Near $\frac{1}{2}$-level, $c|x - \theta^*|^{\kappa-1} \leq |\eta(x) - 1/2| \leq C|x - \theta^*|^{\kappa-1}, \quad \kappa \geq 1$



similar conditions are commonly employed in nonparametric statistics, Tsybakov (2004)

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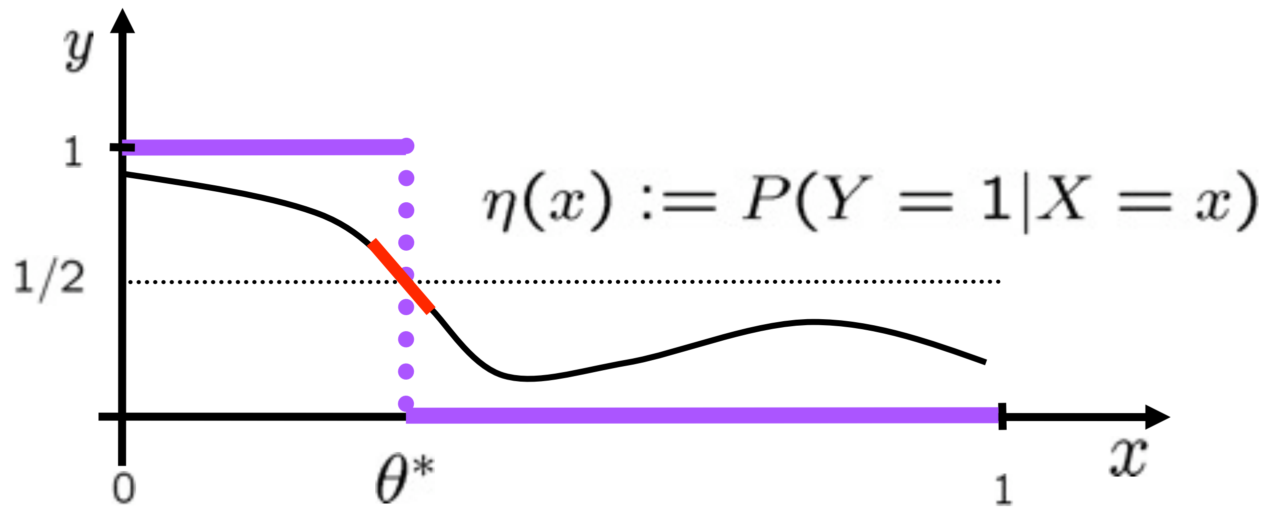

$\kappa = 2$

similar conditions are commonly employed in nonparametric statistics, Tsybakov (2004)

Unbounded Noise Effects

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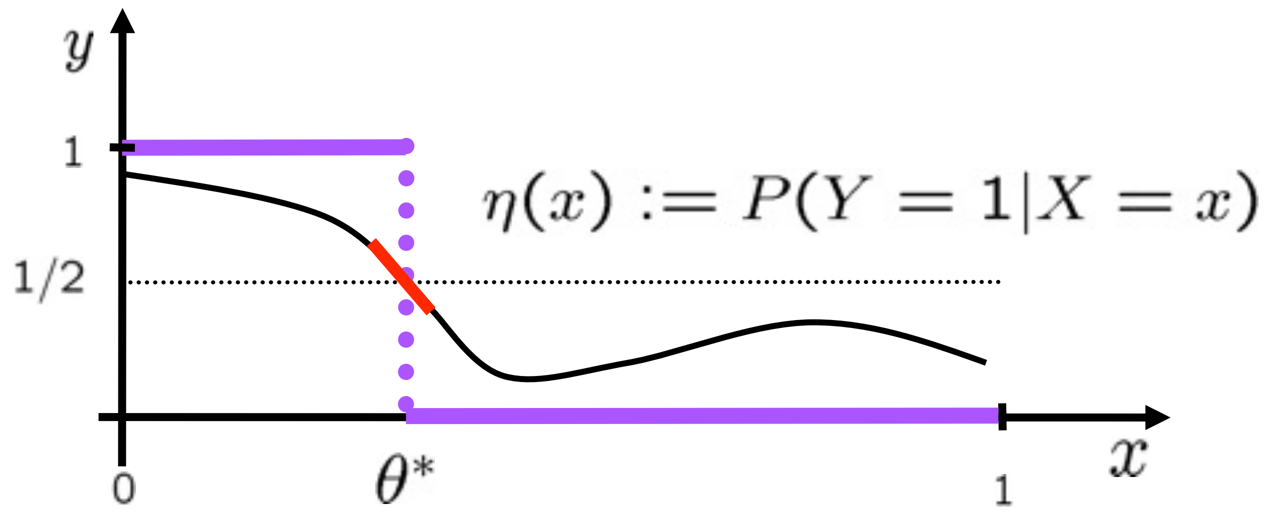

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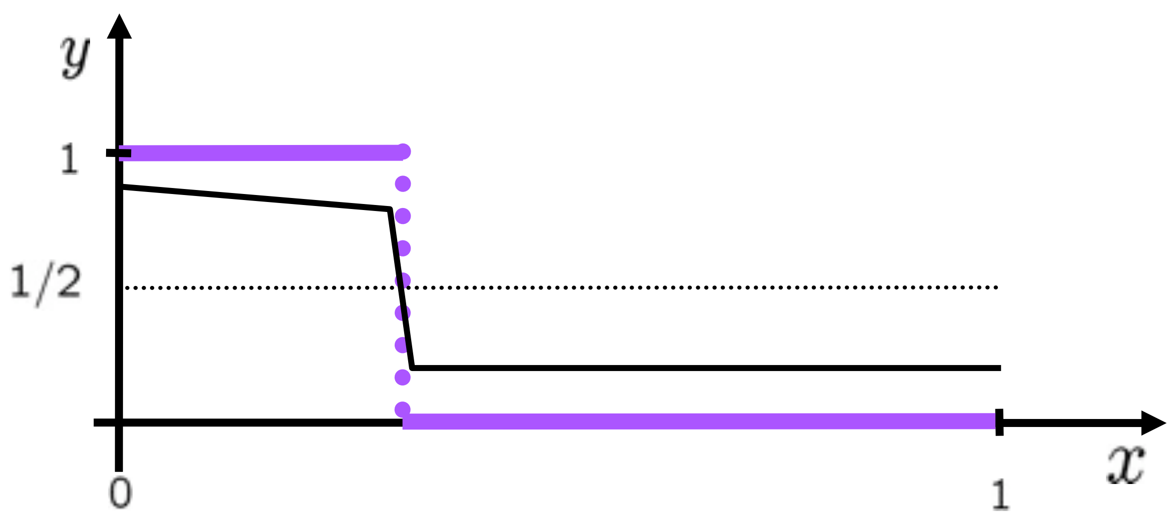

$\kappa \to 1$

similar conditions are commonly employed in nonparametric statistics, Tsybakov (2004)

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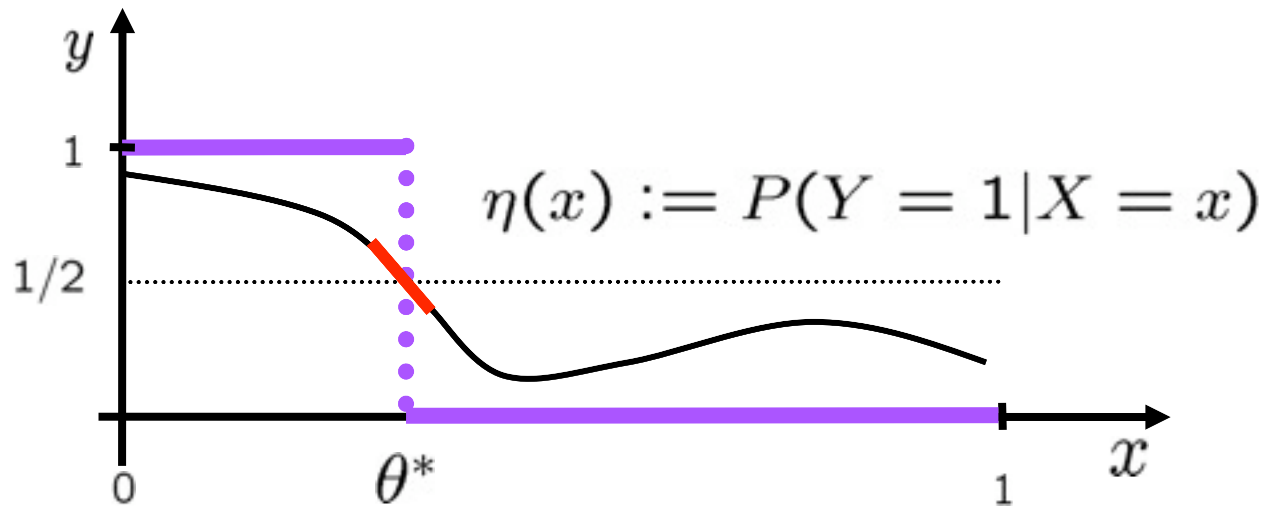
Near $\frac{1}{2}$-level, $c|x-\theta^*|^{\kappa-1} \leq |\eta(x) - 1/2| \leq C|x-\theta^*|^{\kappa-1}$, $\kappa \geq 1$



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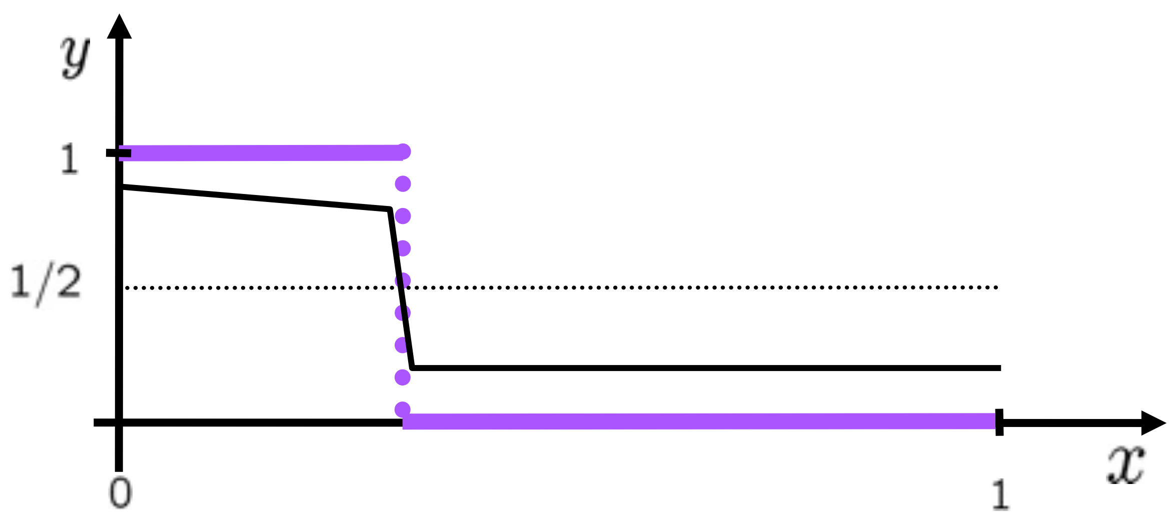
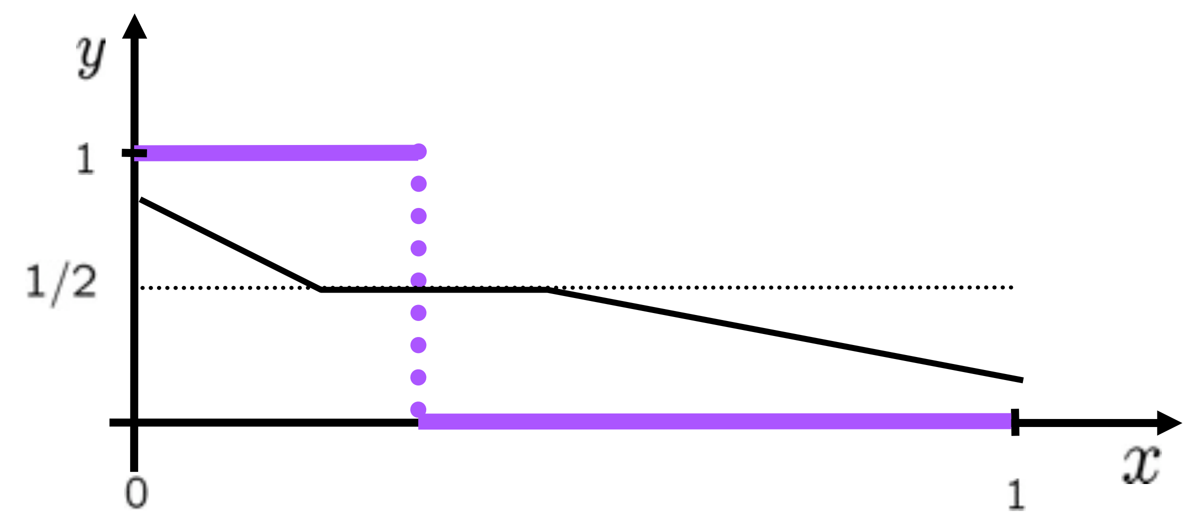


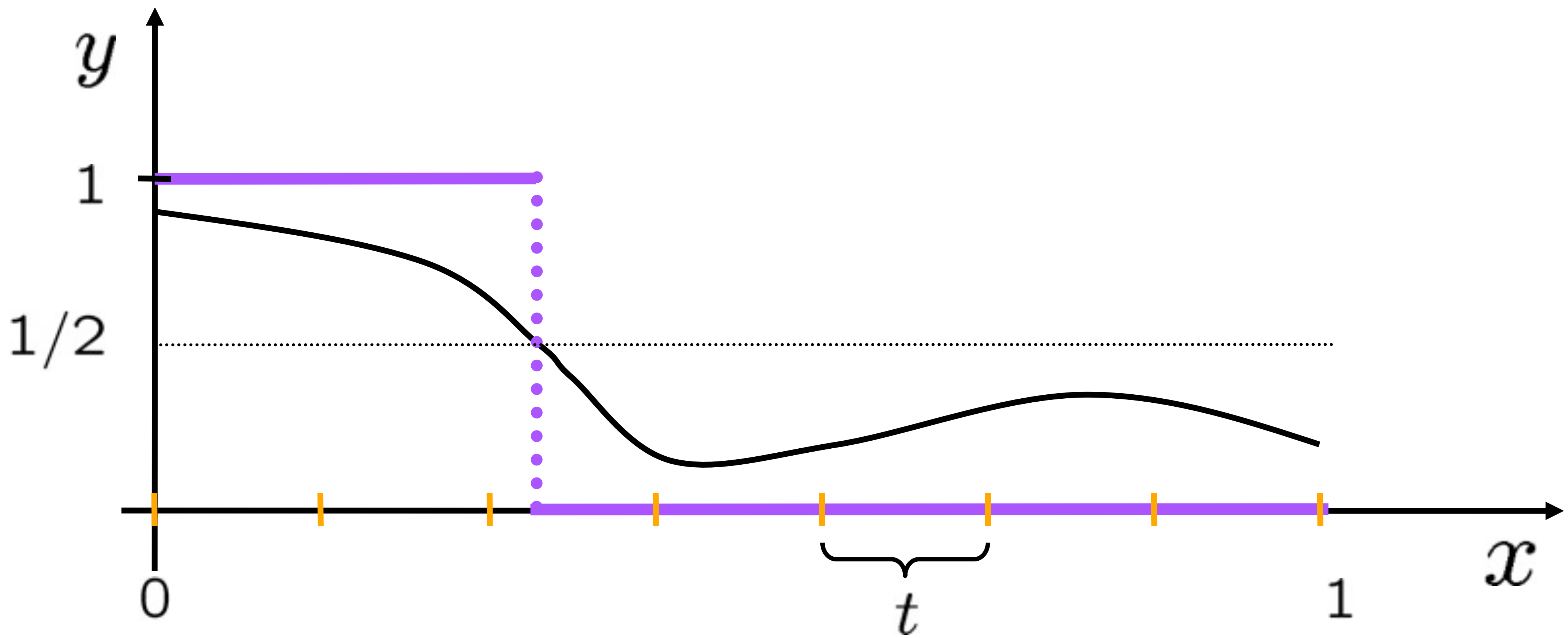
$\kappa \rightarrow 1$



$\kappa \rightarrow \infty$

similar conditions are commonly employed in nonparametric statistics, Tsybakov (2004)

Horstein's Algorithm in Unbounded Noise



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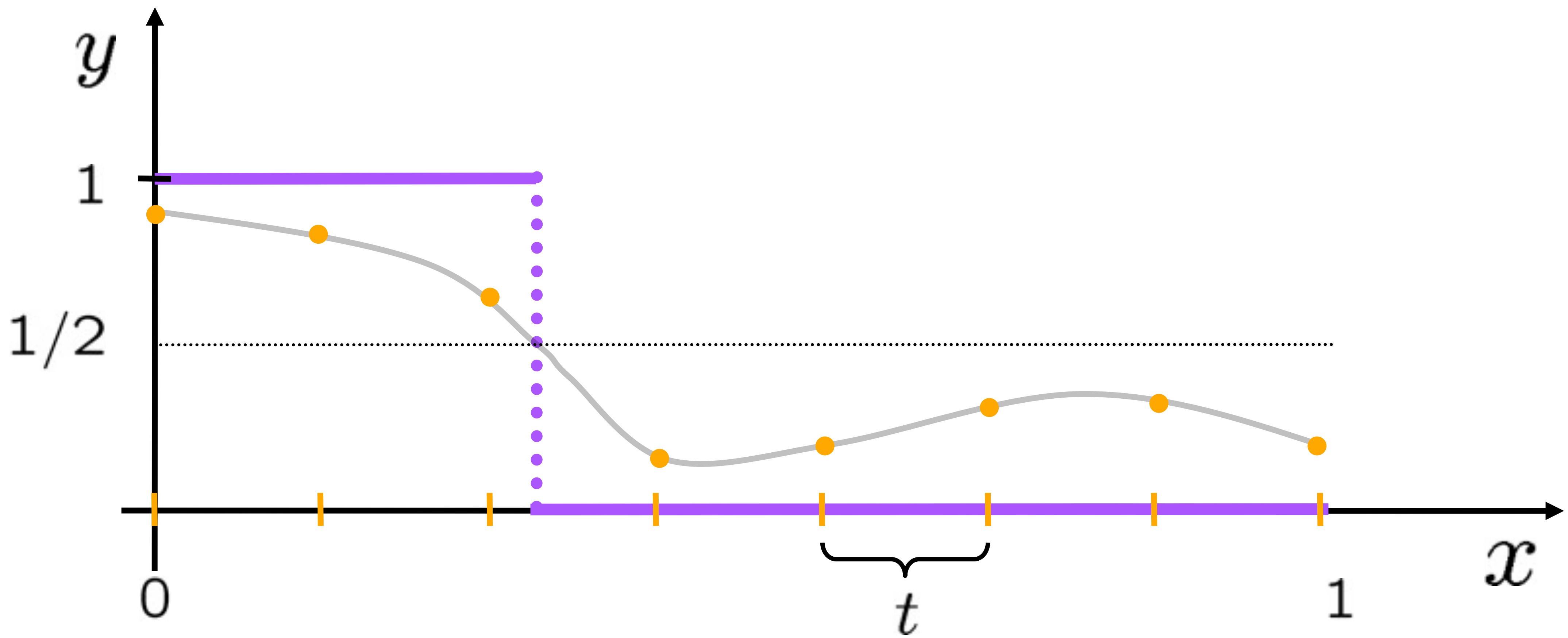
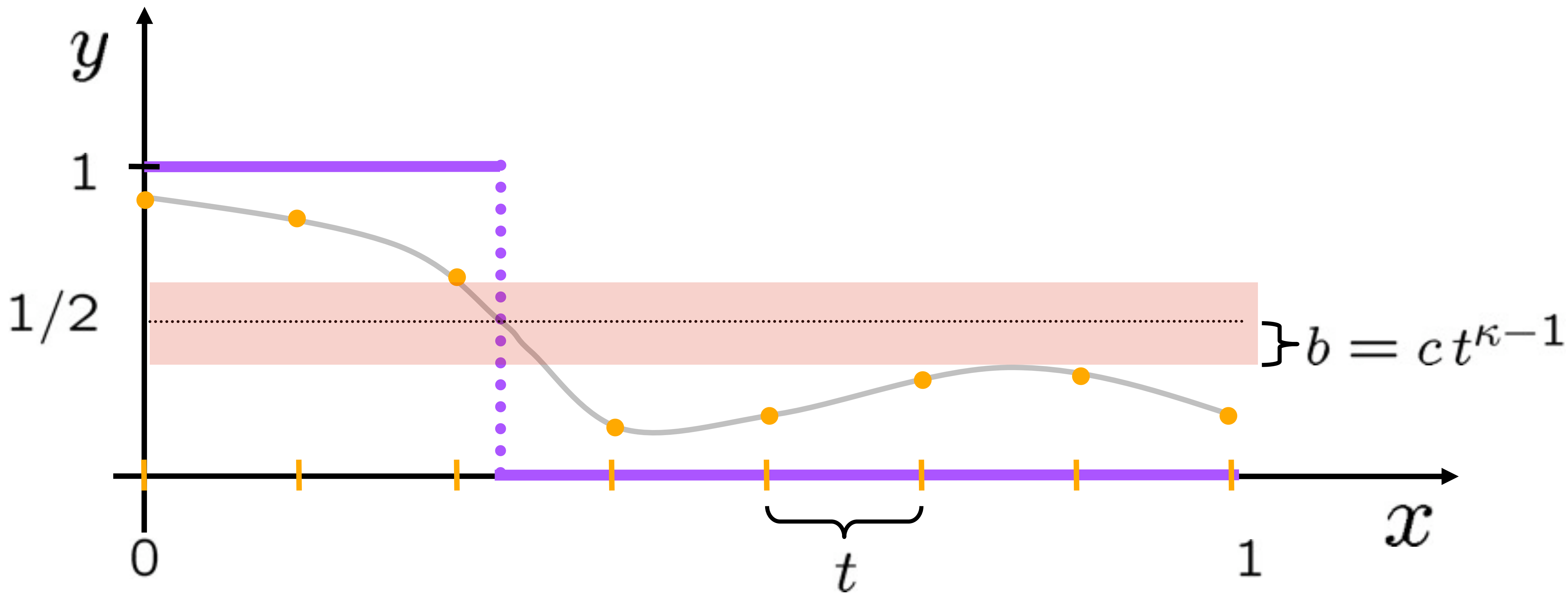


Consider discrete set of thresholds and discretized version of P(Y=1|X=x)

Horstein's Algorithm in Unbounded Noise



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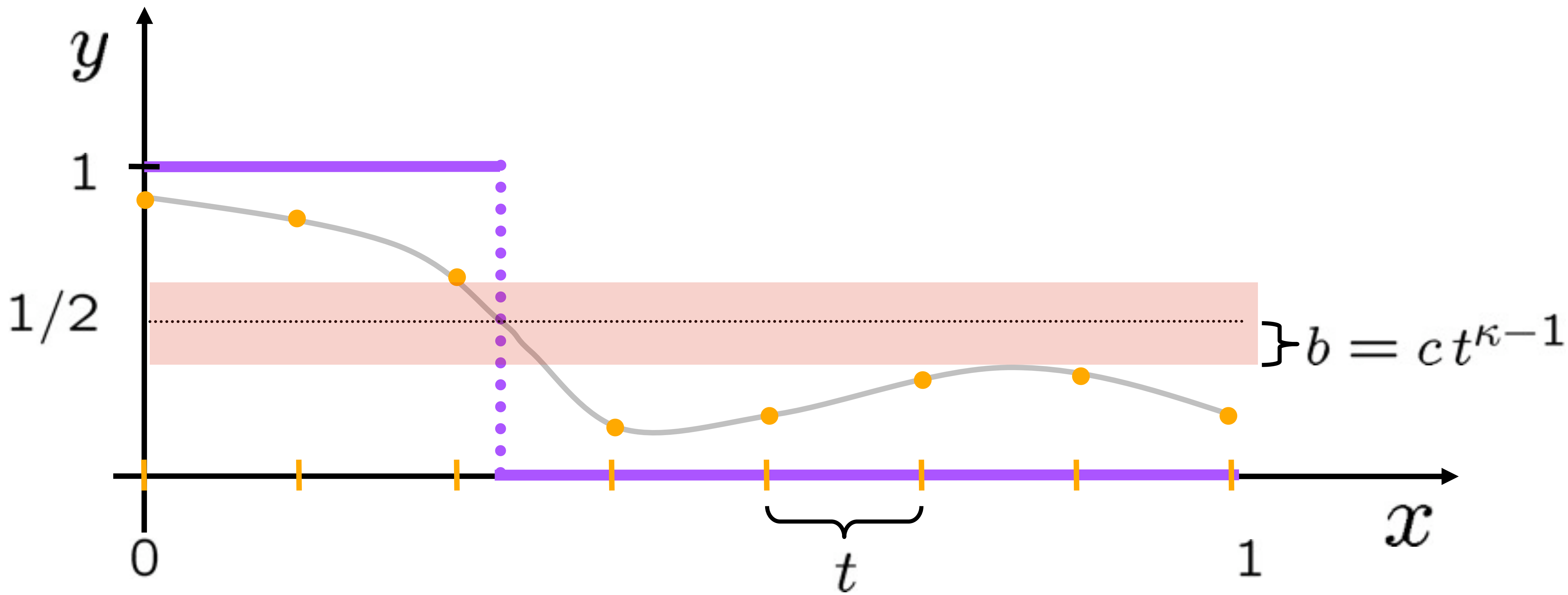
If ½ level is not aligned with discrete thresholds, then noise of discretized problem is bounded, but depends on resolution of discretization t and the behavior of P(Y=1|X=x) at the ½ level

Horstein's Algorithm in Unbounded Noise



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$$\mathbb{P}[h_n(X) \neq Y] - \mathbb{P}[h^*(X) \neq Y] \quad \leq \quad t^{\kappa} \; + \; t^{-1}\exp(-nc^2 t^{2\kappa-2})$$

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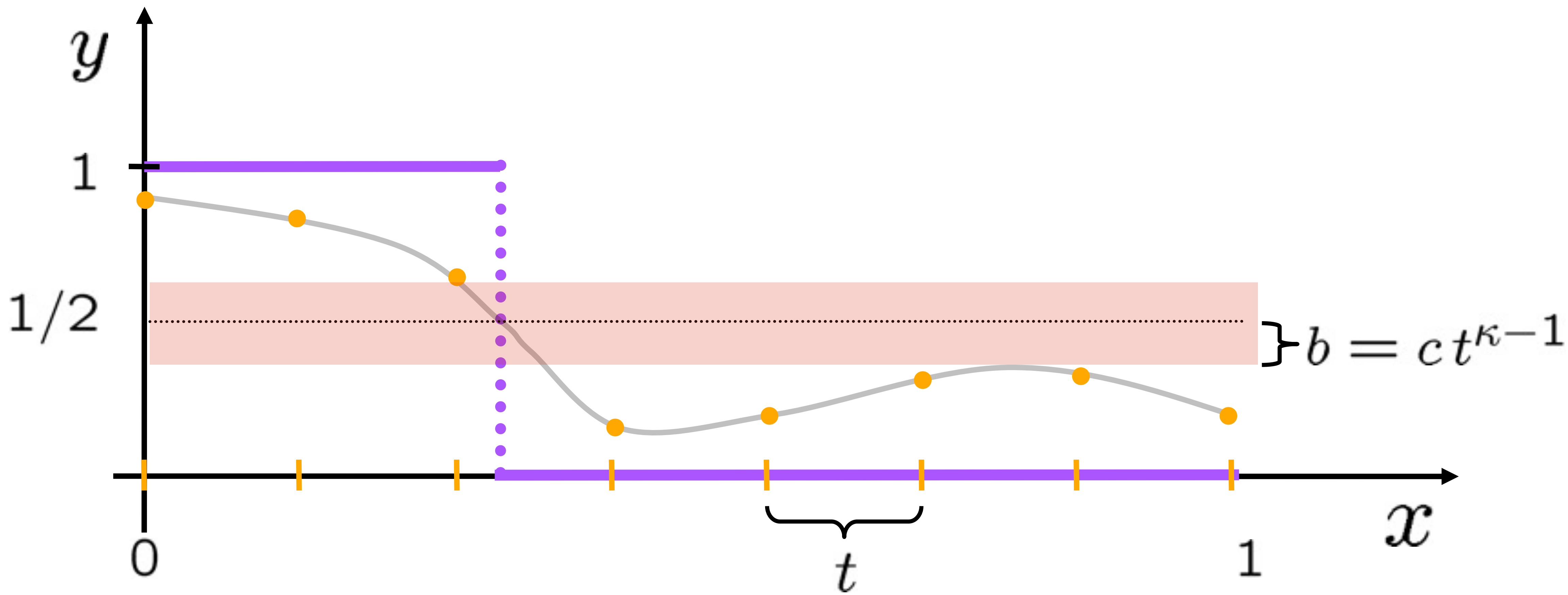

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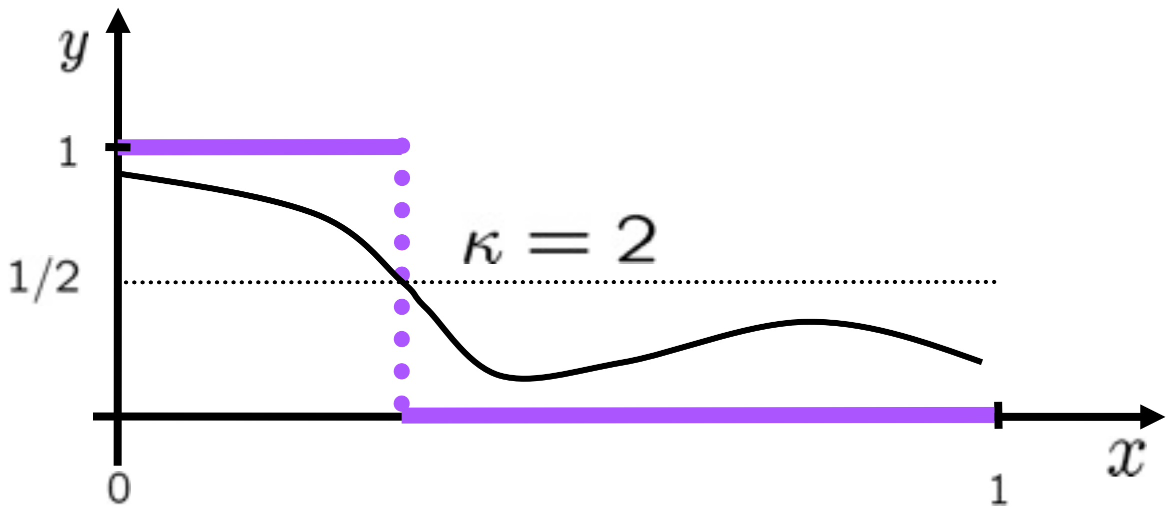
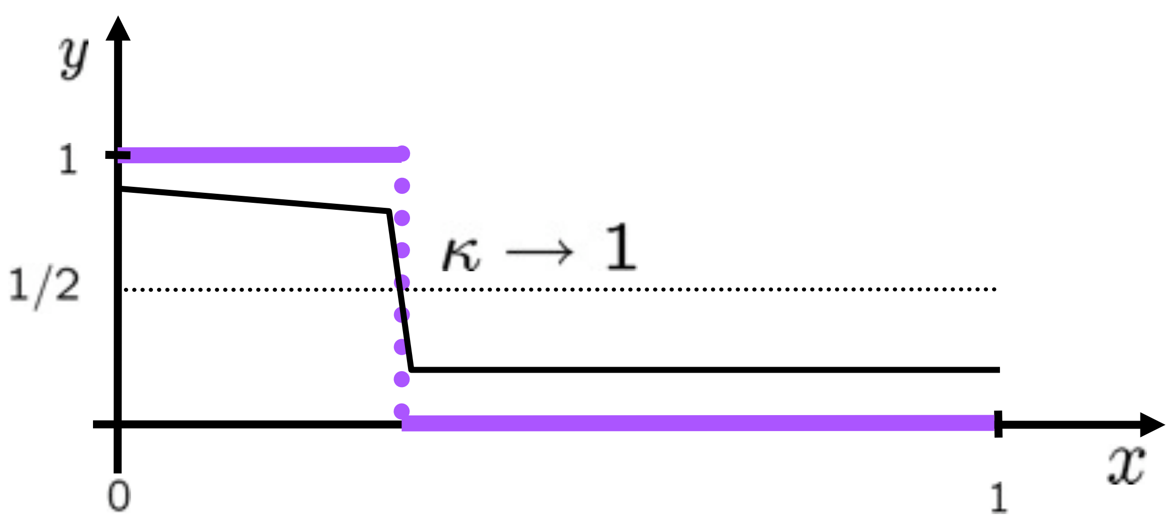
$$\begin{aligned}\mathbb{P}[h_n(X) \neq Y] - \mathbb{P}[h^*(X) \neq Y] &\leq t^{\kappa} + t^{-1}\exp(-nc^2 t^{2\kappa-2}) \\ &= O\left(\left[\frac{\log n}{n}\right]^{\frac{\kappa}{2\kappa-2}}\right)\end{aligned}$$

Rates of Convergence



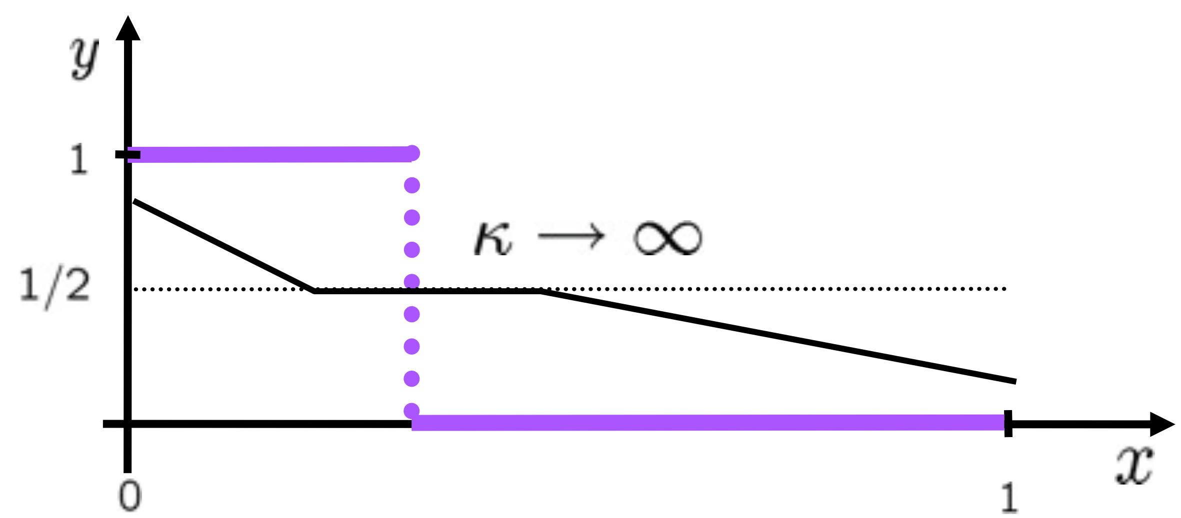
passive: $n^{-2/3}$

active: n^{-1}



passive: $\to n^{-1}$

active: $\to e^{-cn}$



passive: $\to n^{-1/2}$

active: $\to n^{-1/2}$

Are you a good active learner ?

Castro, Kalish, Nowak, Qian, Rogers & Zhu (NIPS 2008)

Investigate human active learning in task analogous to 1-d threshold problem

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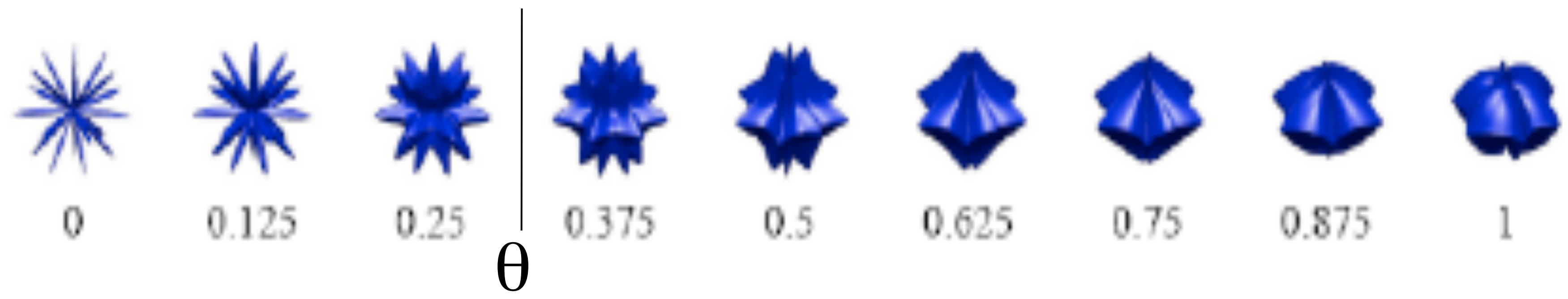

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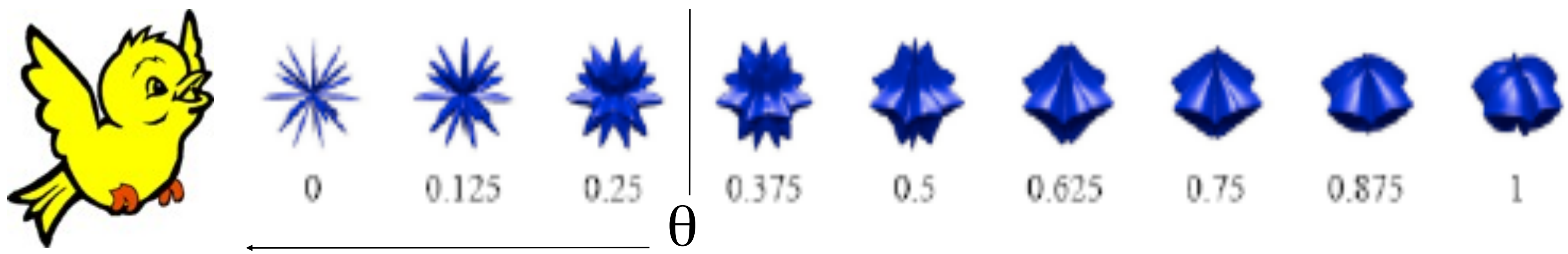

more probably birds

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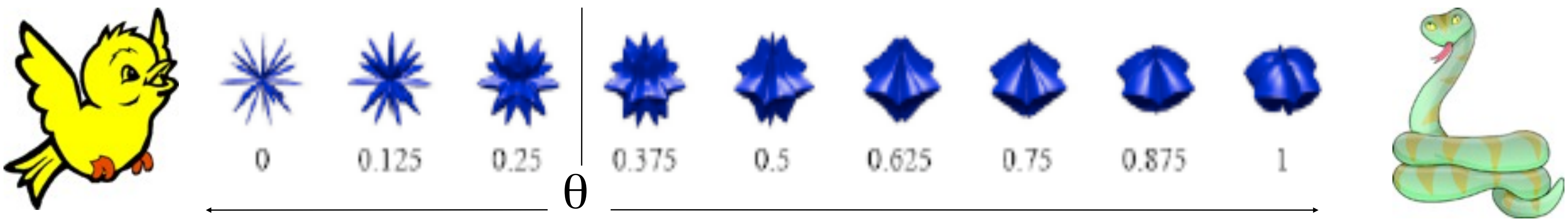

more probably birds

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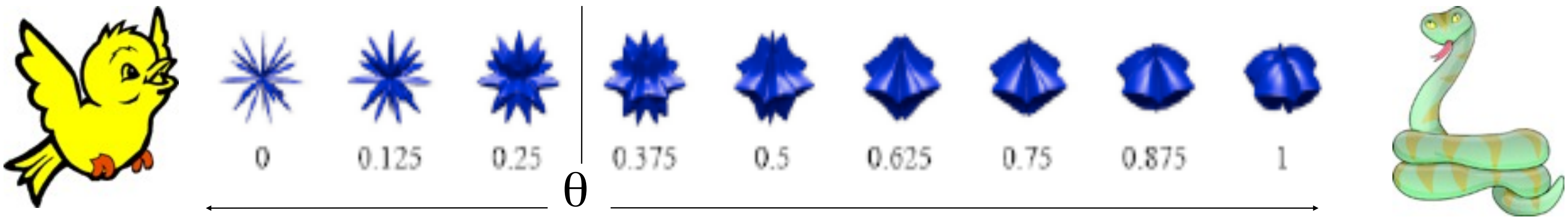

more probably birds

more probably snakes

Subjects observe random egg hatchings (passive learning) or they can select eggs to hatch (active learning).

They are asked to determine the egg shape where snakes become more probable than birds.

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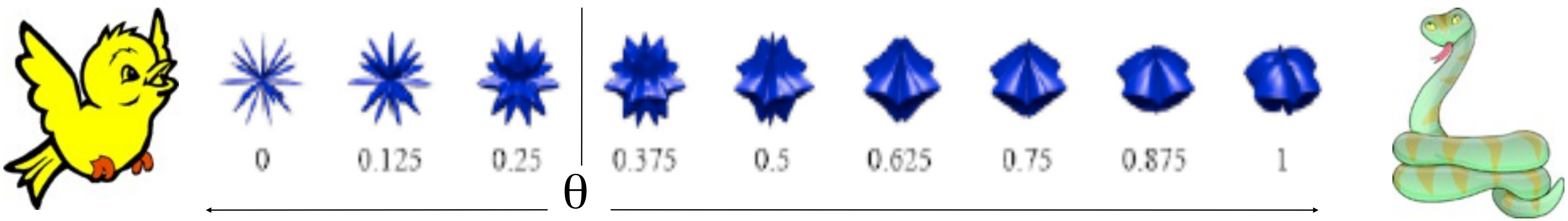

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Subjects observe random egg hatchings (passive learning) or they can select eggs to hatch (active learning).

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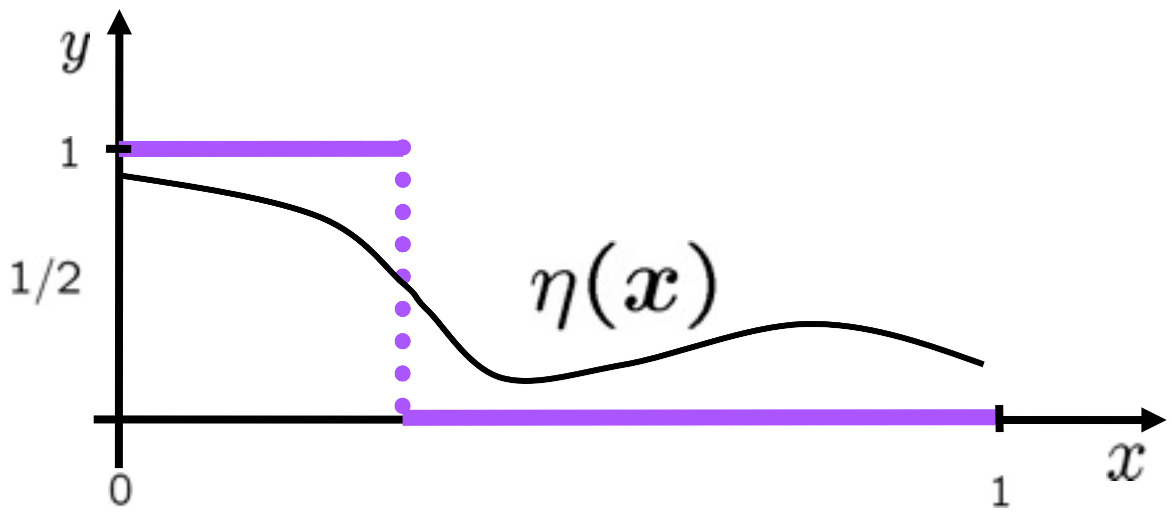
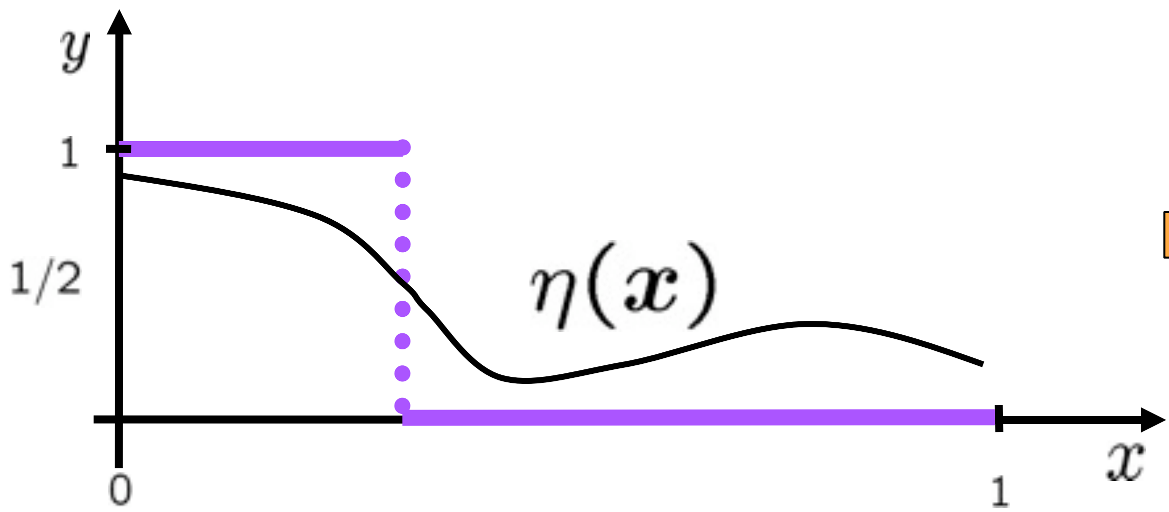
Results: Human learning rates agree with theory, $1/n$ in passive mode and $\exp(-cn)$ in active mode.

Learning Multidimensional Threshold Functions



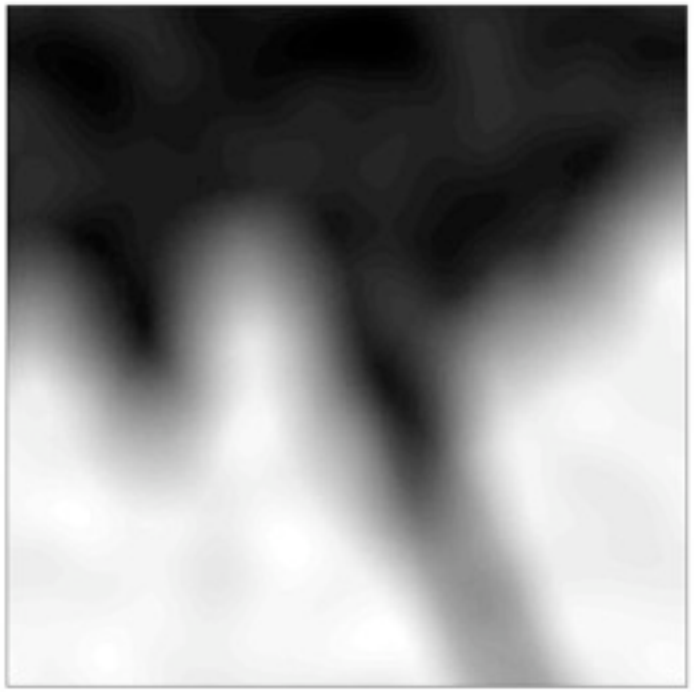
$$d = 1$$

Learning Multidimensional Threshold Functions



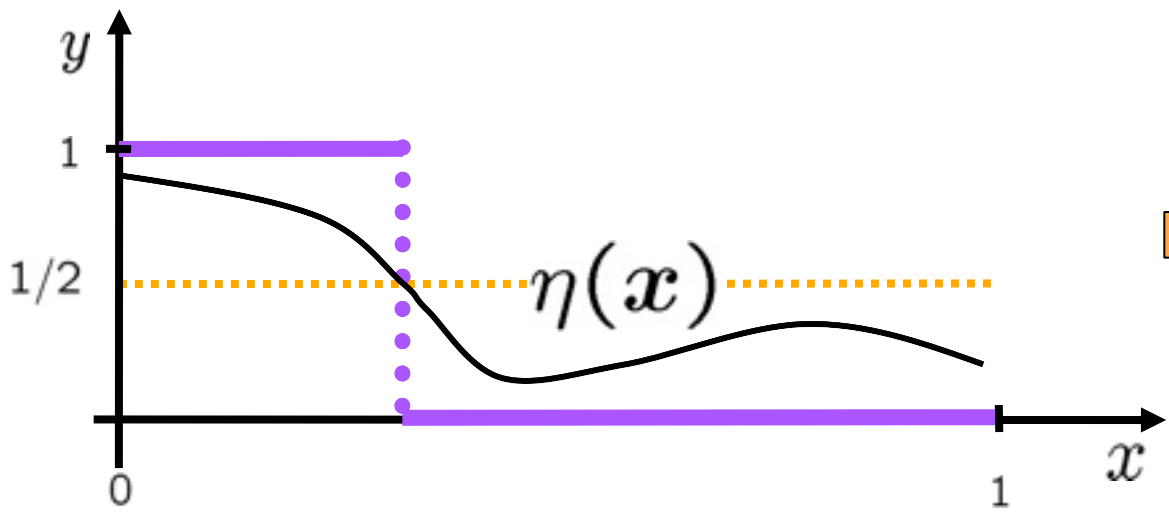
$d = 1$

$\eta(\boldsymbol{x})$

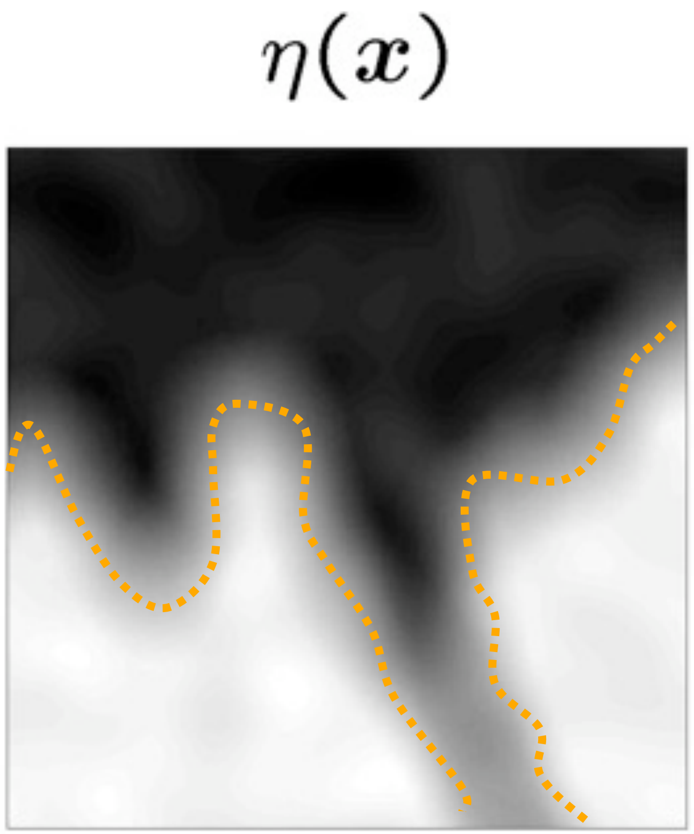


$d > 1$

Learning Multidimensional Threshold Functions



$d = 1$



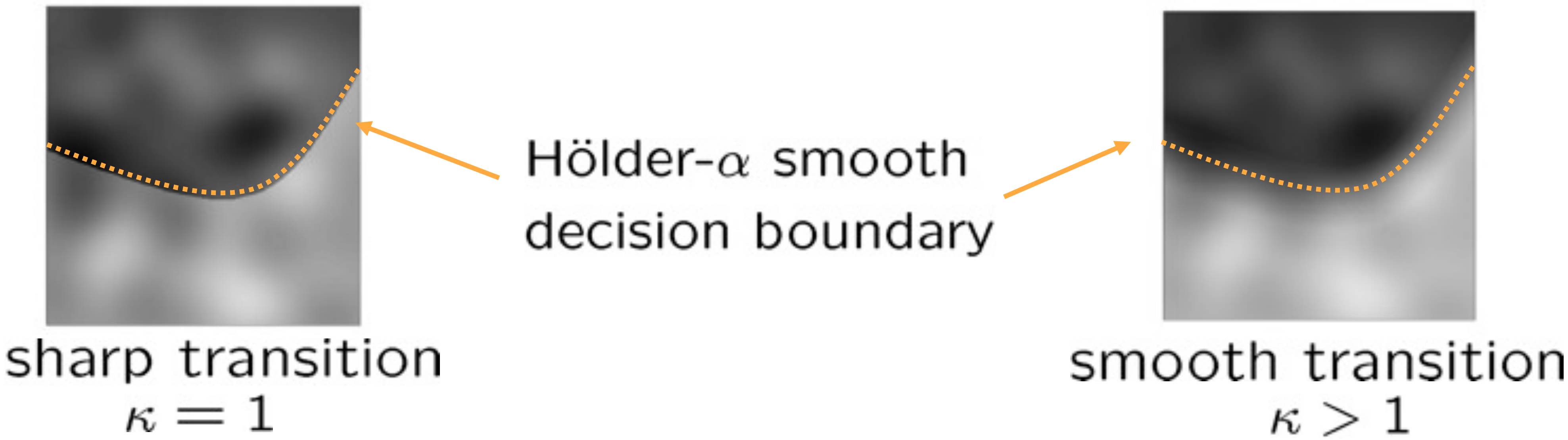
$d > 1$

Learning Rates for Multidimensional Thresholds

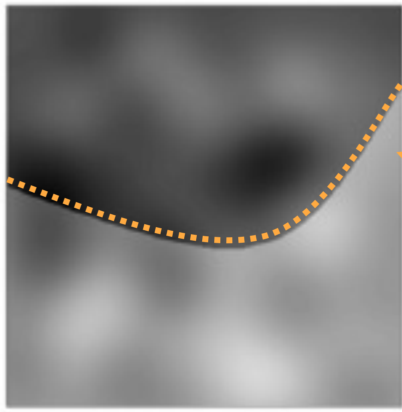
sharp transition
$\kappa = 1$

smooth transition
$\kappa > 1$

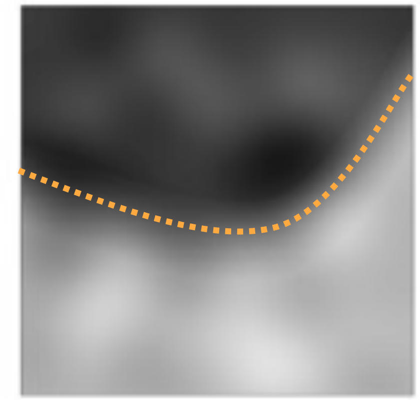
Learning Rates for Multidimensional Thresholds



Learning Rates for Multidimensional Thresholds



sharp transition
$\kappa = 1$



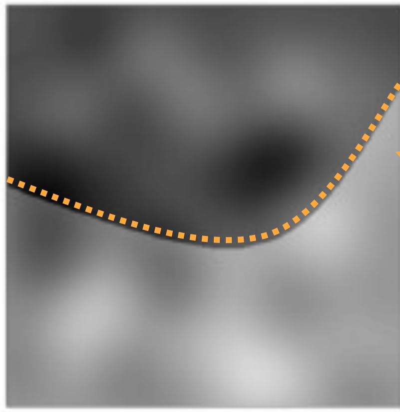
smooth transition
$\kappa > 1$

Active Learning: Theorem (R. Castro and RN '07)

$$\left(\frac{1}{n}\right)^{\frac{\kappa}{2\kappa+\rho-2}} \preceq \inf_{h_n, S_n} \sup_{P_{XY} \in \mathrm{BF}(\alpha,\kappa)} \mathcal{E}(h_n) \preceq \left(\frac{\log n}{n}\right)^{\frac{\kappa}{2\kappa+\rho-2}}$$

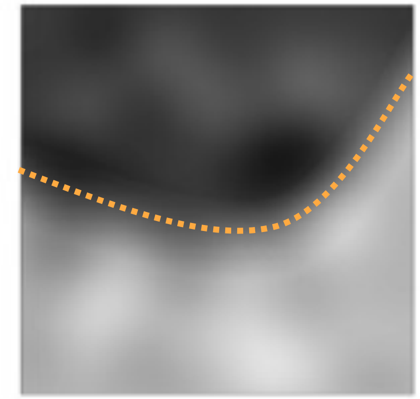
$(\rho = (d-1)/\alpha)$

Learning Rates for Multidimensional Thresholds



sharp transition
$\kappa = 1$

Hölder-α smooth
decision boundary



smooth transition
$\kappa > 1$

Active Learning: Theorem (R. Castro and RN '07)

$$\left(\frac{1}{n}\right)^{\frac{\kappa}{2\kappa+\rho-2}} \preceq \inf_{h_n, S_n} \sup_{P_{XY} \in \mathrm{BF}(\alpha,\kappa)} \mathcal{E}(h_n) \preceq \left(\frac{\log n}{n}\right)^{\frac{\kappa}{2\kappa+\rho-2}}$$

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Compare with passive learning

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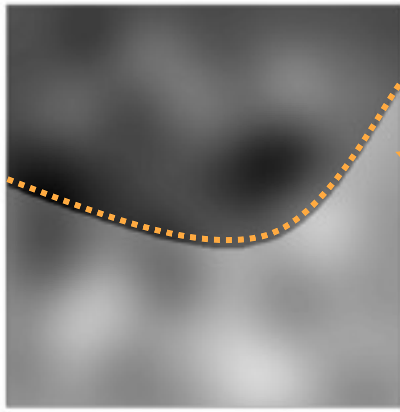
Learning Rates for Multidimensional Thresholds



sharp transition
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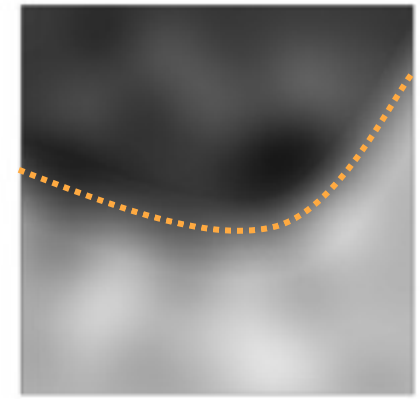

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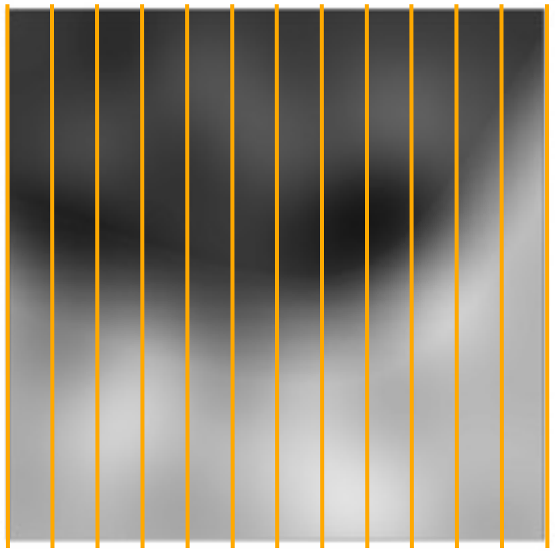
Learning Rates for Multidimensional Thresholds

Main idea: reduce multidimensional problem to a sequence of 1-dim problems



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Algorithms for Active Learning

$\mathcal{X}$:= domain or *query space*

$\mathcal{Y} := \{-1, +1\}$

$\mathcal{H}$:= *hypothesis space* $\quad \forall h \in H, \ \ h : \mathcal{X} \to \mathcal{Y}$

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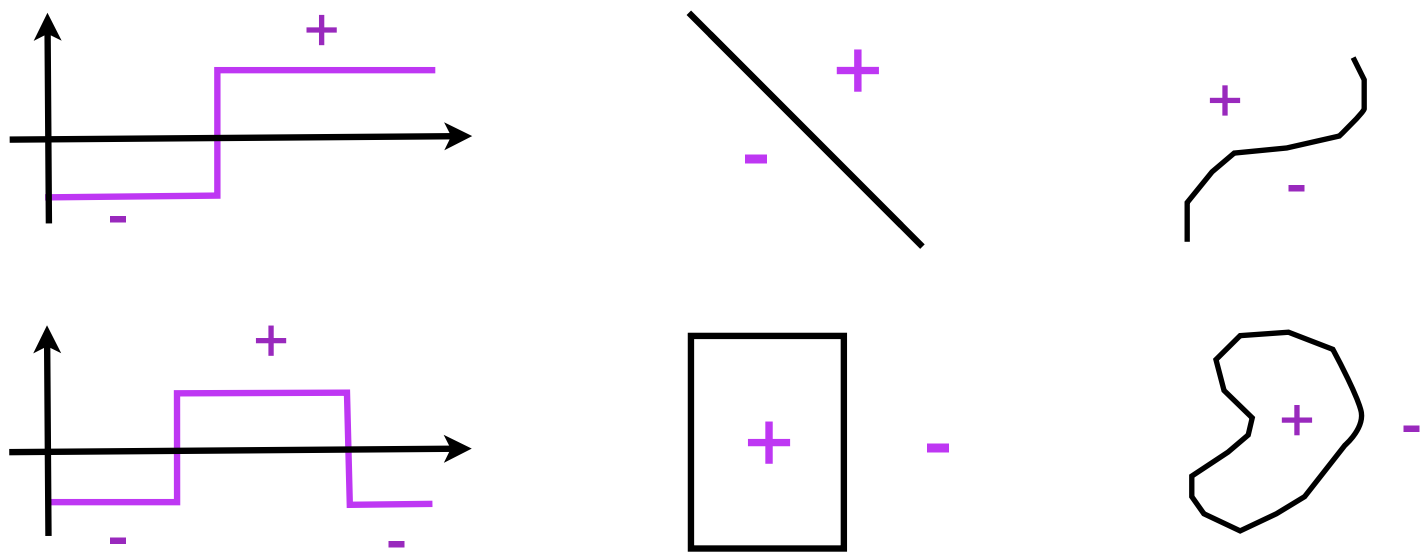

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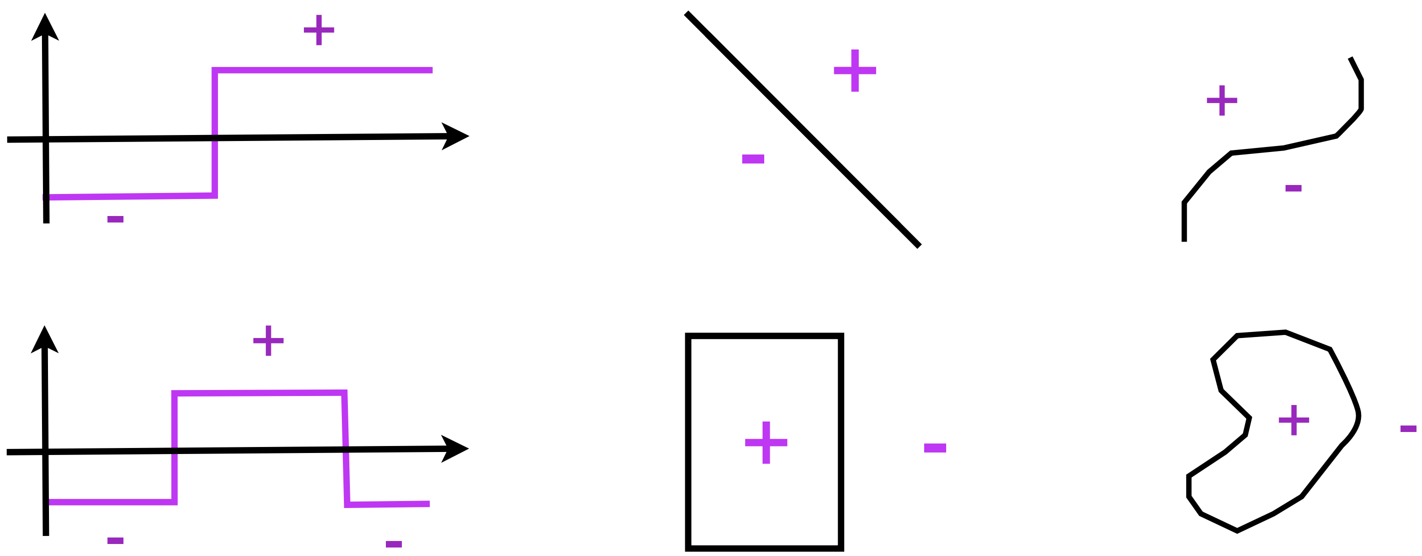

Question: How many queries are required to determine h^*?

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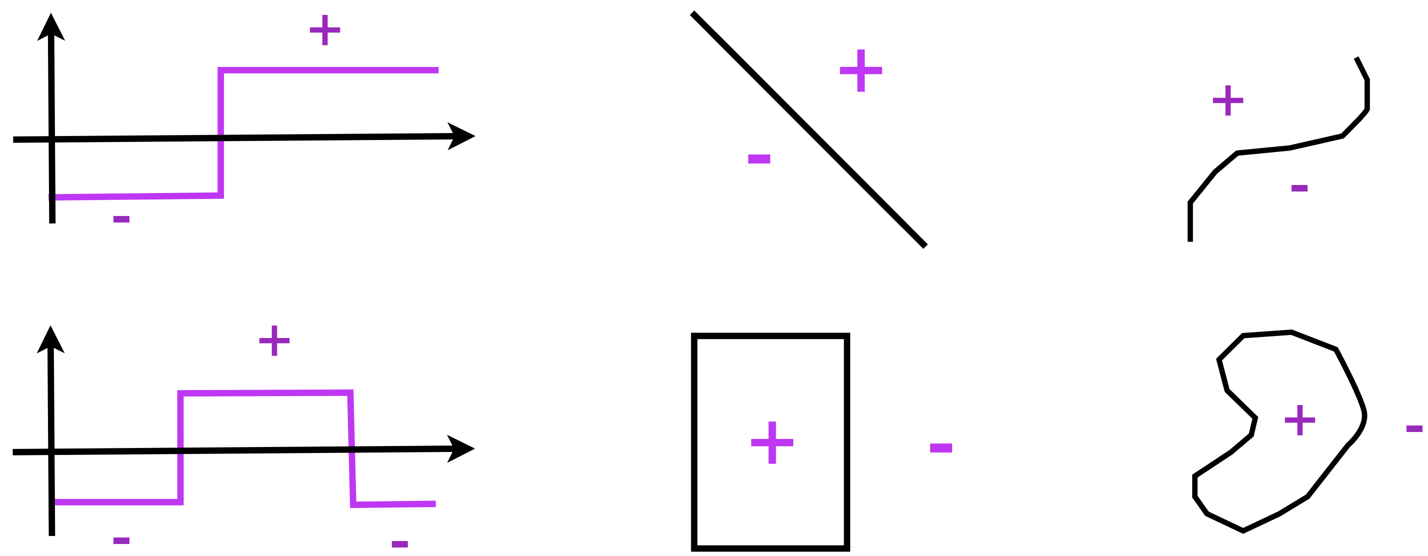

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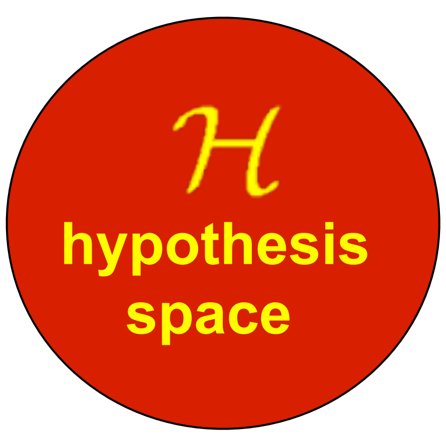
If $\mathcal{H}$ is finite with $N := |\mathcal{H}|$, then identification of h^* requires at least $\log_2 N$ bits/queries.

Generalized Binary Search (aka Splitting Algorithm)

initialize: $n = 0$, $\mathcal{H}_0 = \mathcal{H}$

while $|\mathcal{H}_n| > 1$



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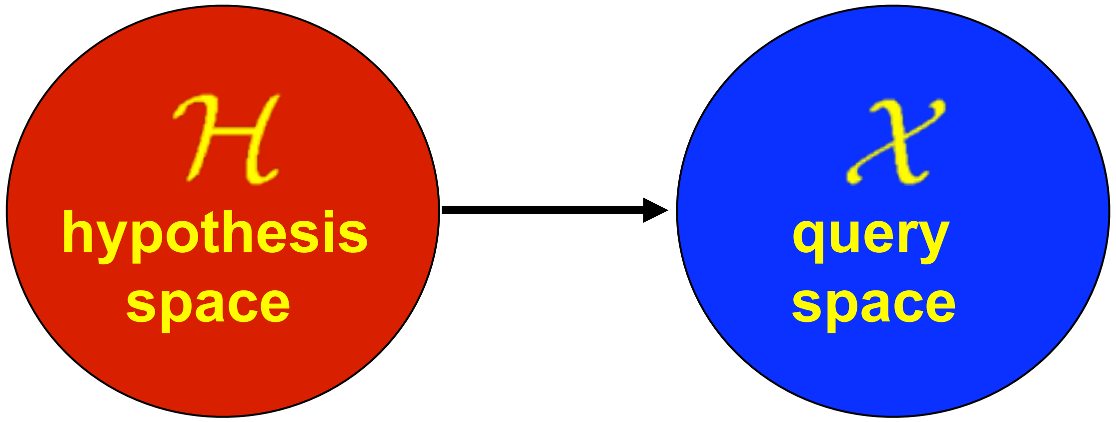

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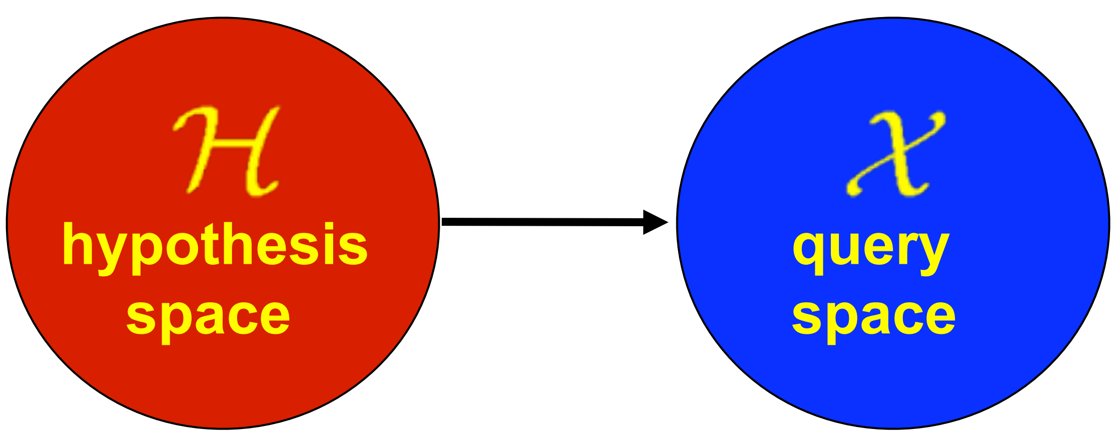
Selects a query for which disagreement among hypotheses is maximal



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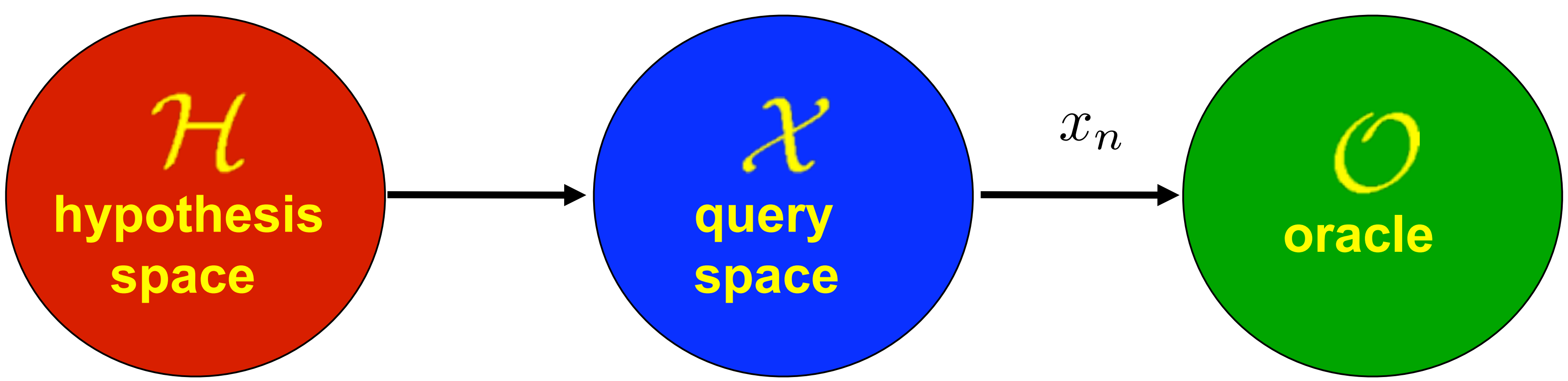

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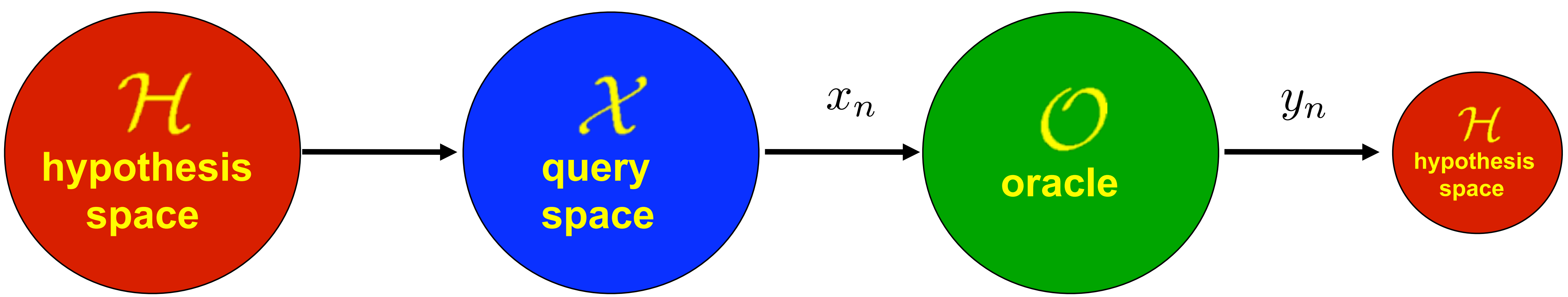
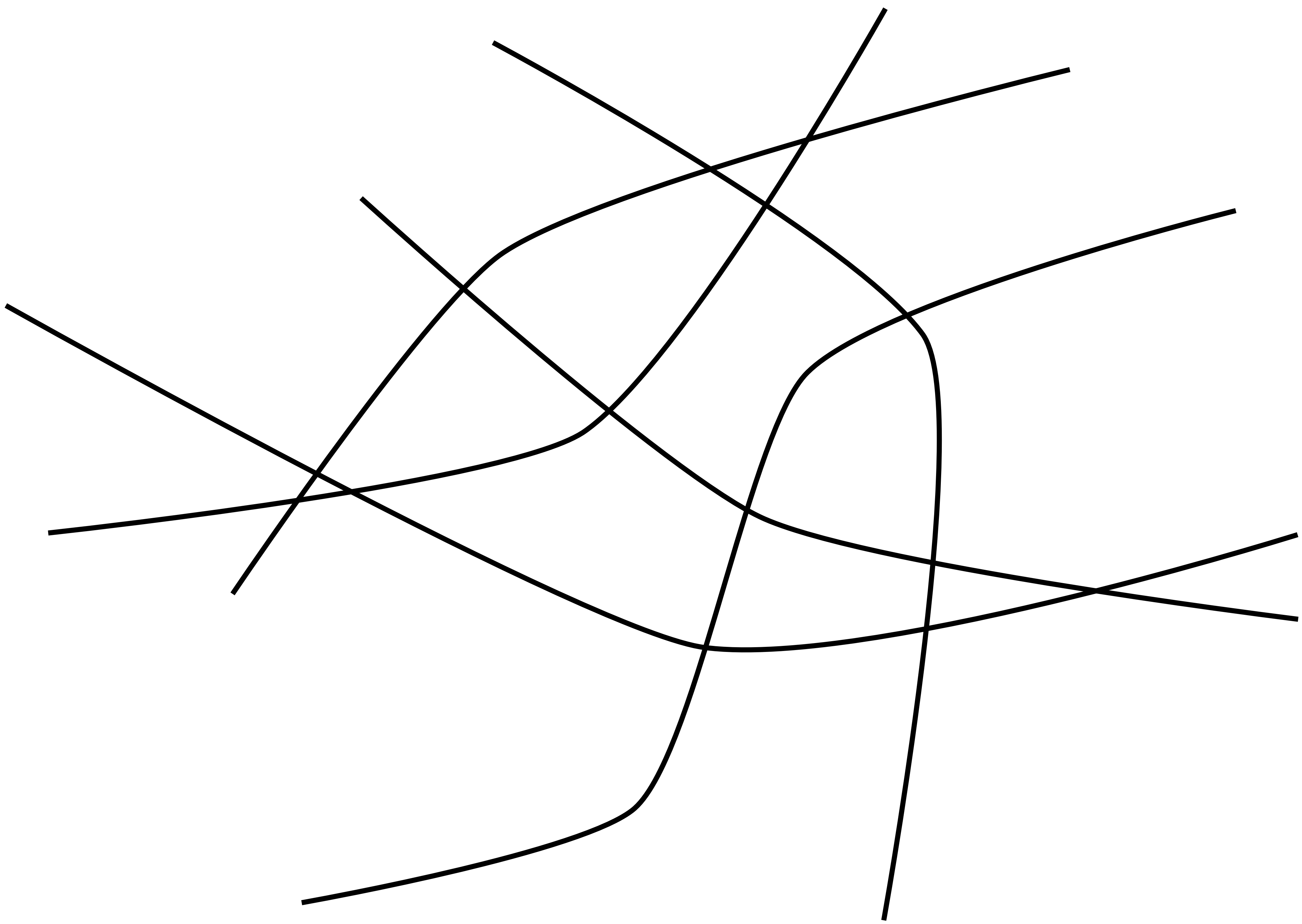


Bisection in Higher Dimensions

Consider the decision boundaries of a collection of classifiers in a multidimensional feature space



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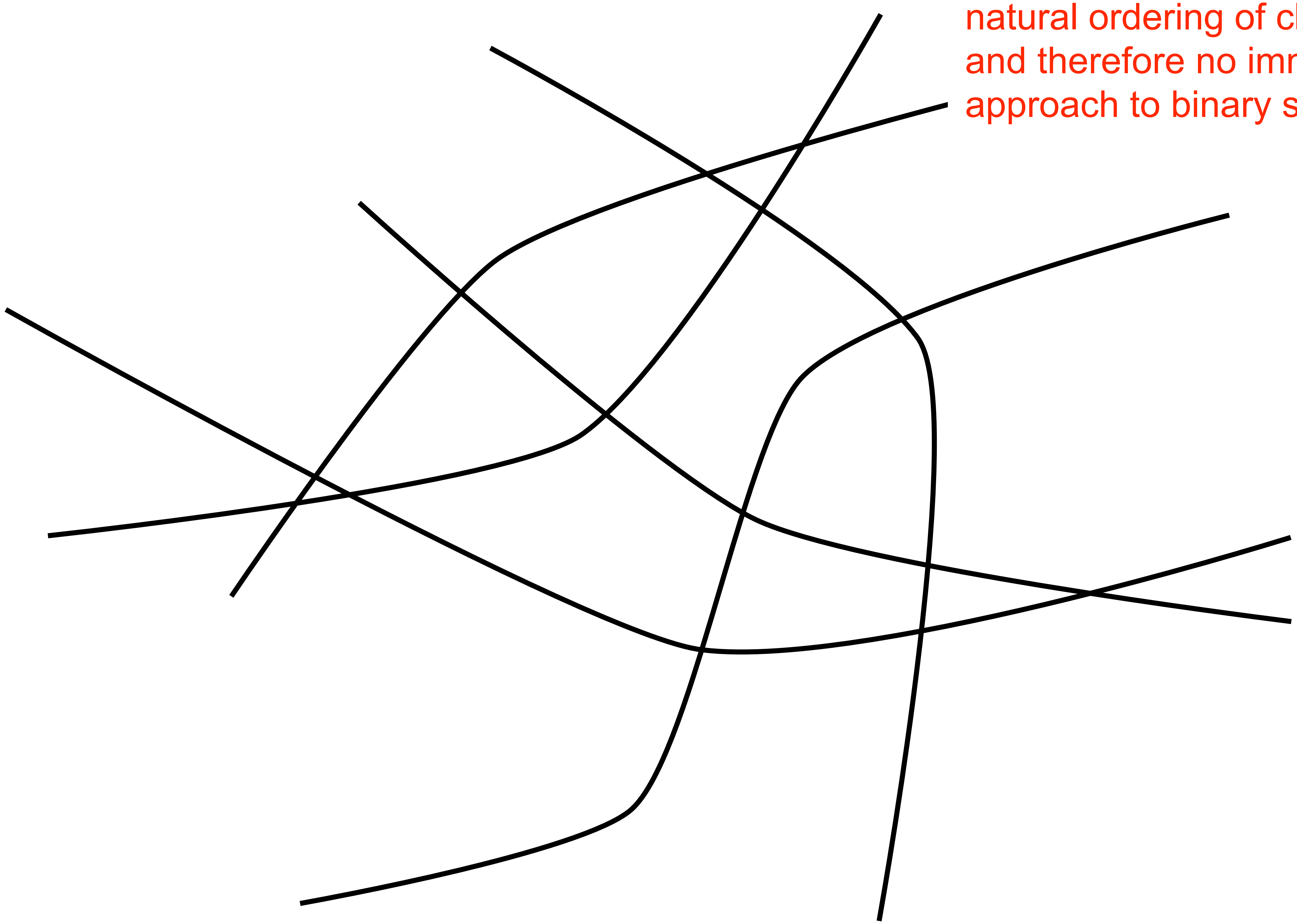
Unlike the situation in 1-d, there is no natural ordering of classifiers/boundaries and therefore no immediately obvious approach to binary search



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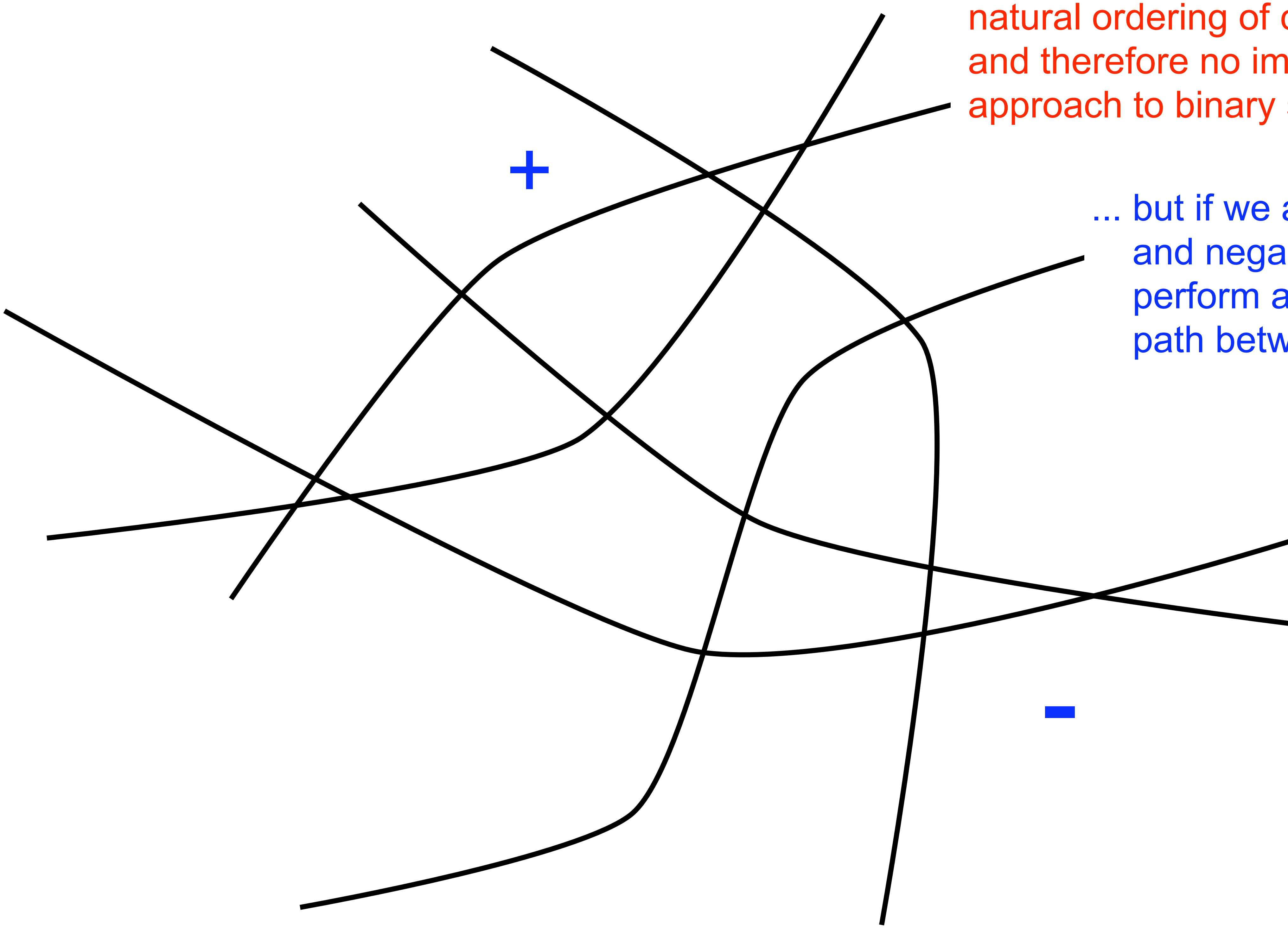

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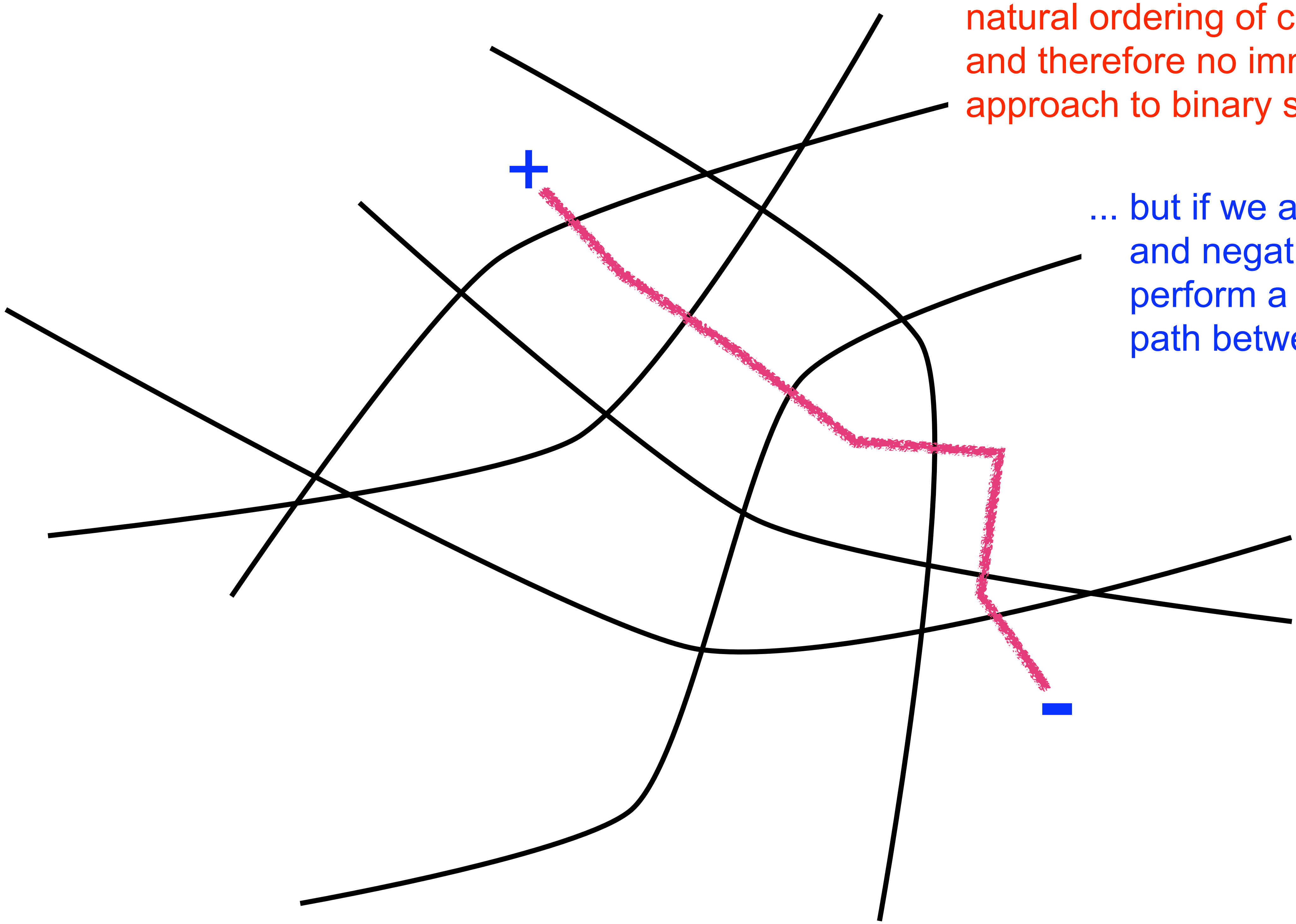

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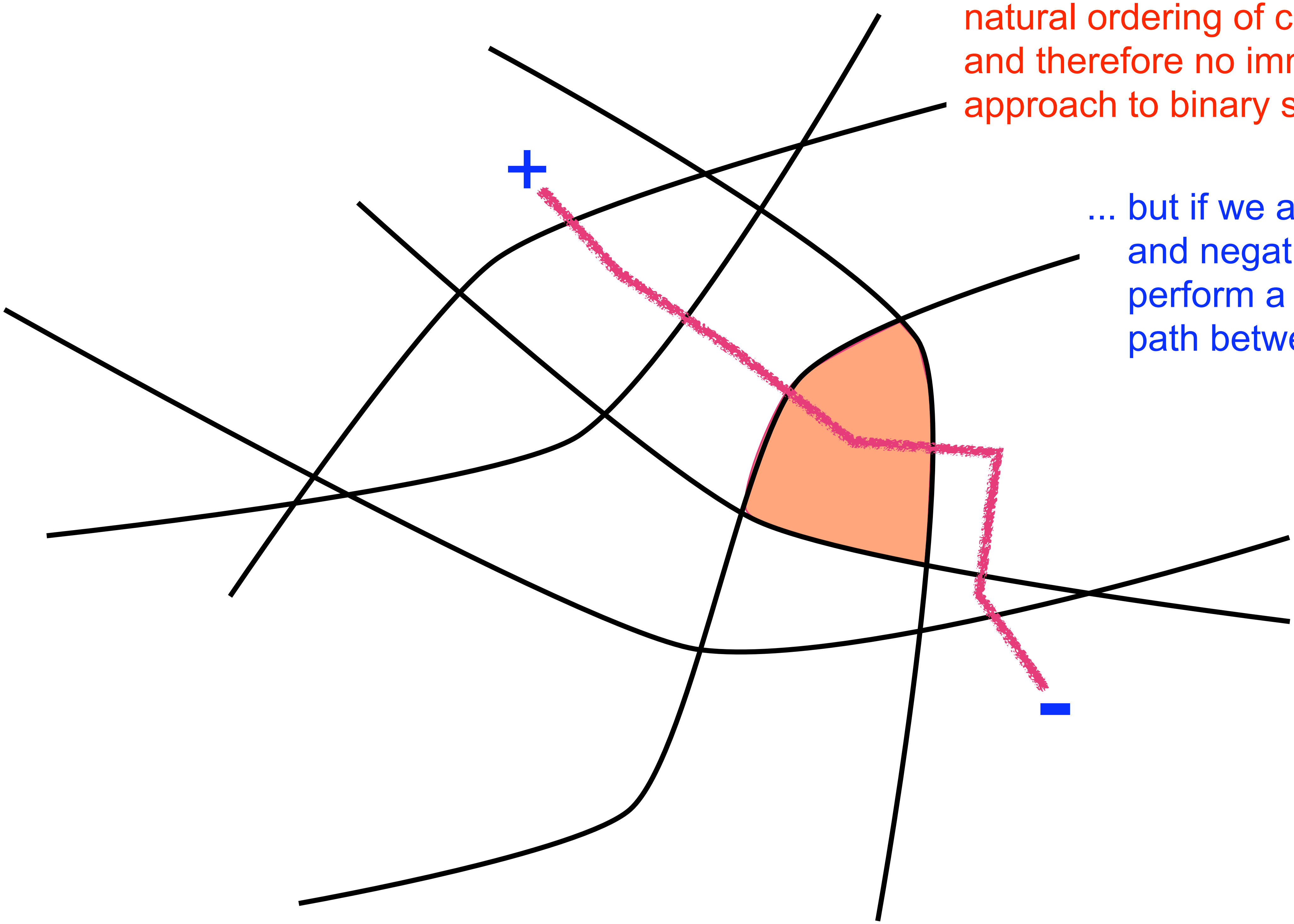

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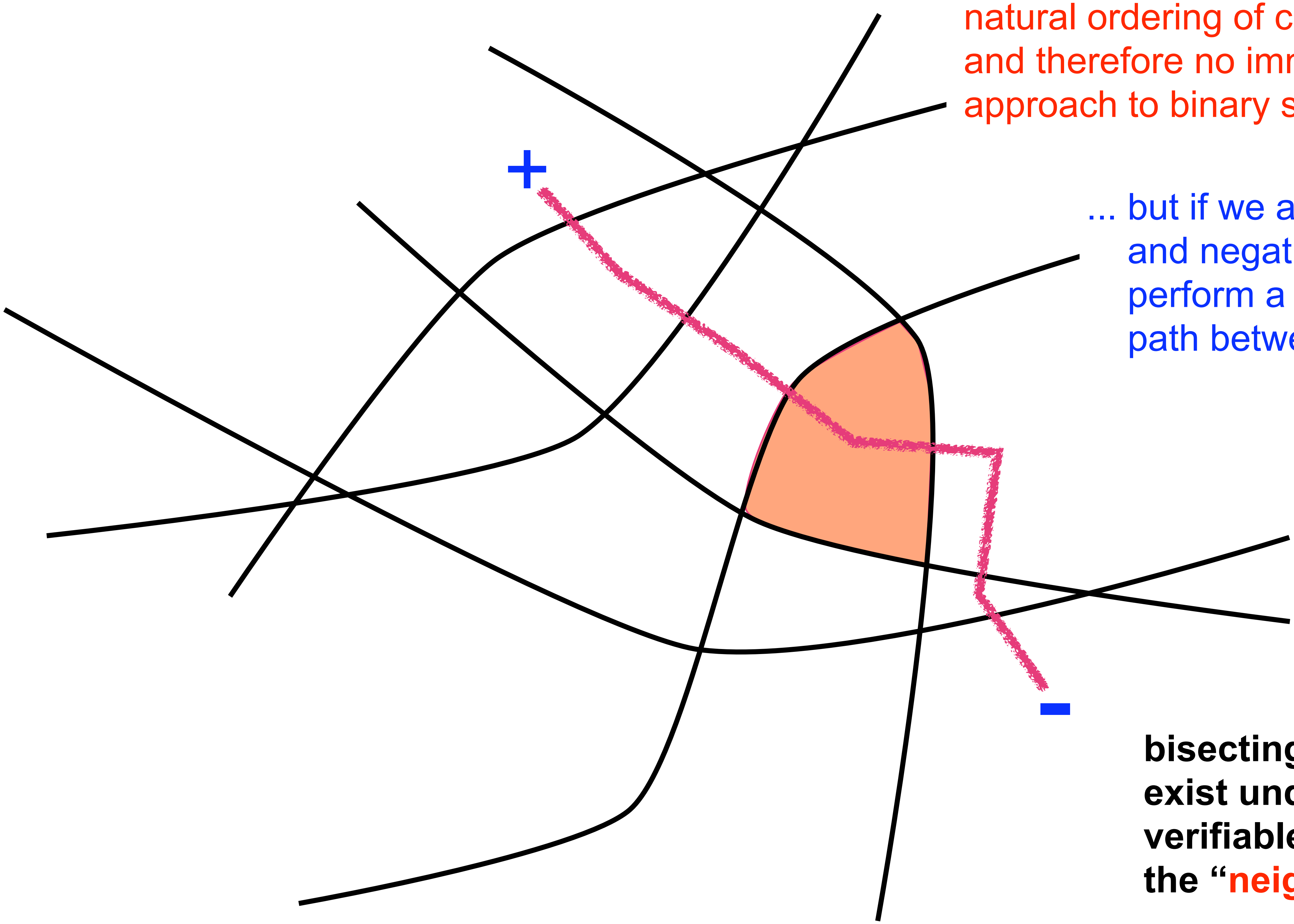

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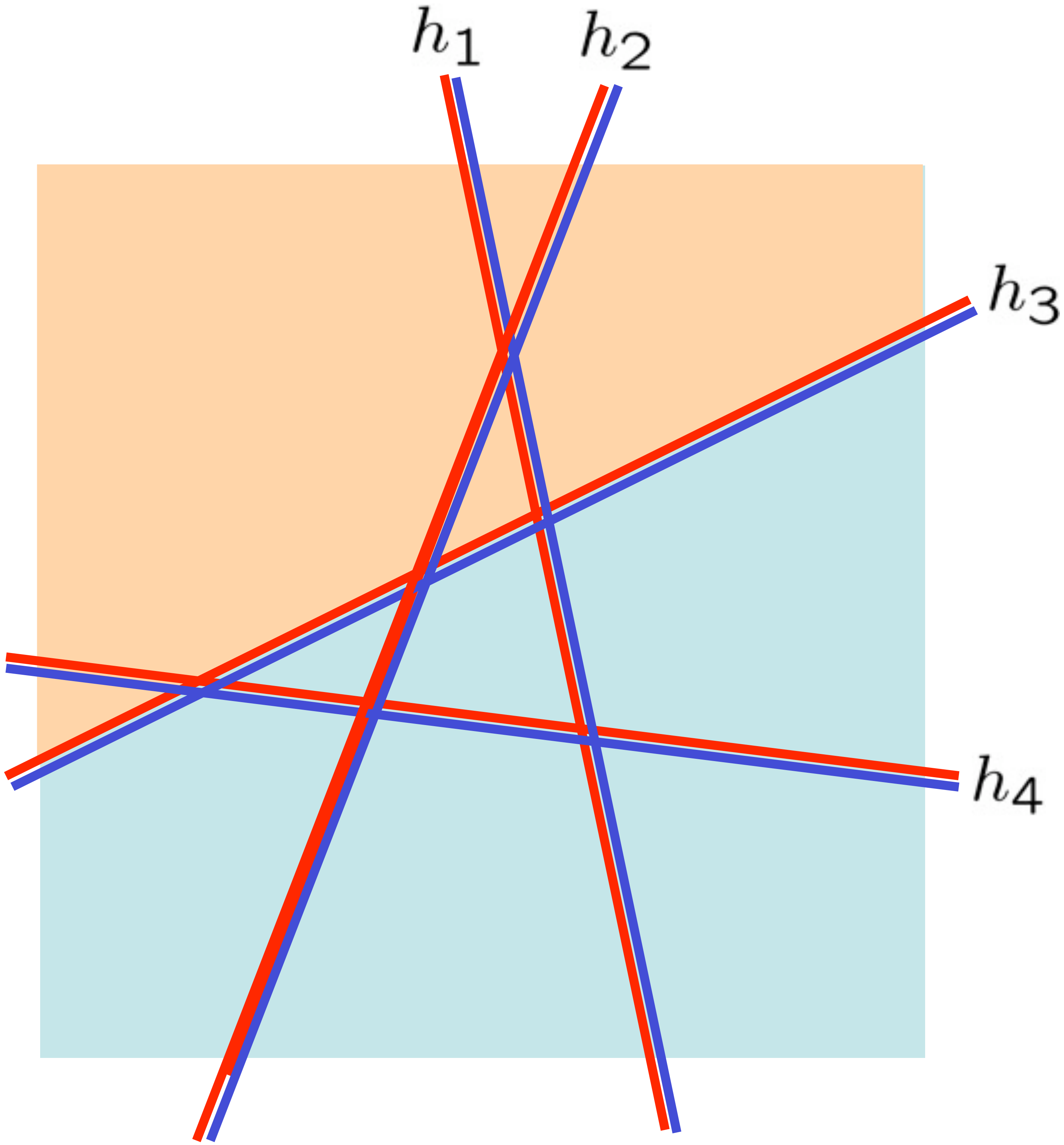
bisecting paths of this sort exist under a mild and verifiable property we call the "neighborly condition"

Learning Halfspaces in $\mathbb{R}^d$



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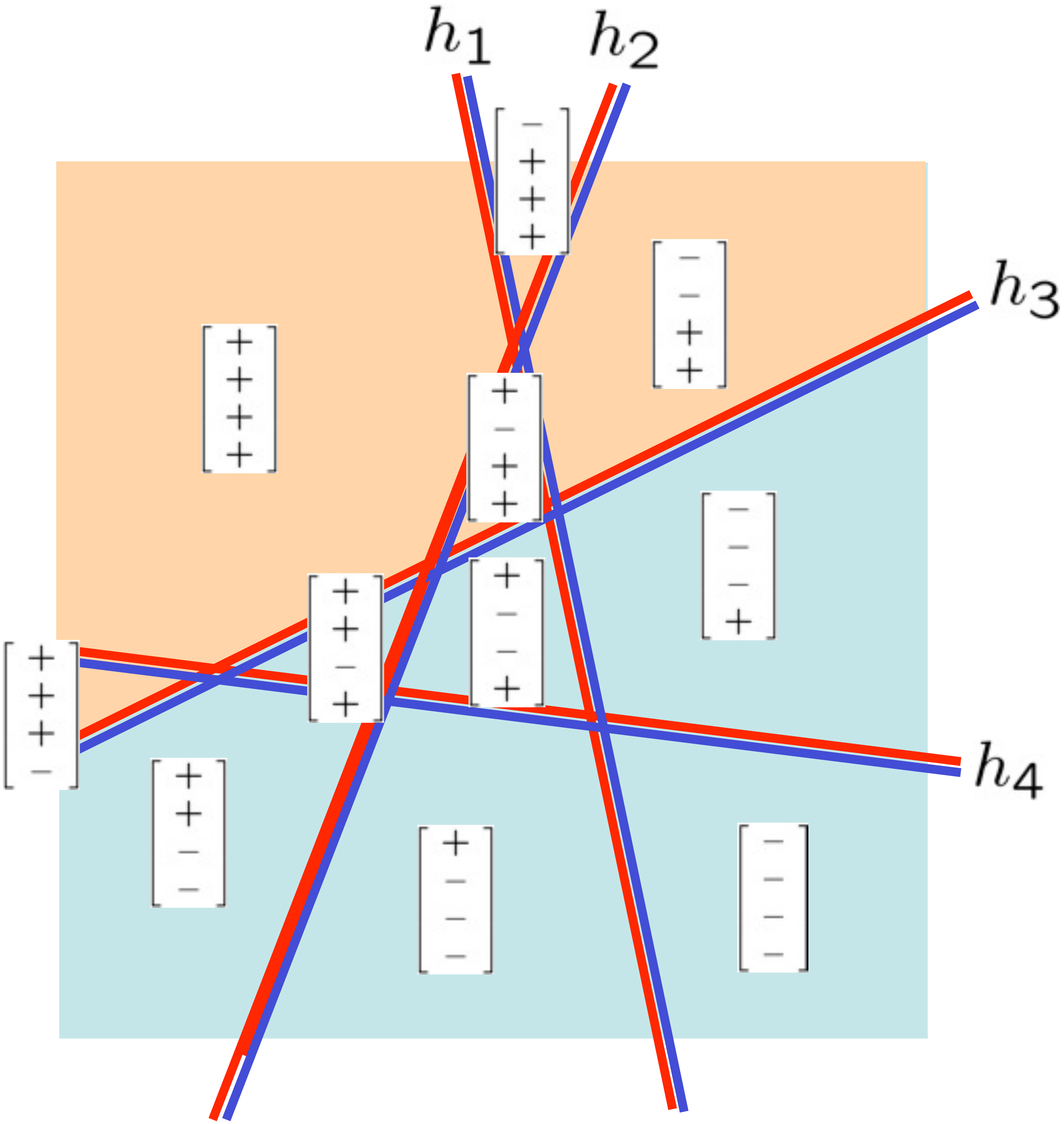

$$
\begin{array}{c}
\\ h_1 \\ h_2 \\ h_3 \\ h_4
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} & A_{11} \end{array} \\
\left[\begin{array}{ccccccccccc}
+ & - & - & - & - & + & + & + & + & + & + \\
+ & + & - & - & - & - & + & + & + & - & - \\
+ & + & + & - & - & - & - & + & - & - & + \\
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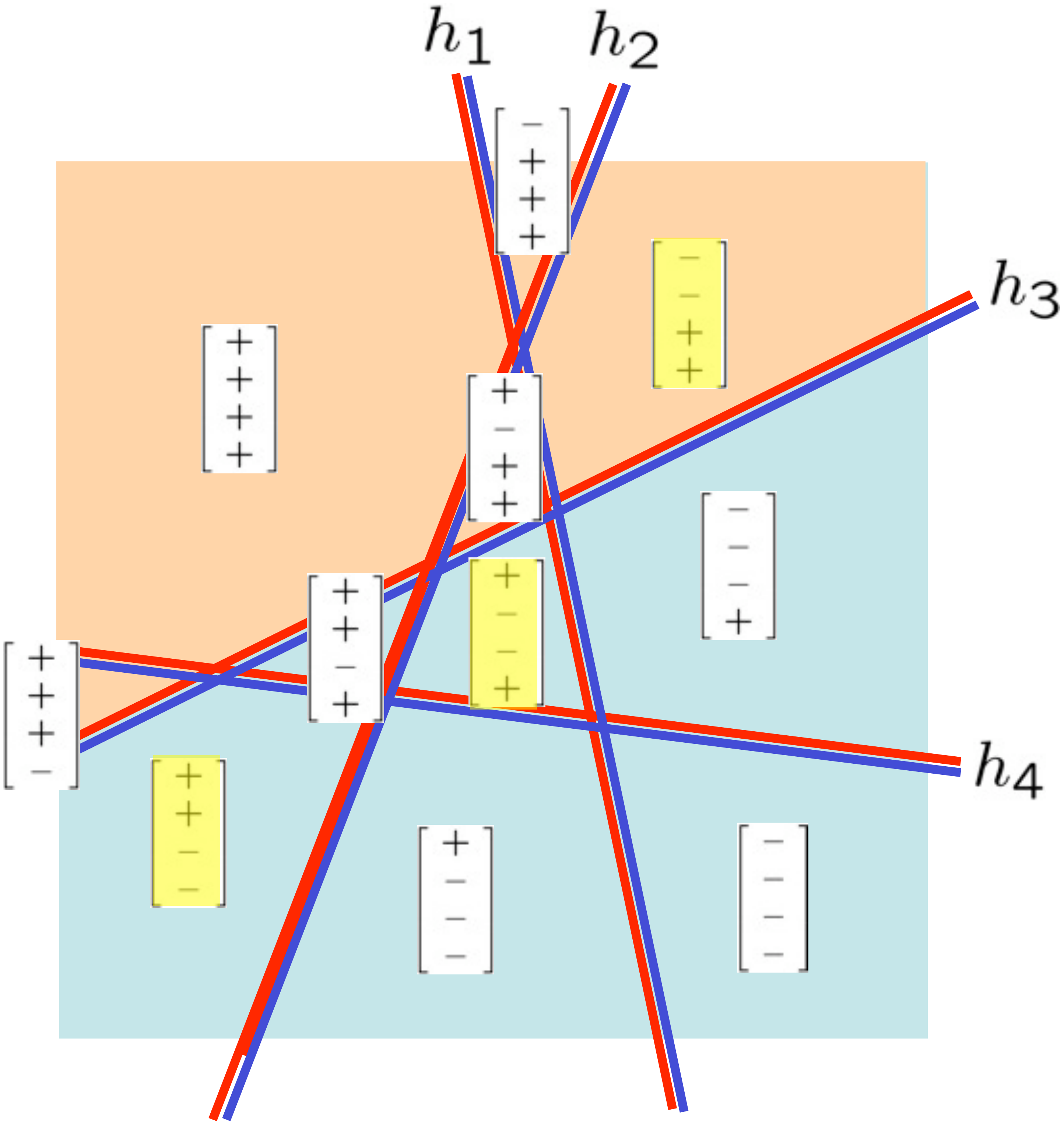

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bisecting queries

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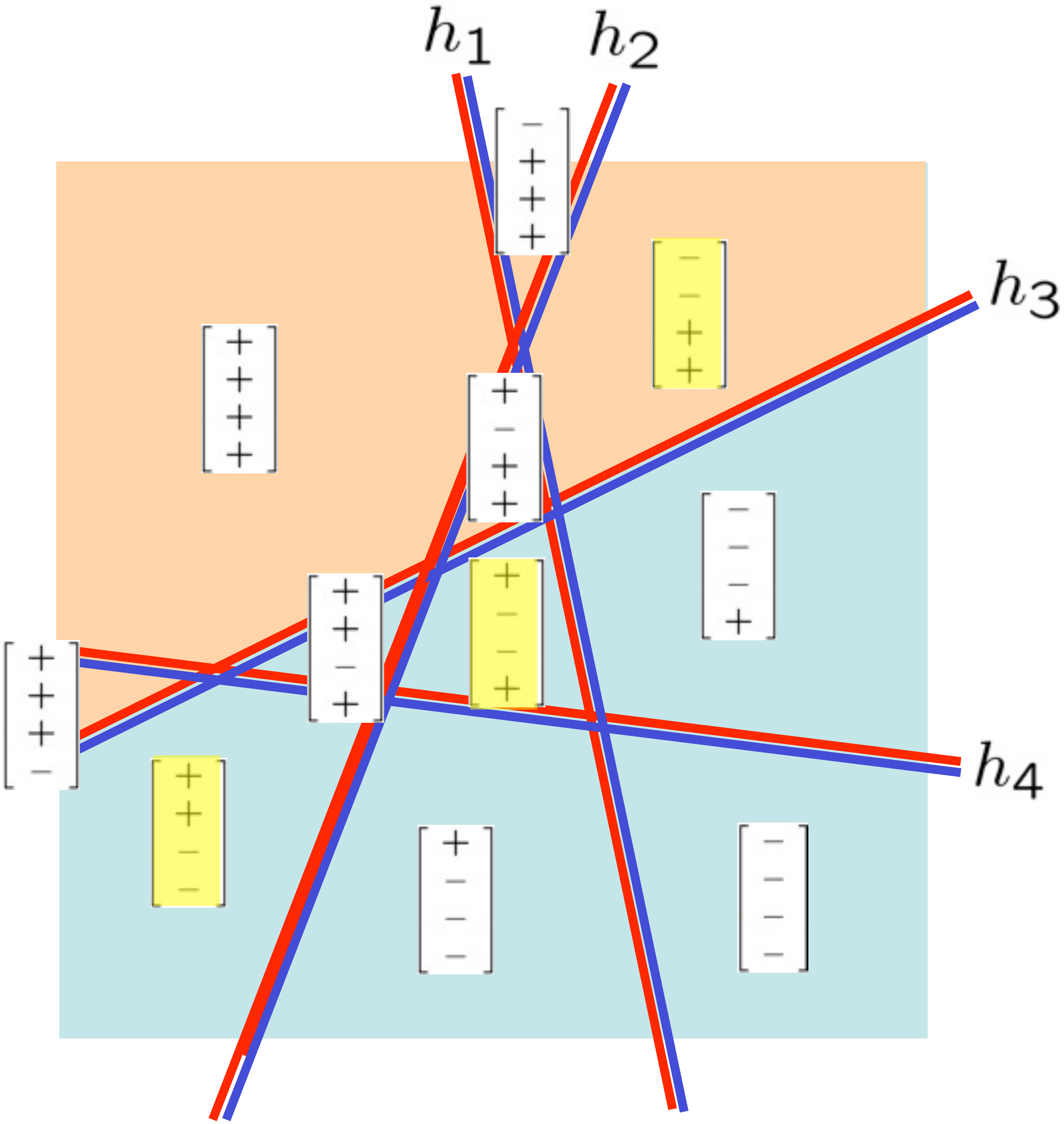

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bisecting queries

queries generate only $O(N^d)$ of the possible 2^N binary patterns!

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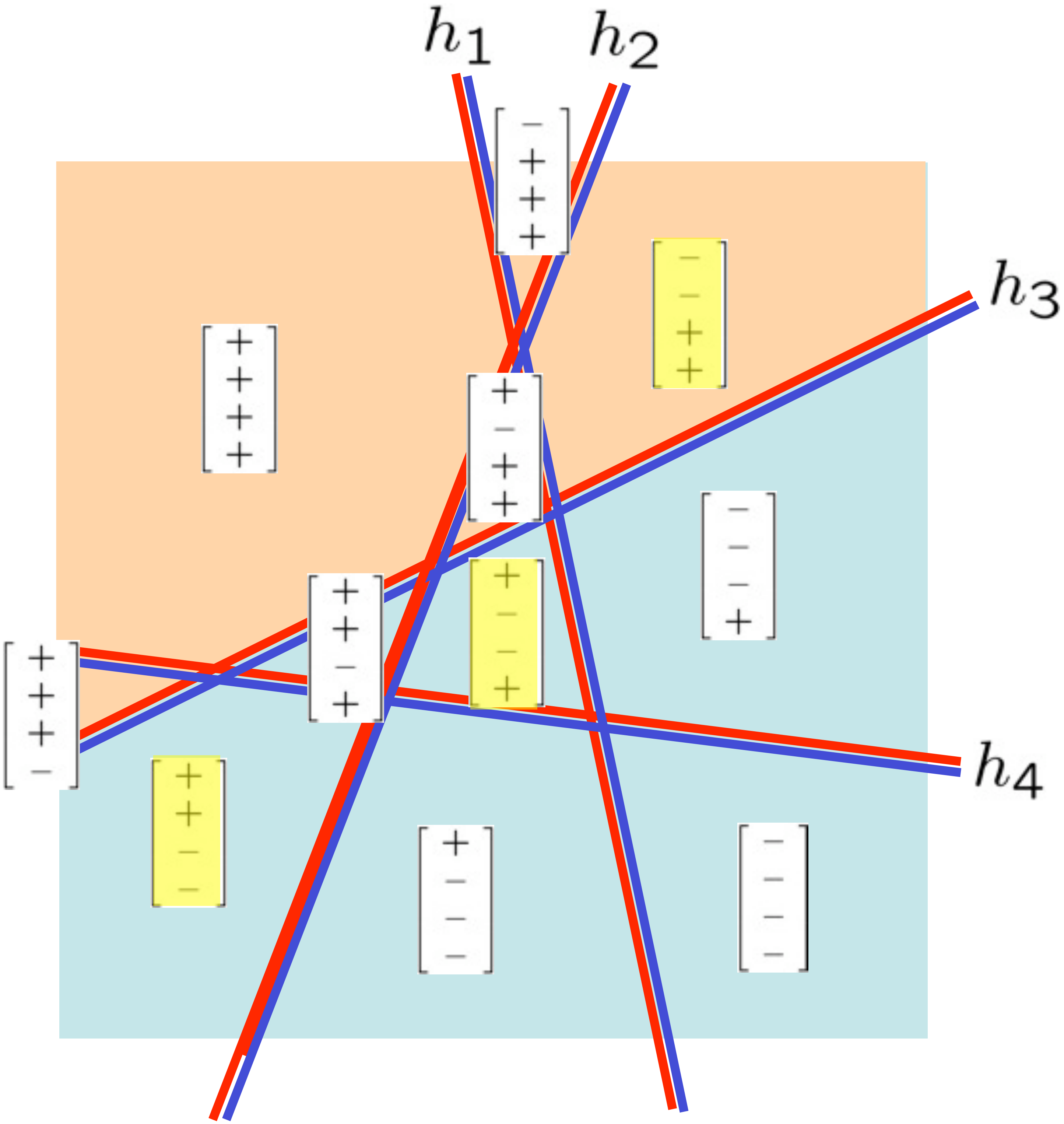

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Can GBS find near-bisecting queries in general?

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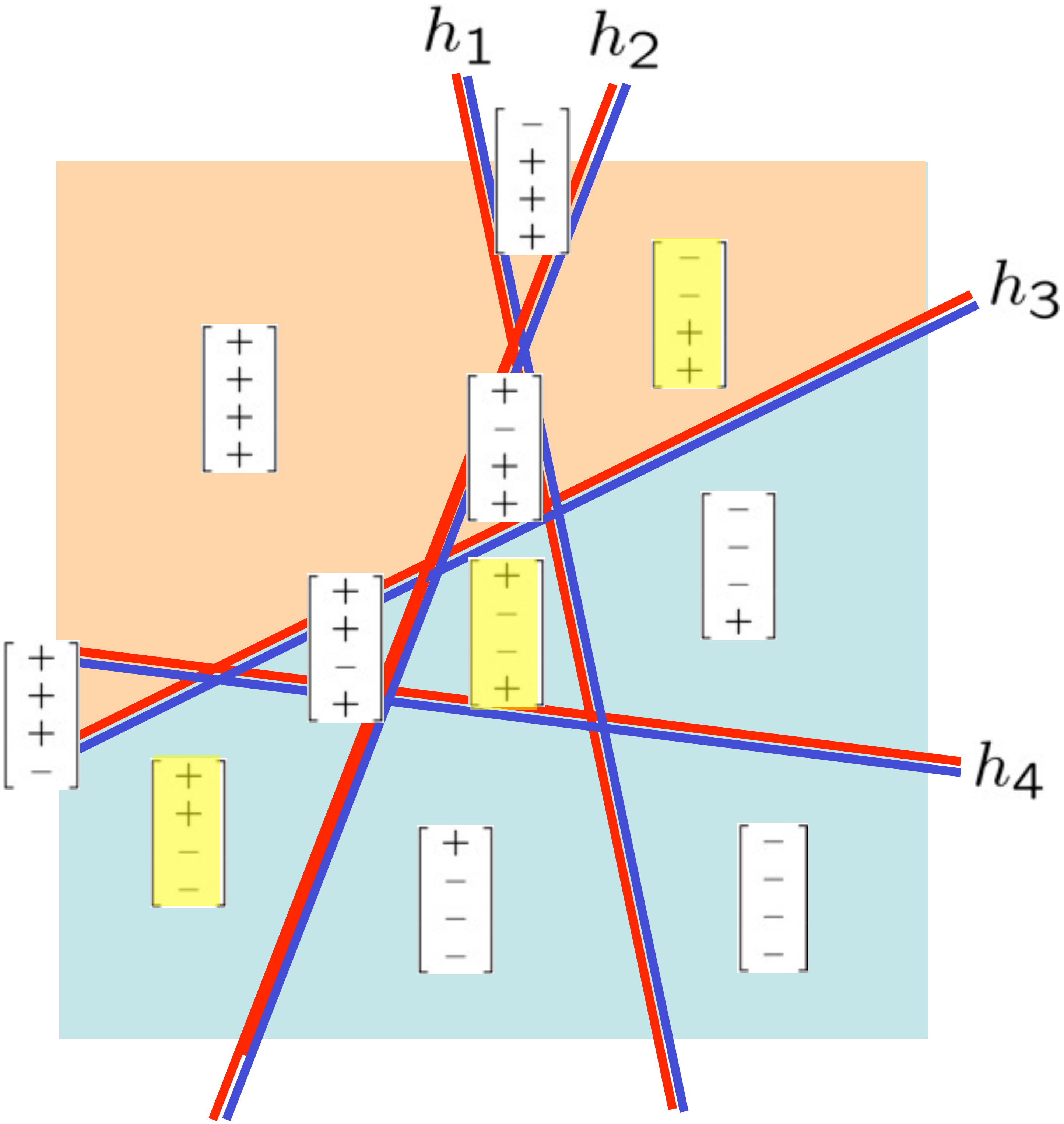

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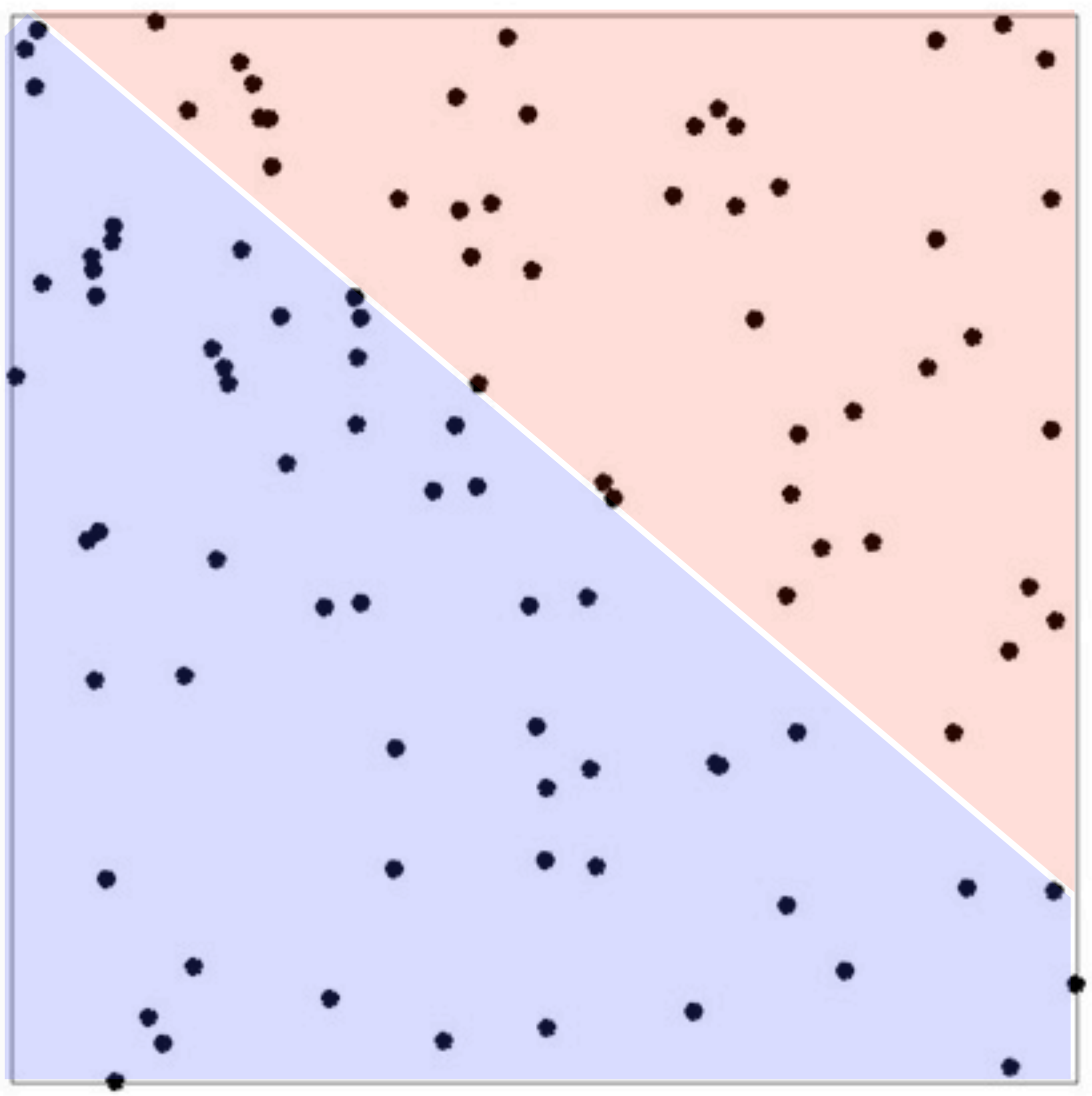
Can GBS find near-bisecting queries in general?

If $\mathcal{H}$ is a collection of N halfspaces on $\mathcal{X} = \mathbb{R}^d$, then GBS terminates with the correct halfspace after $O(\log N)$ queries.

Example

Suppose we have a sensor network observing a binary activation pattern with a linear boundary. How many sensors must be queried to determine the pattern?



100 sensors, 9900 possible linear boundaries

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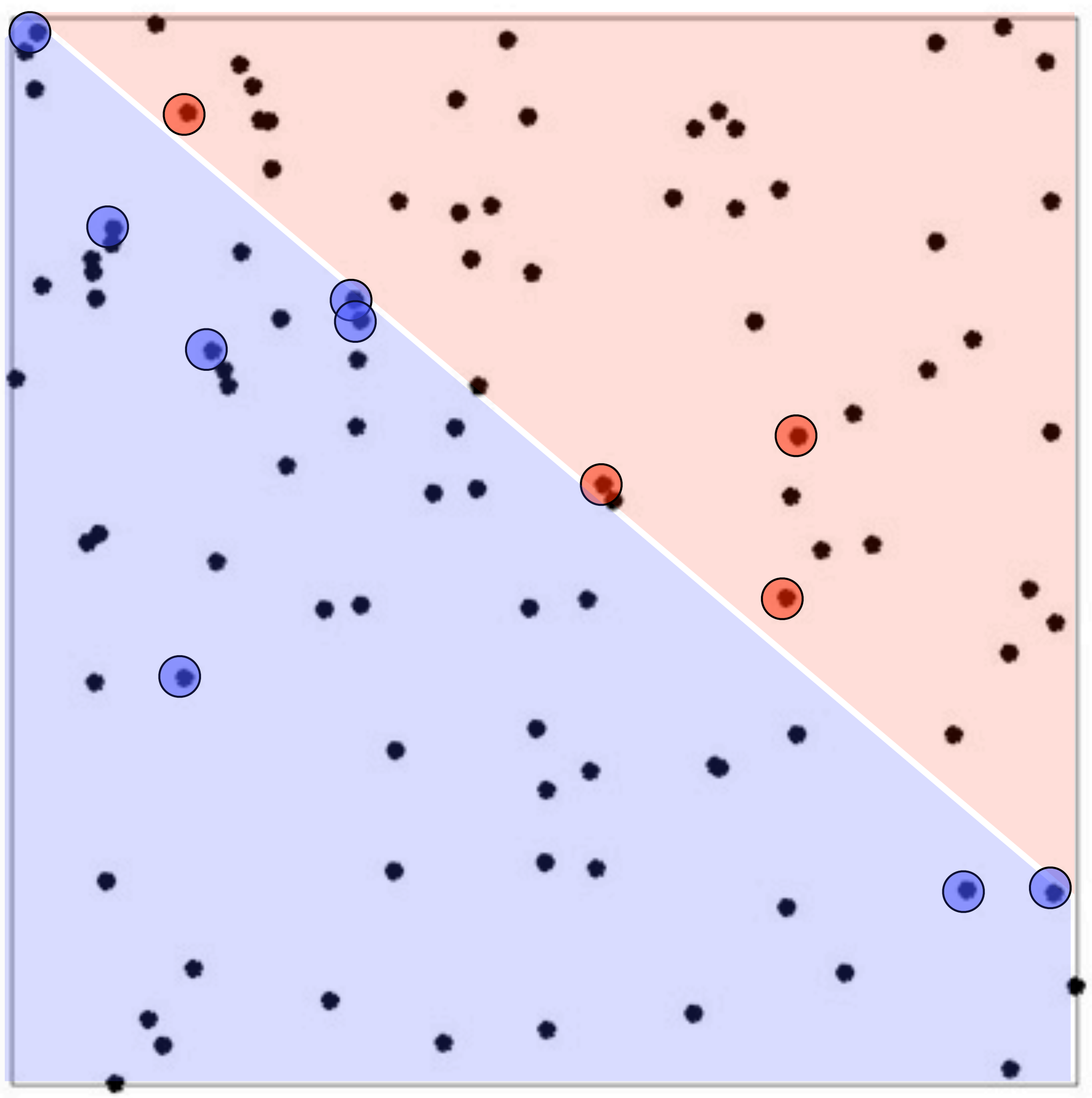

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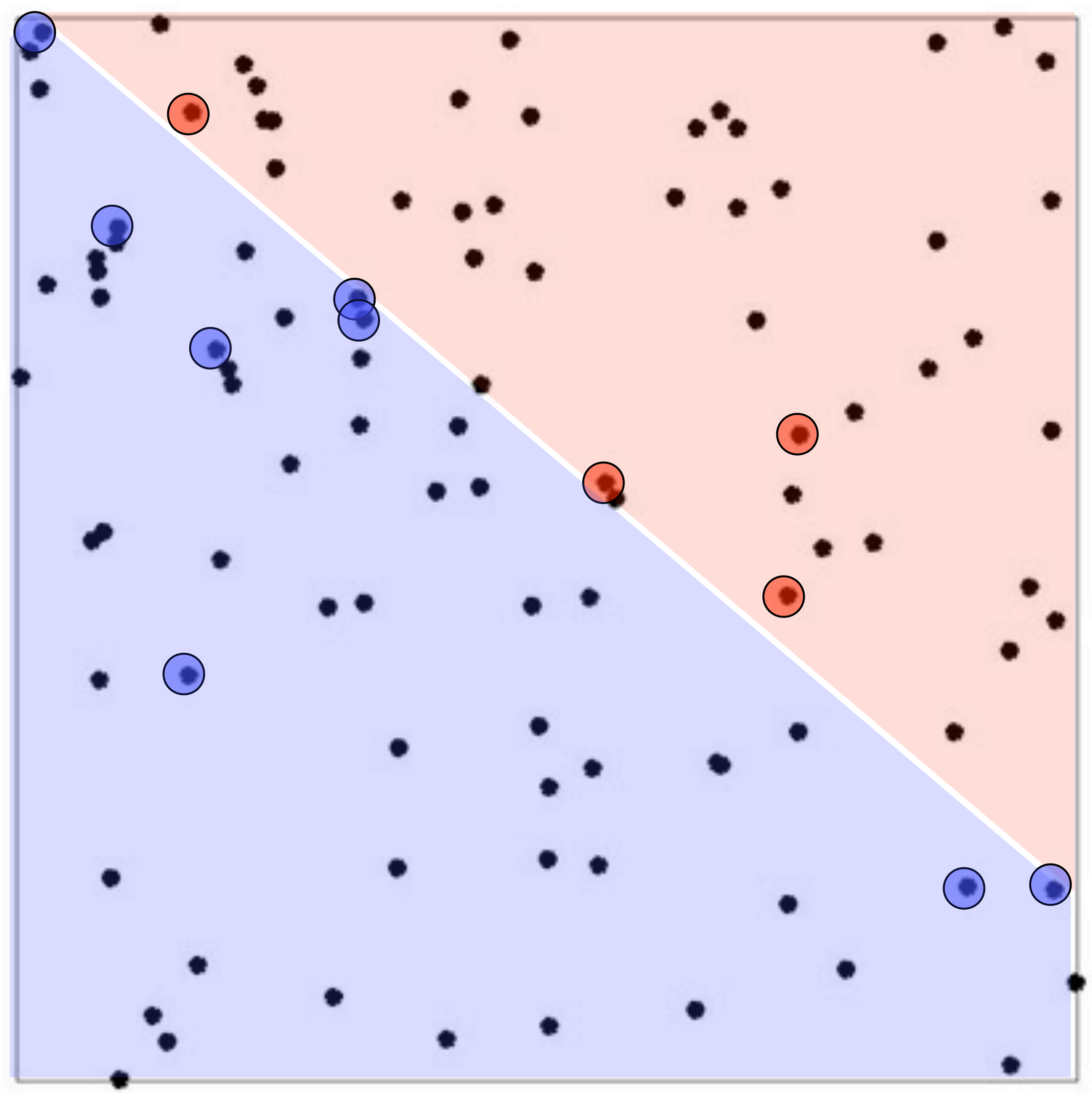
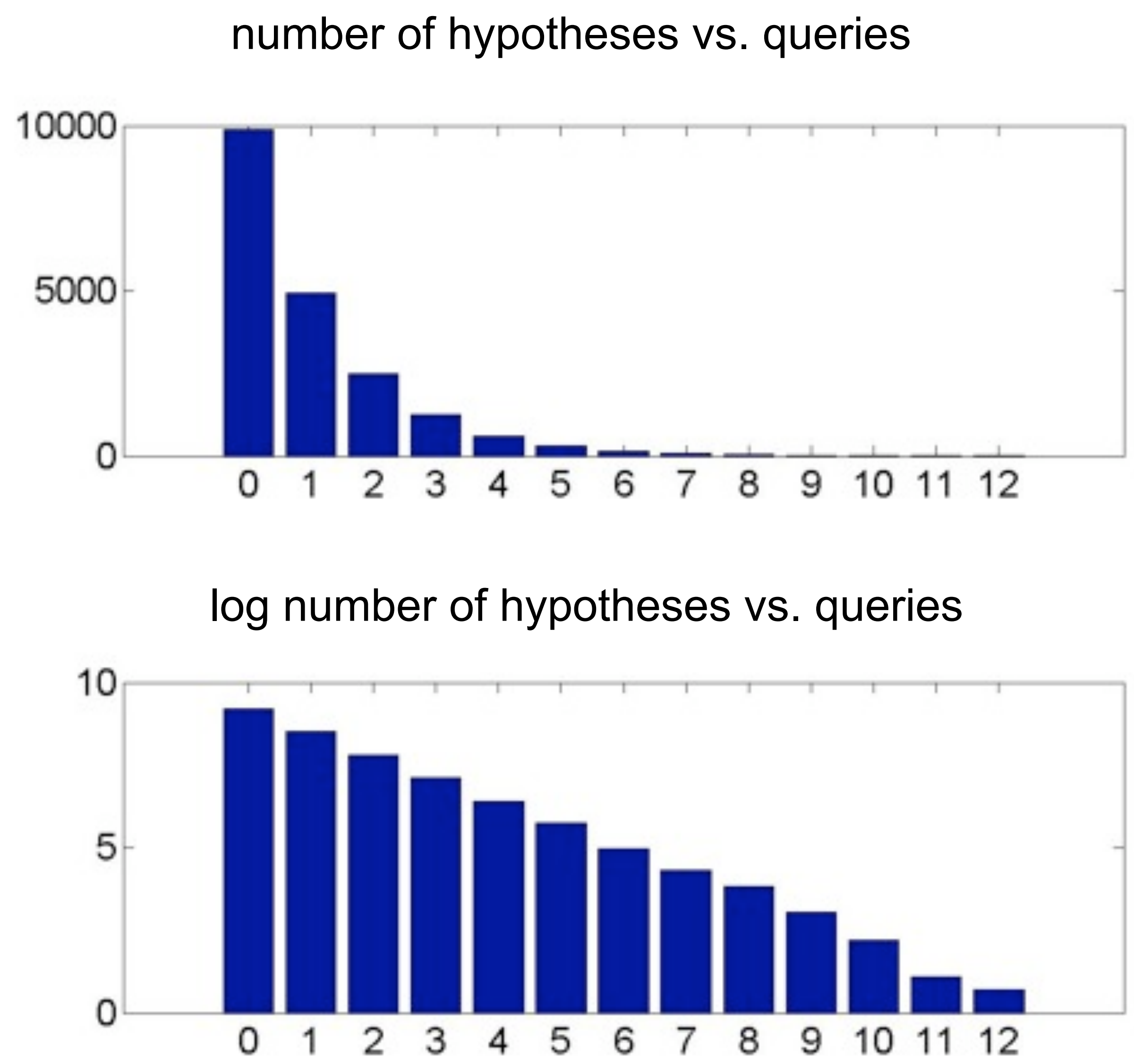


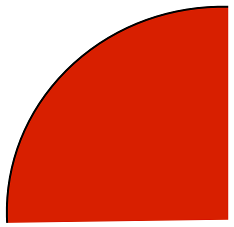


Correct boundary determined after querying 12 sensors

$\mathcal{H}$

hypothesis space

“Is the person wearing a hat ?”



“Is the person wearing a hat ?”

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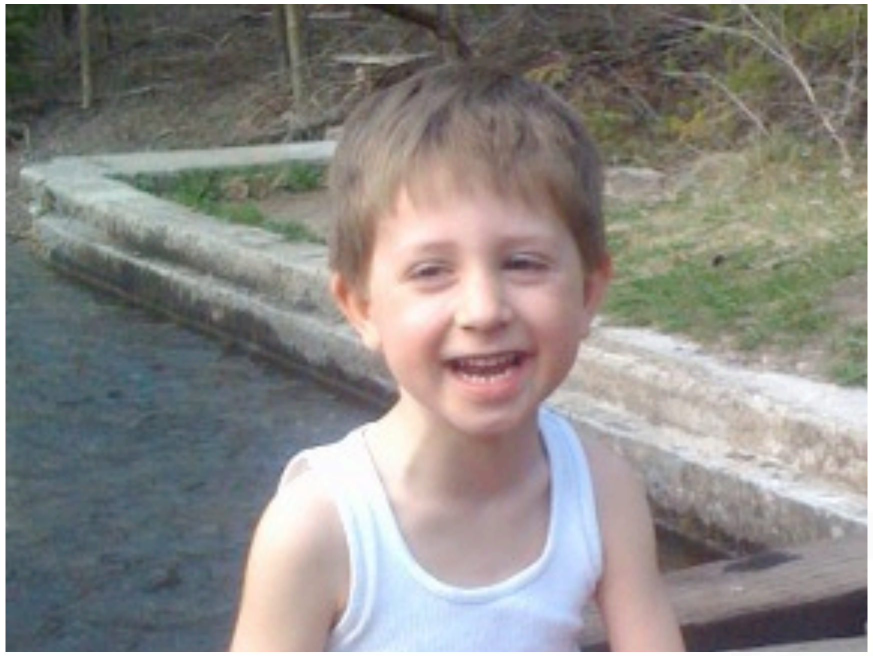
GBS is quite effective if responses are reliable



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Generalized Binary Search with Noise

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The *noise bound* is defined as $\alpha := \sup_{x \in \mathcal{X}} \mathbb{P}(Y \neq h^*(x))$

Generalized Binary Search with Noise

Noise-tolerant GBS

initialize: p_0 uniform over $\mathcal{H}$ and $\alpha < \beta < 1/2$.

for $n = 0, 1, 2, \ldots$

1) $x_n = \arg\min_{x \in \mathcal{X}} |\sum_{h \in \mathcal{H}} p_n(h) h(x)|$

2) Obtain noisy response y_n

3) Bayes update: $\forall\, h$

$$p_{n+1}(h) \;\propto\; p_n(h) \times \begin{cases} 1-\beta & , h(x_n) = y_n \\ \beta & , h(x_n) \neq y_n \end{cases}$$

hypothesis selected at each step:

$\widehat{h}_n := \arg\max_{h \in H} p_n(h)$

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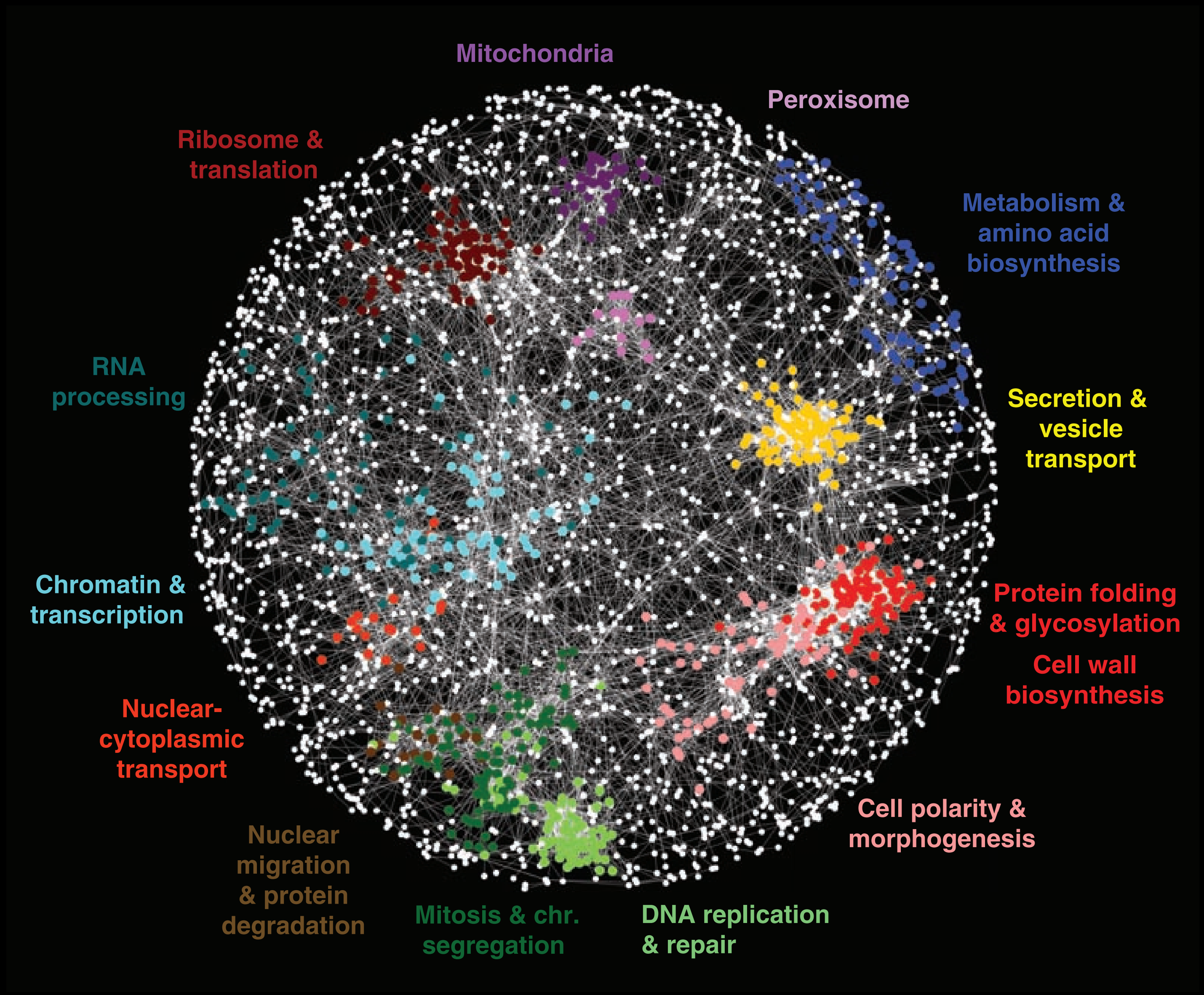
If we desire $\mathbb{P}(\widehat{h}_n \neq h^*) < \delta$, then we require only $n = \frac{1}{\lambda} \log \frac{N}{\delta}$ queries.

Active Clustering

Clustering in Large-Scale Networked Systems

Difficult or impossible to measure/observe everything in large systems

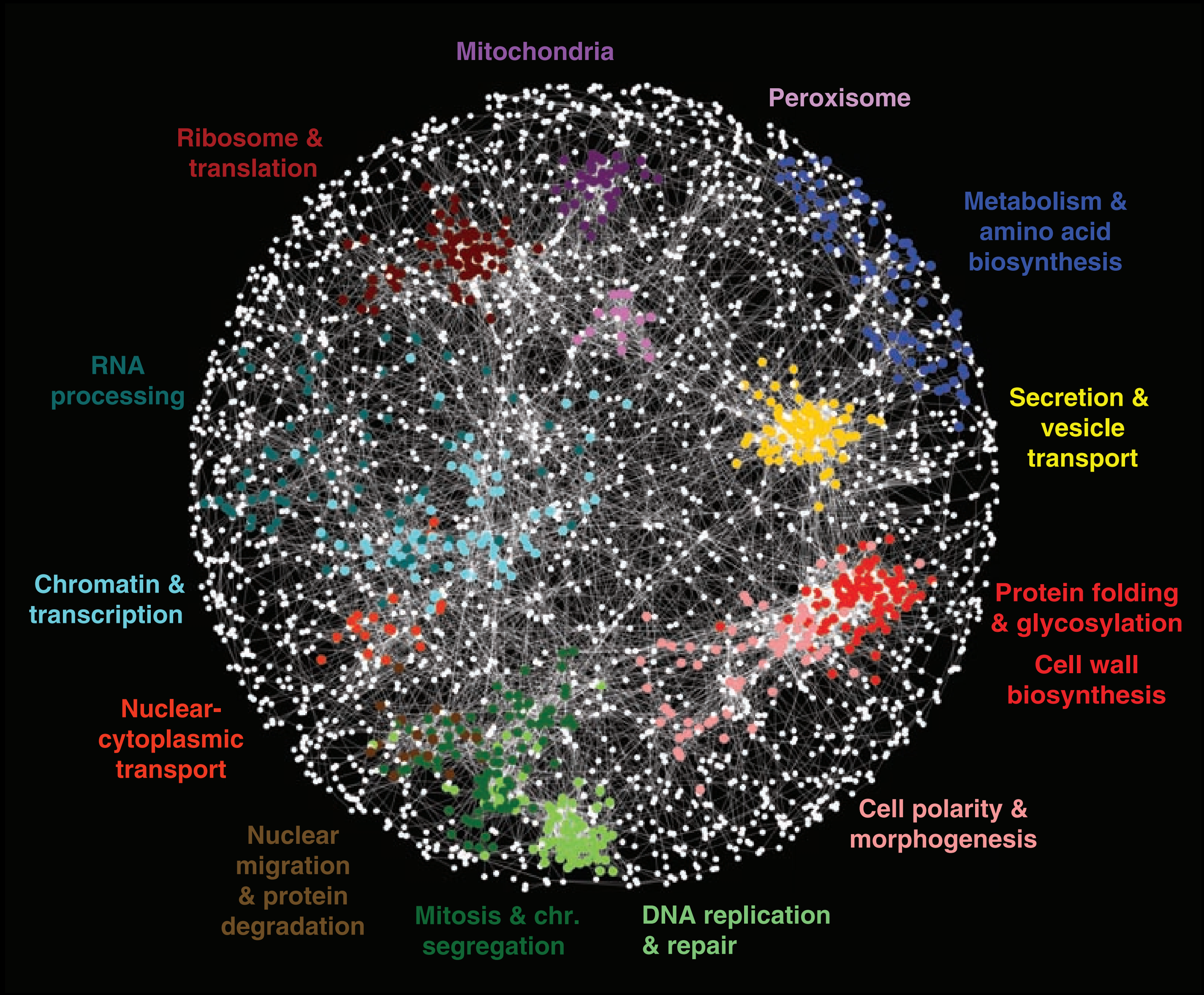
Clustering in Large-Scale Networked Systems



Genetic Landscape of a Cell
Boone Lab - Toronto

Difficult or impossible to measure/observe everything in large systems

Clustering in Large-Scale Networked Systems



Genetic Landscape of a Cell
Boone Lab - Toronto

State of the Internet
Akamai

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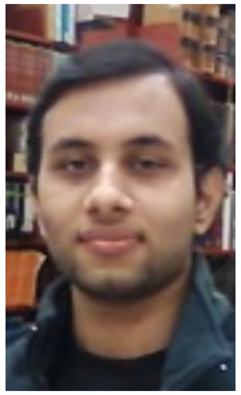
Network Structure and Clustering

Complex systems are not defined by the independent functions of individual components, rather they depend on the orchestrated interactions of these elements.



Gautam Dasarathy

Brian Eriksson

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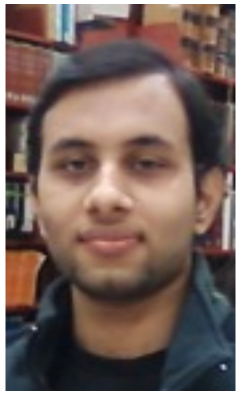
Network(s) of interactions can be revealed via **clustering** based on measured features



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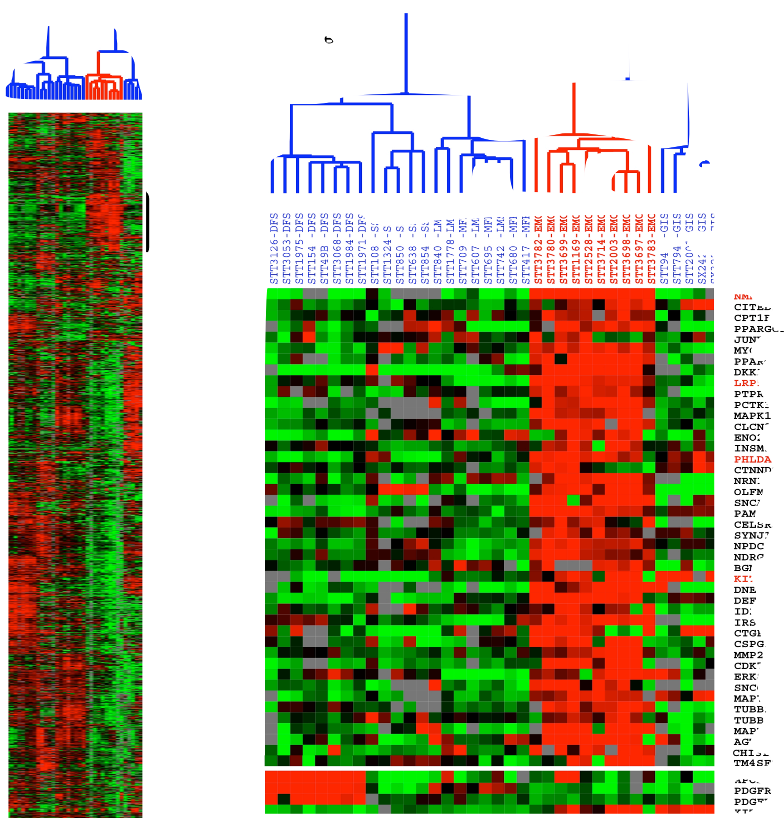
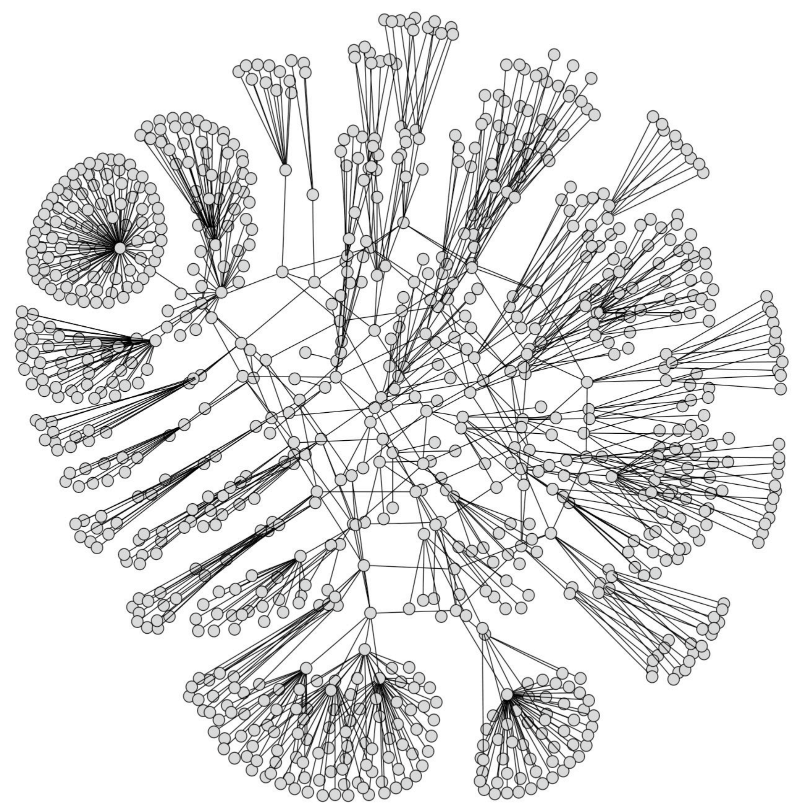
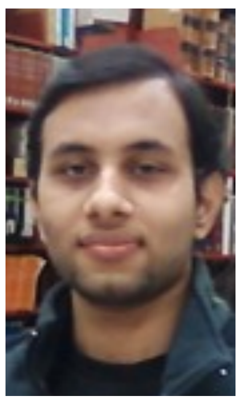


genes and expression/ interaction profiles



network routers and traffic/distance profiles



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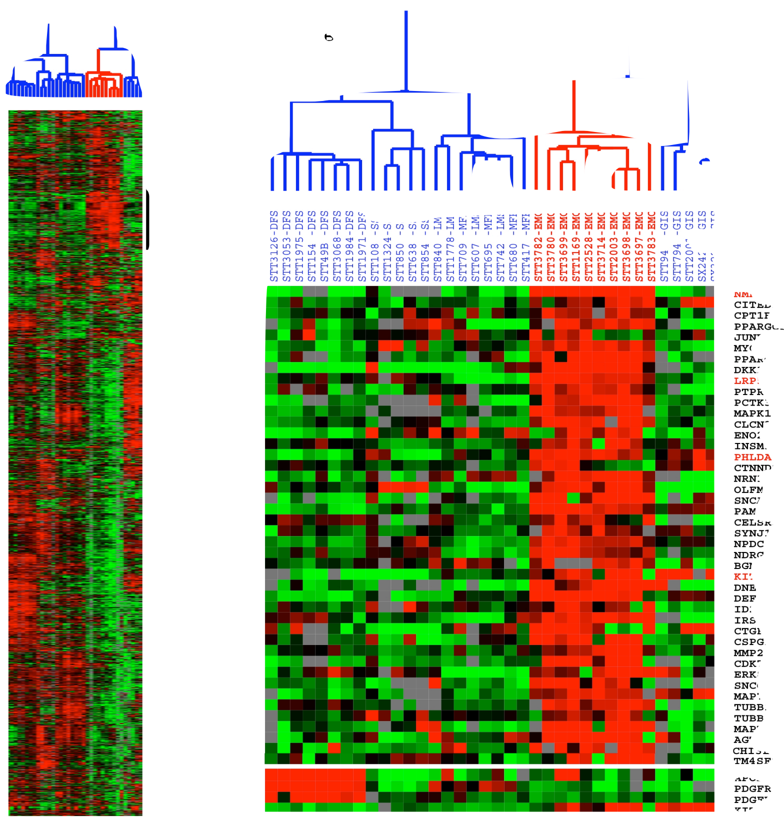

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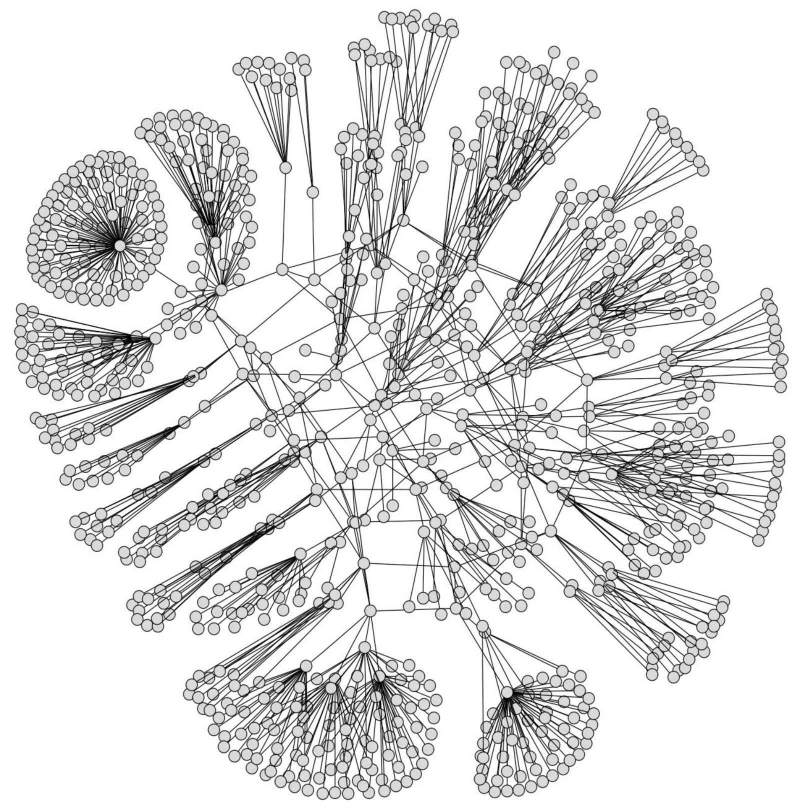

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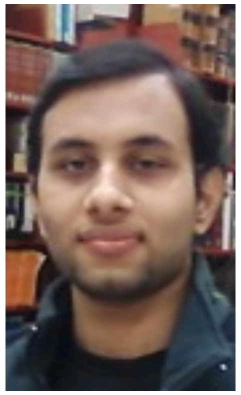

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Similarity-Based Clustering: Each component (gene/router) has an associated feature (measurement profile). Components can be clustered based on feature similarities.

Internet Topology Inference

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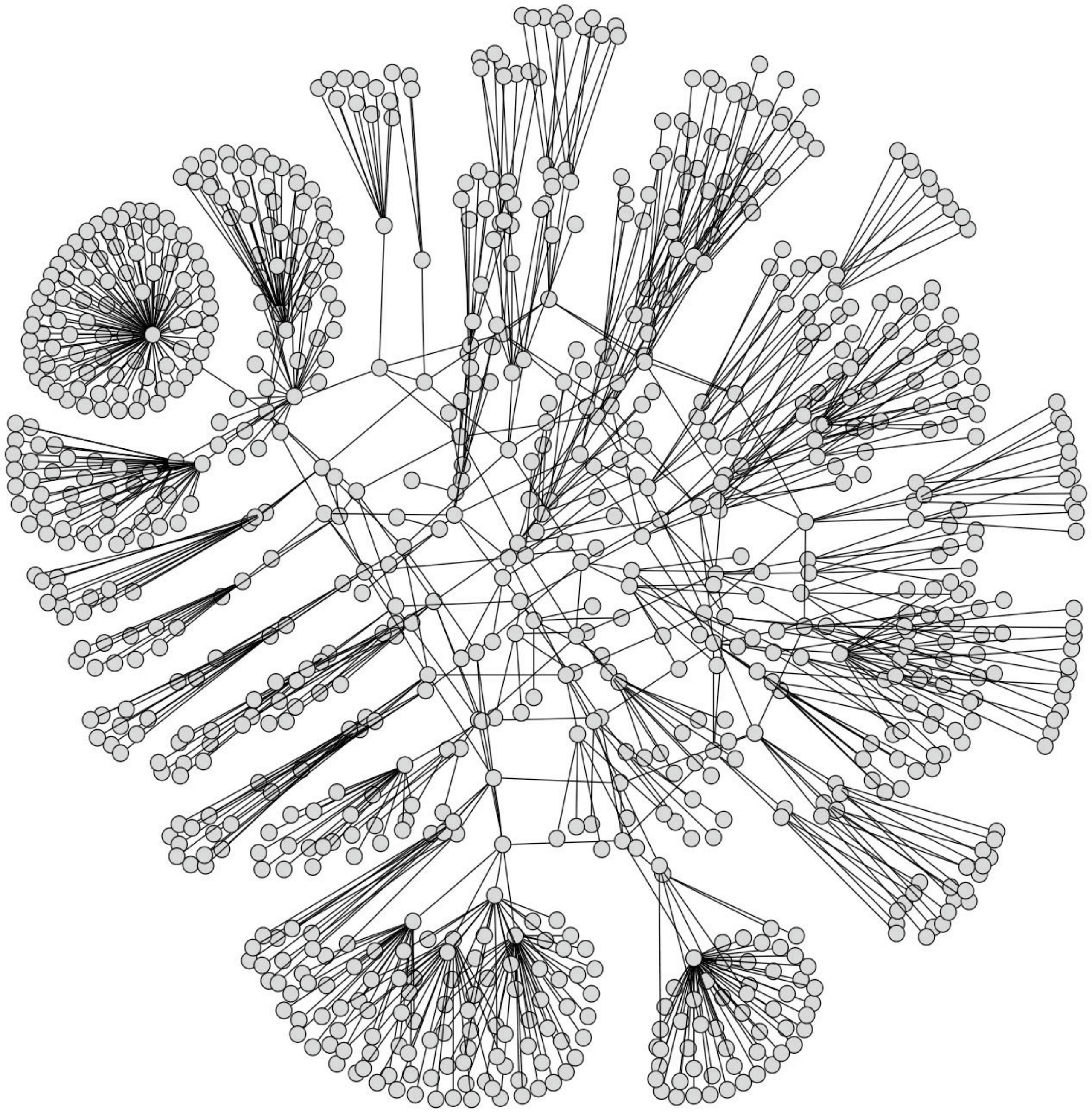

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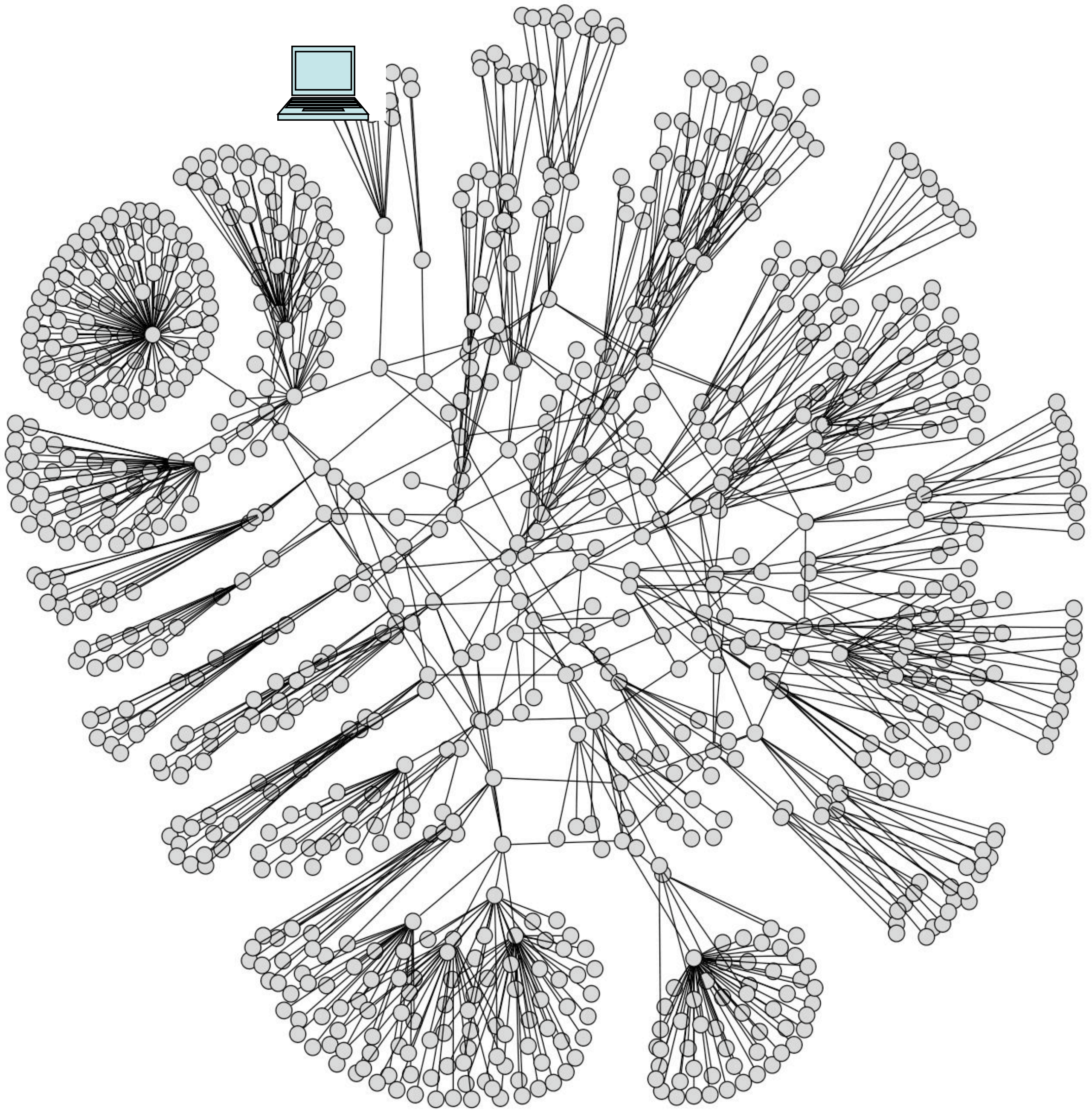

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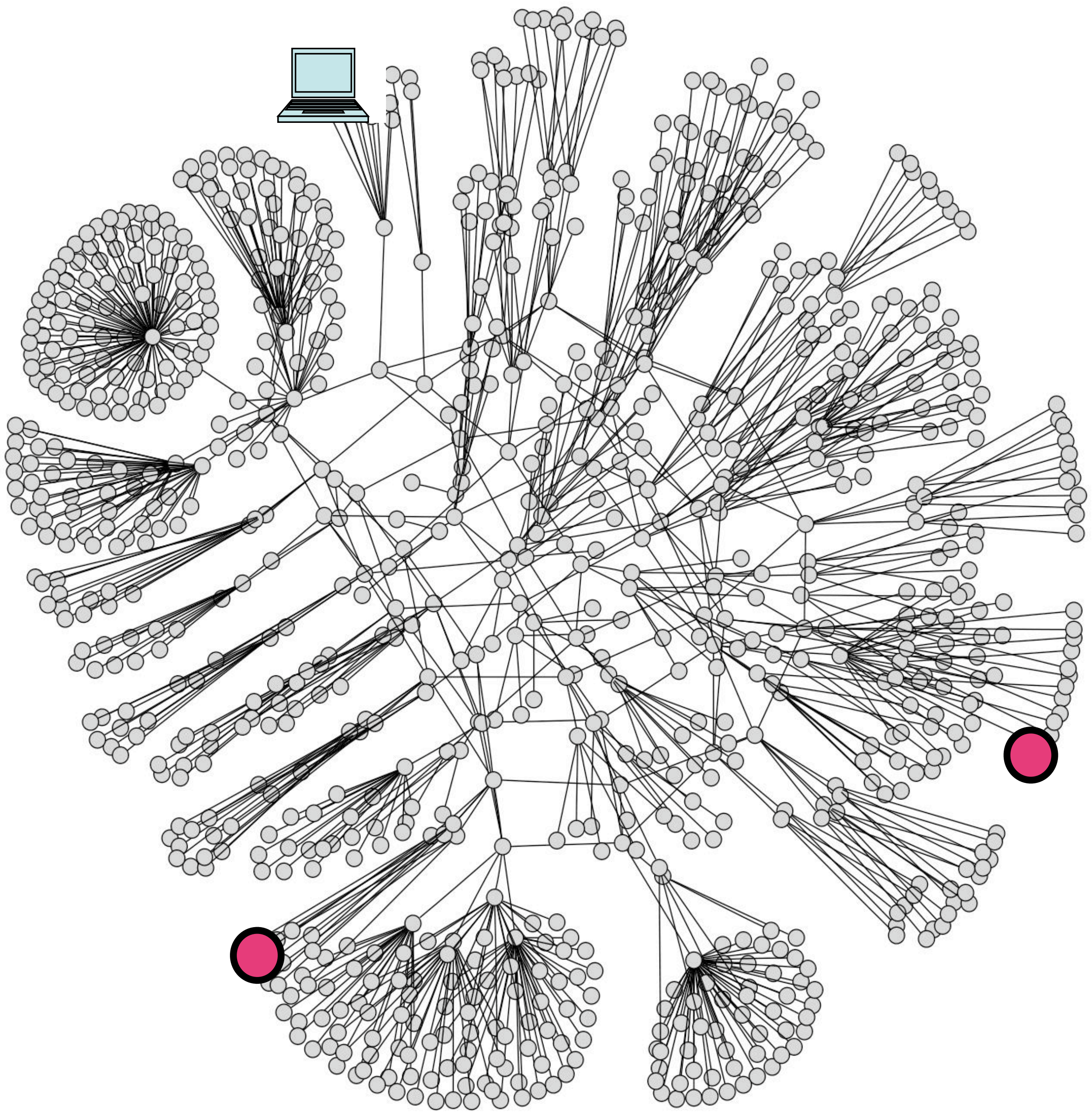

Correlation between traffic patterns at two points can indicate the **similarity** between nodes (e.g., number of shared links in paths)

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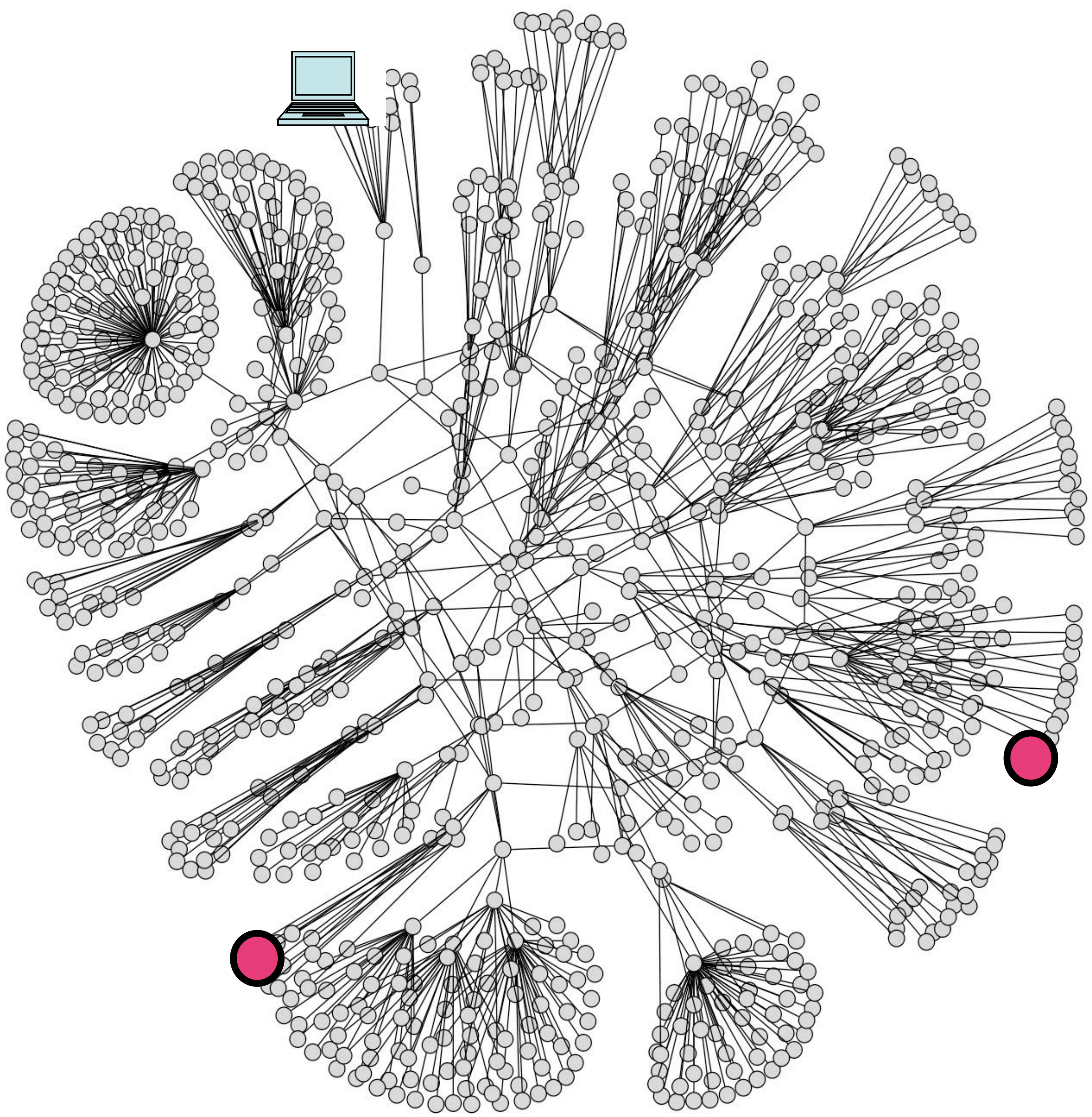

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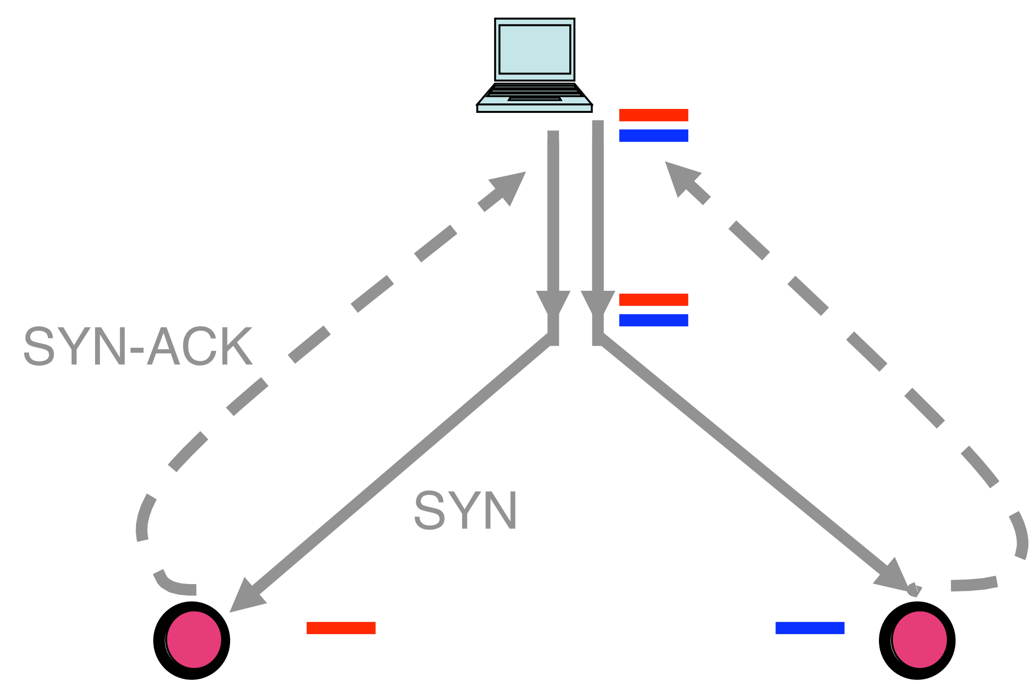
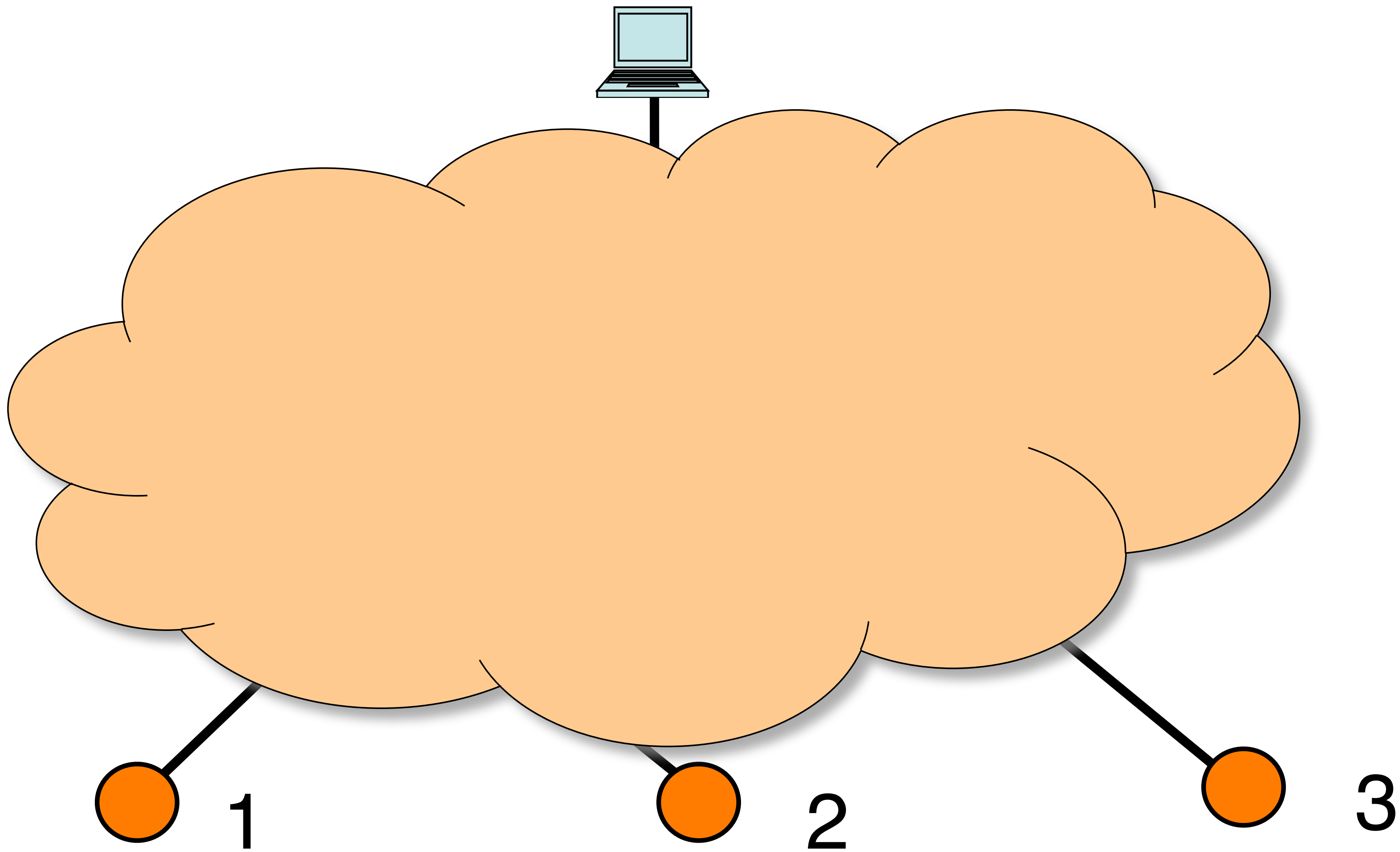


Network Mapping



$$s_{1,2} > s_{1,3},\ s_{2,3}$$

RTT_1 & RTT_2 more correlated than RTT_1 & RTT_3 or RTT_2 & RTT_3

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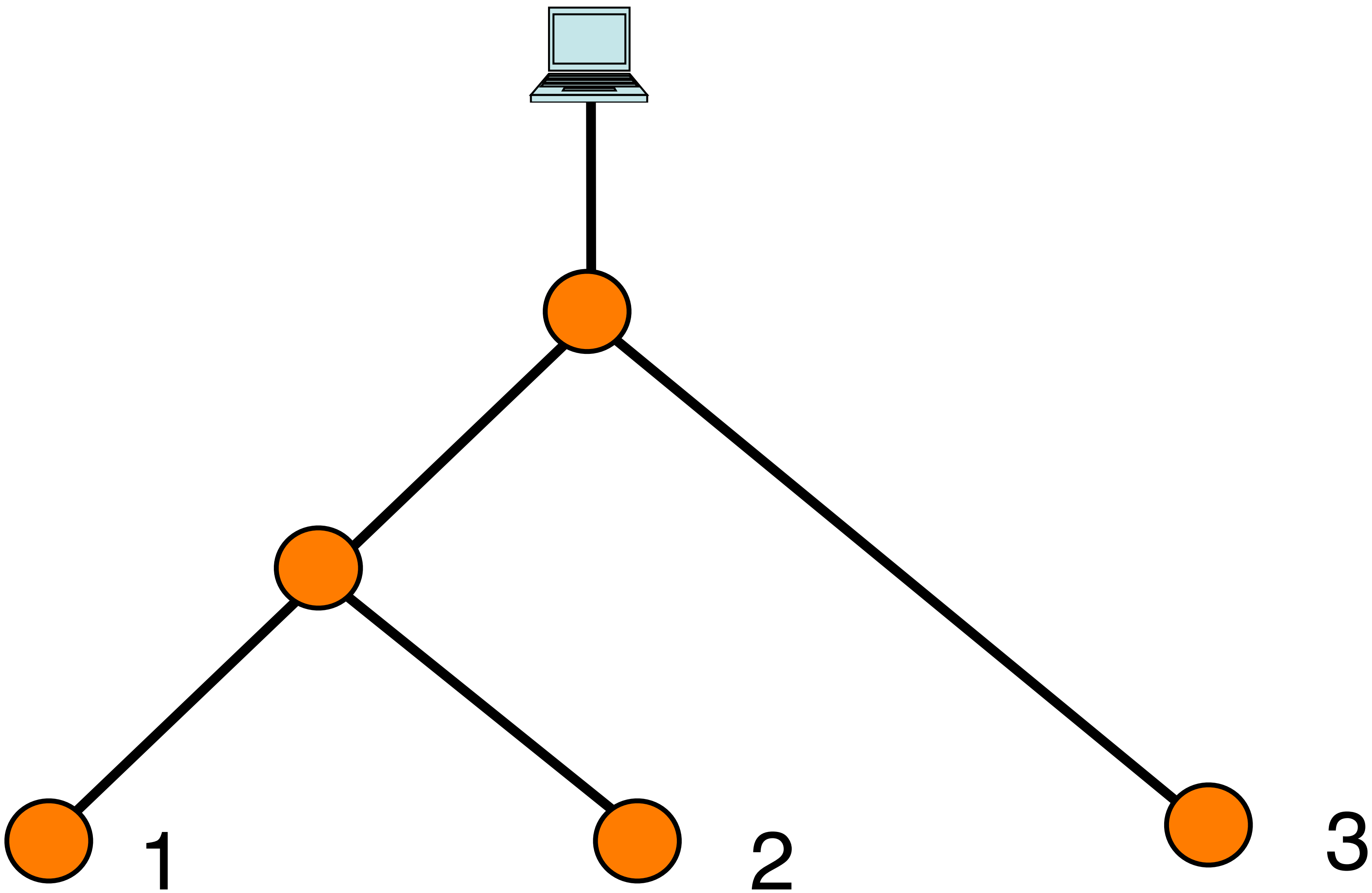

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Active Clustering

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Random subsampling will miss small clusters

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A : Maybe unnecessary to obtain all pairwise similarities

2. Does random subsampling suffice?

A: No ! We will require $O(n^2)$ random similarities

Redundancy

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Active Clustering: Efficient Hierarchical Clustering

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The proposed method **adaptively** selects the most informative pairwise similarities to recover the hierarchical clustering.

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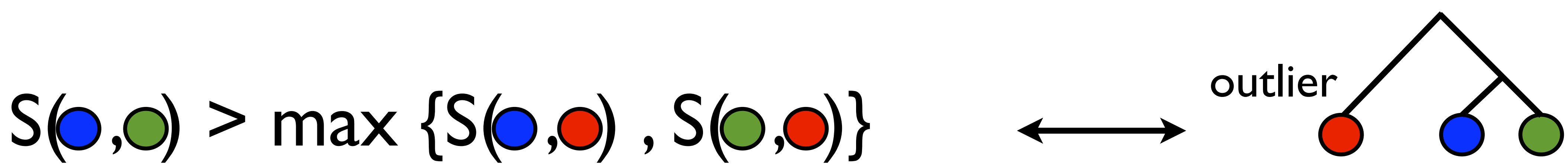

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Inserting a new object into a tree with *i* leaves

- Pick an internal node v with ≈ $i/2$ objects as descendants
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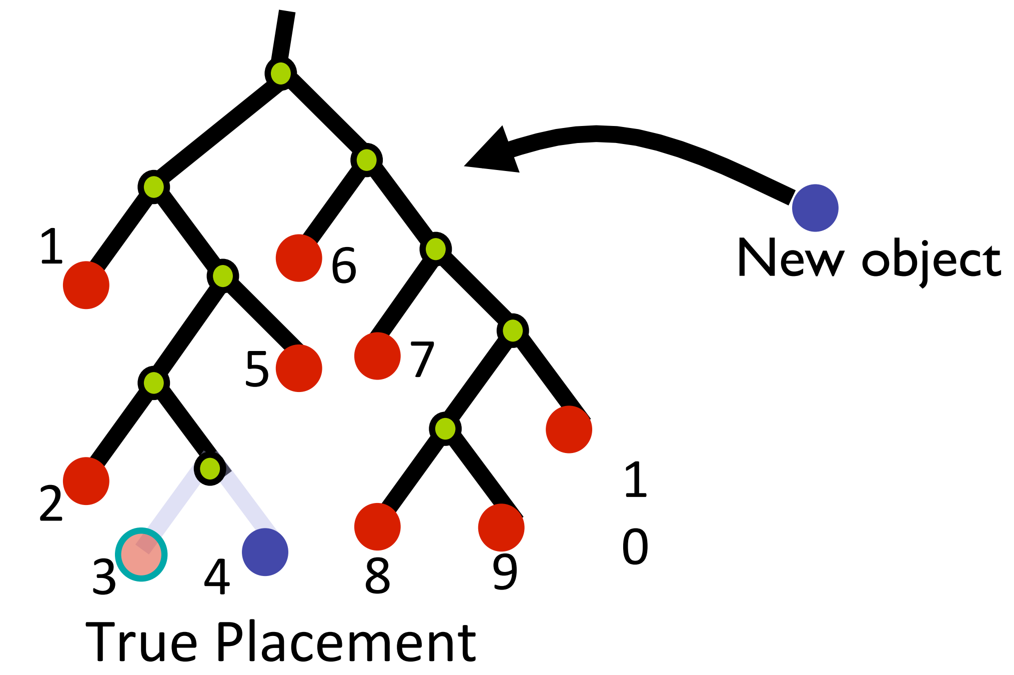

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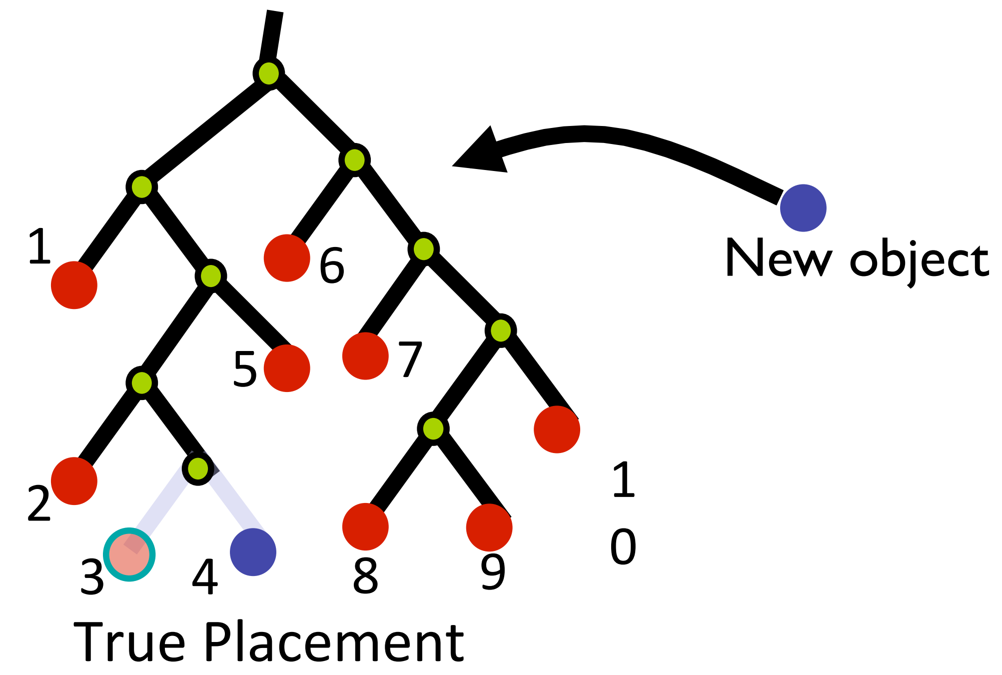

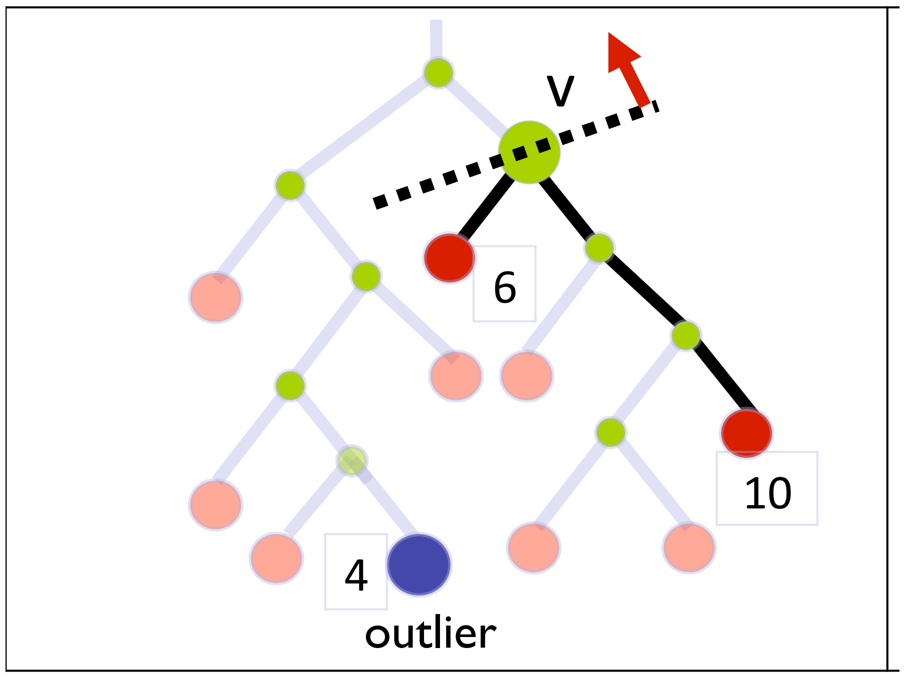

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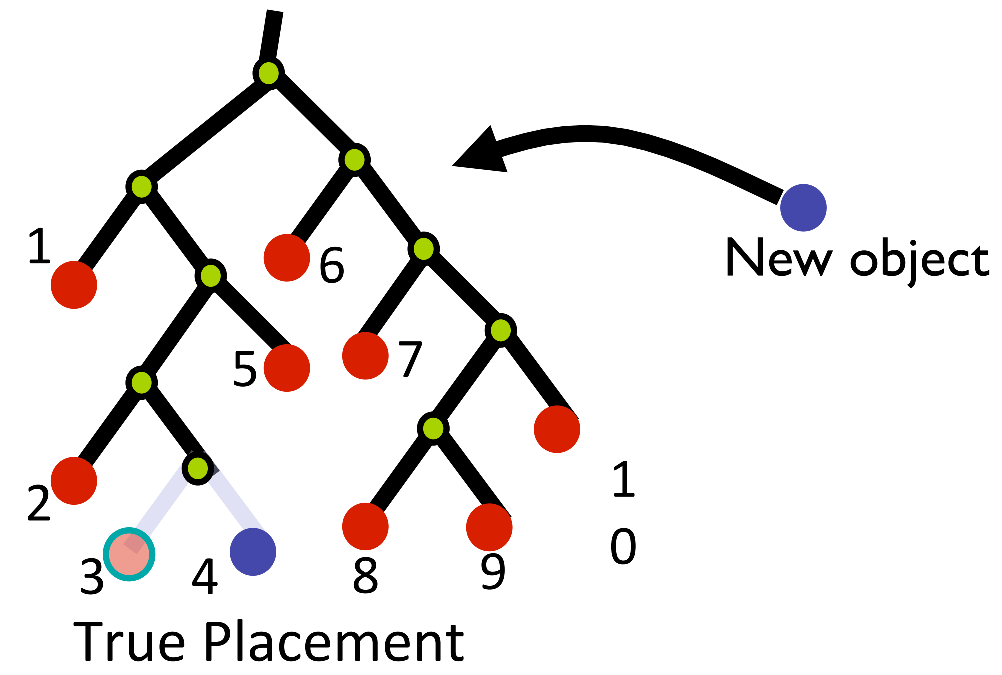

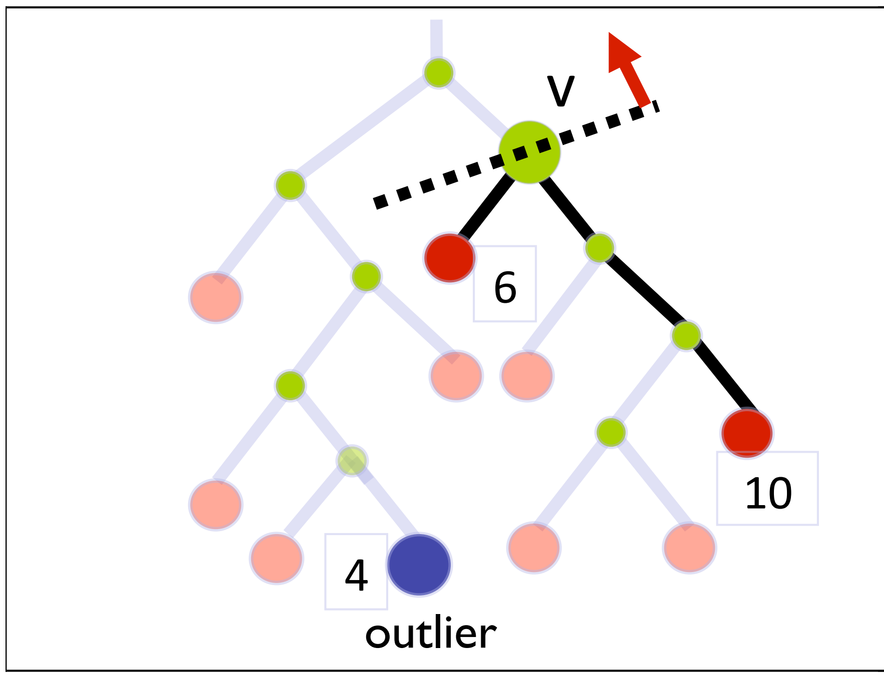

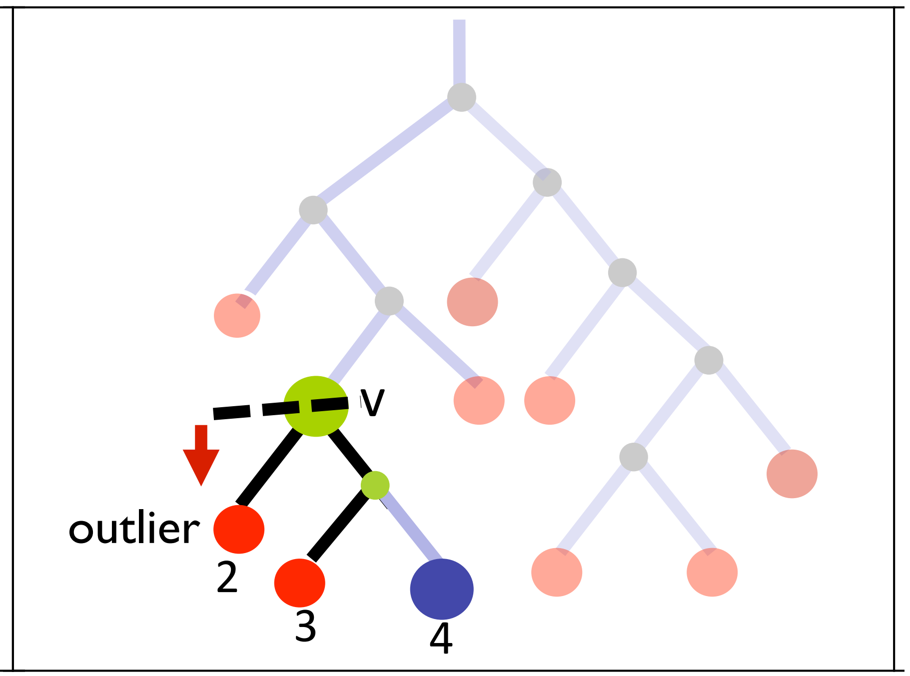

Step 1

Step 2

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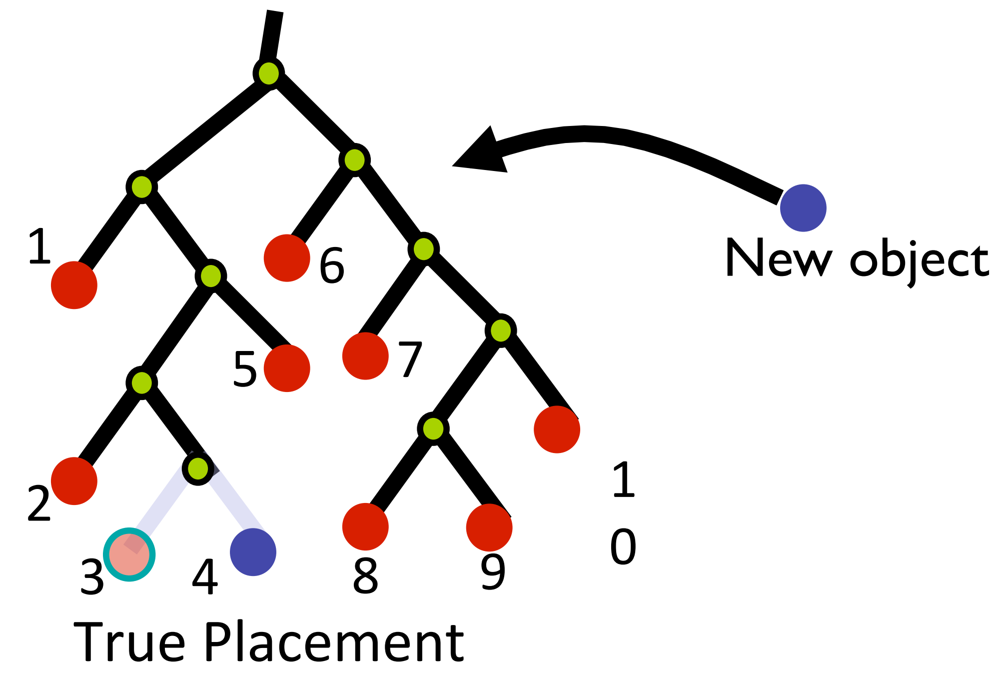

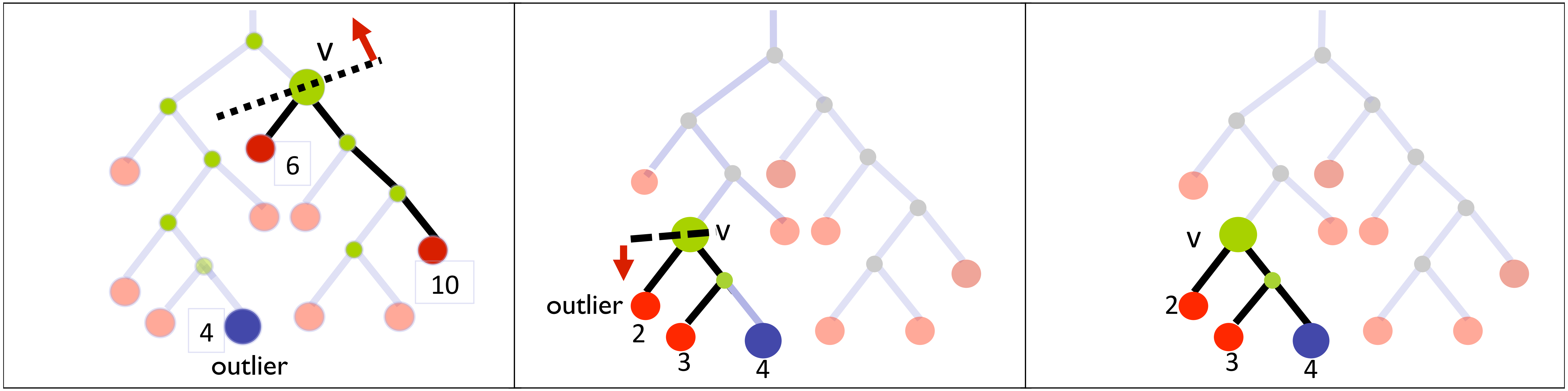

Step 1 Step 2 Step 3

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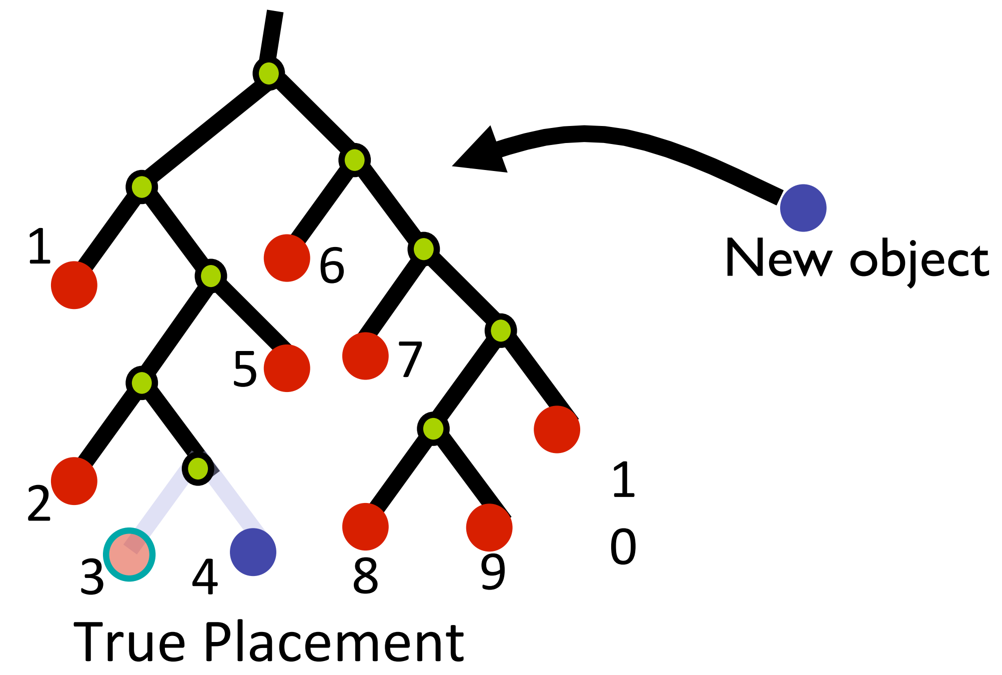

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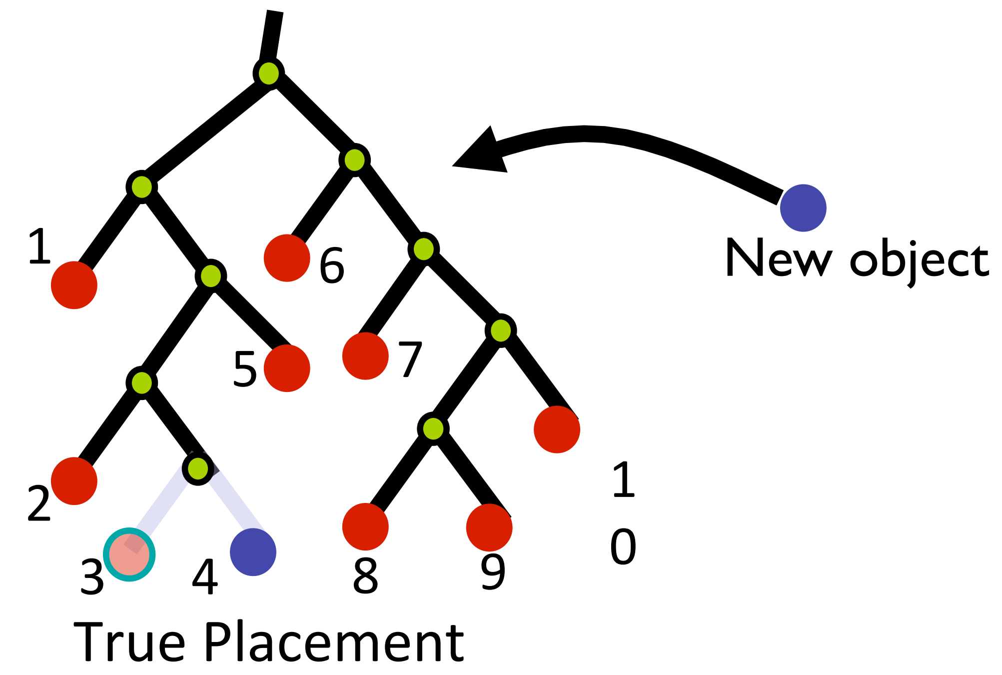


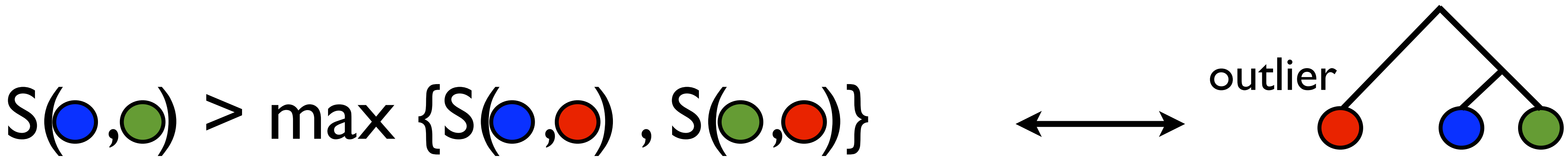
Theorem:
Under certain assumptions, the hierarchical clustering of n objects can be recovered using no more than $\underbrace{3n \log n}$ sequentially and adaptively selected pairwise similarities.

within a constant factor of the *information theoretic lower bound*

Robust Active Clustering

Robust Active Clustering

The previous technique is very sensitive to noise/errors and violations of the assumptions.



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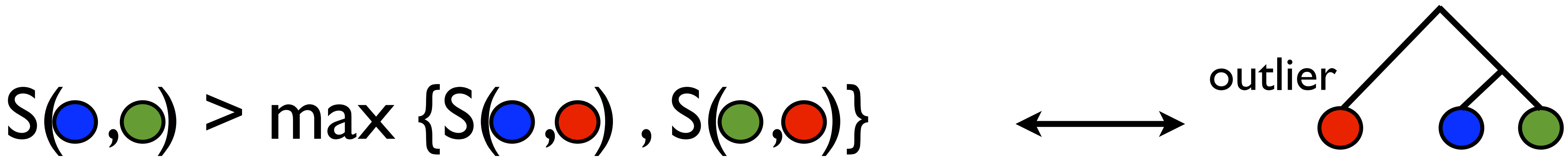

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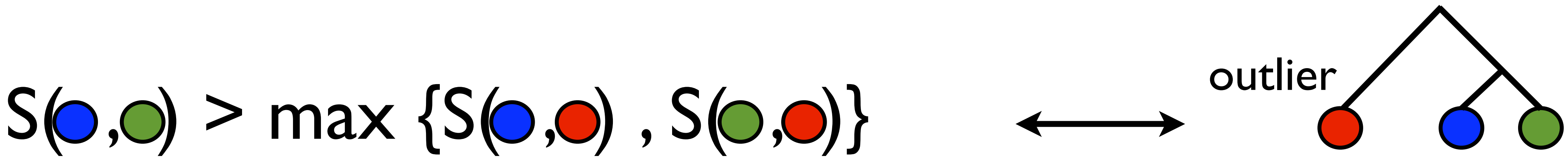

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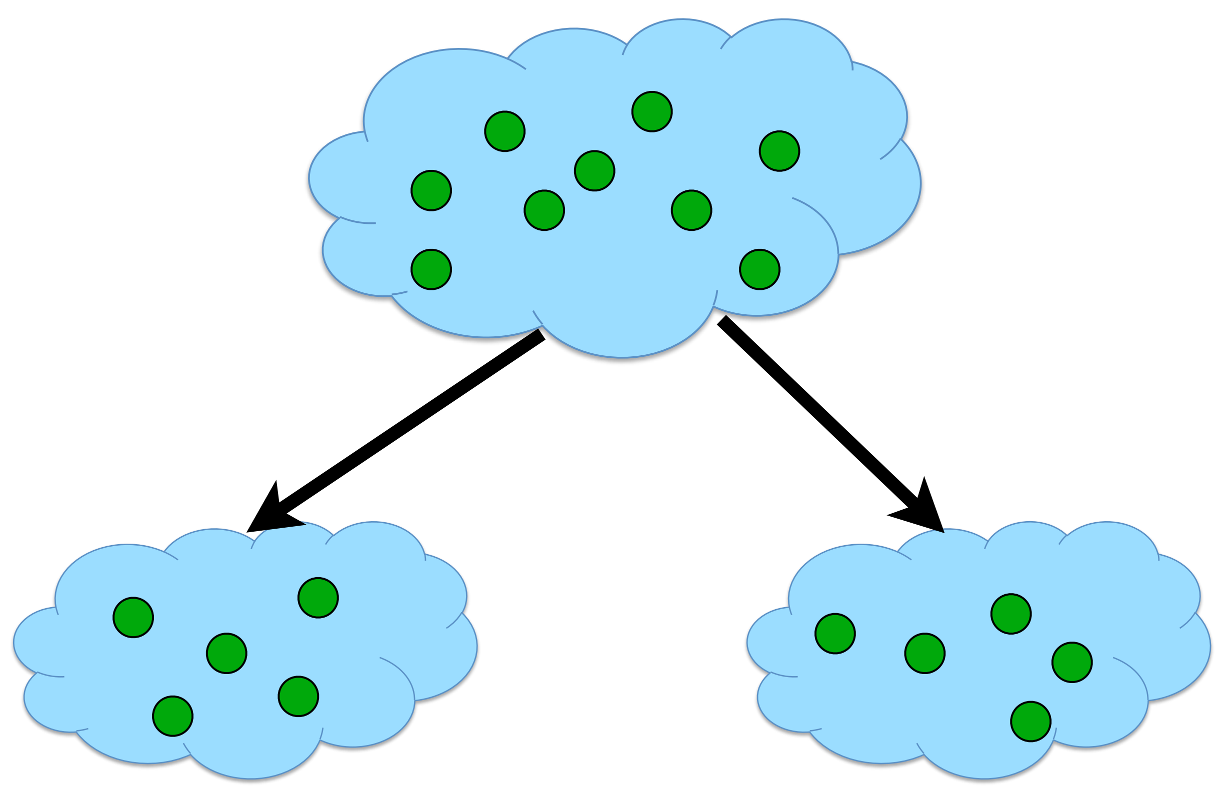
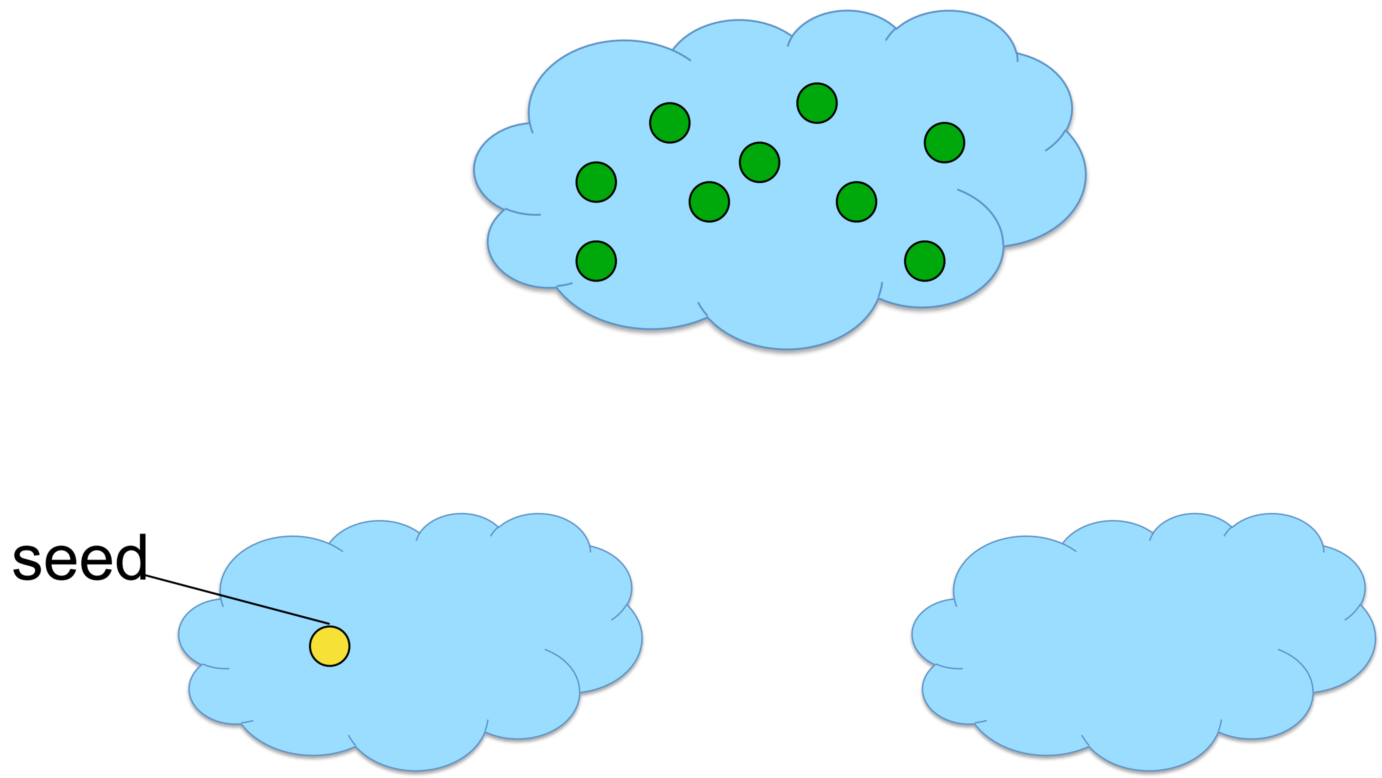


Goal : In each step, split a single cluster into 2 sub-clusters efficiently

Robust Active Clustering Procedure



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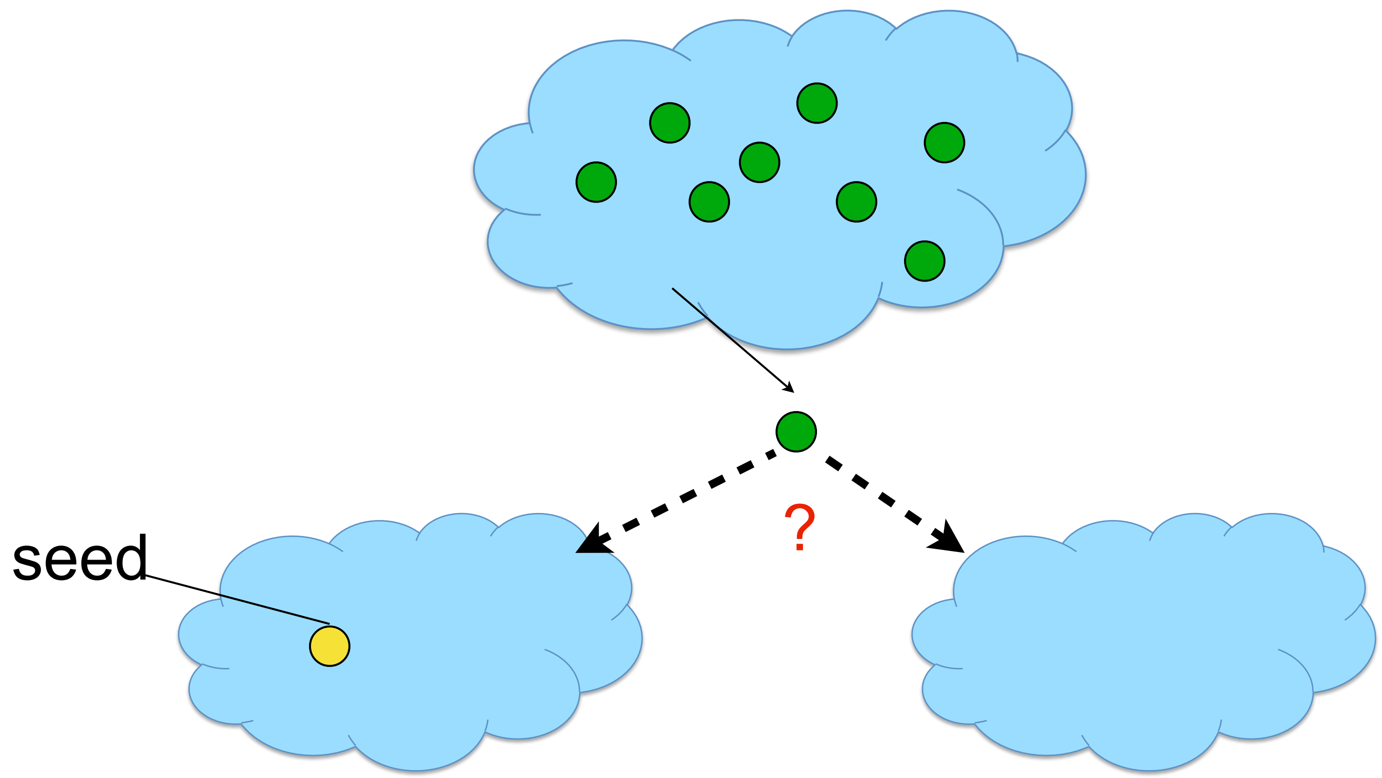

<u>Strategy:</u> Sequentially decide which of the two sub-clusters each ● goes into.

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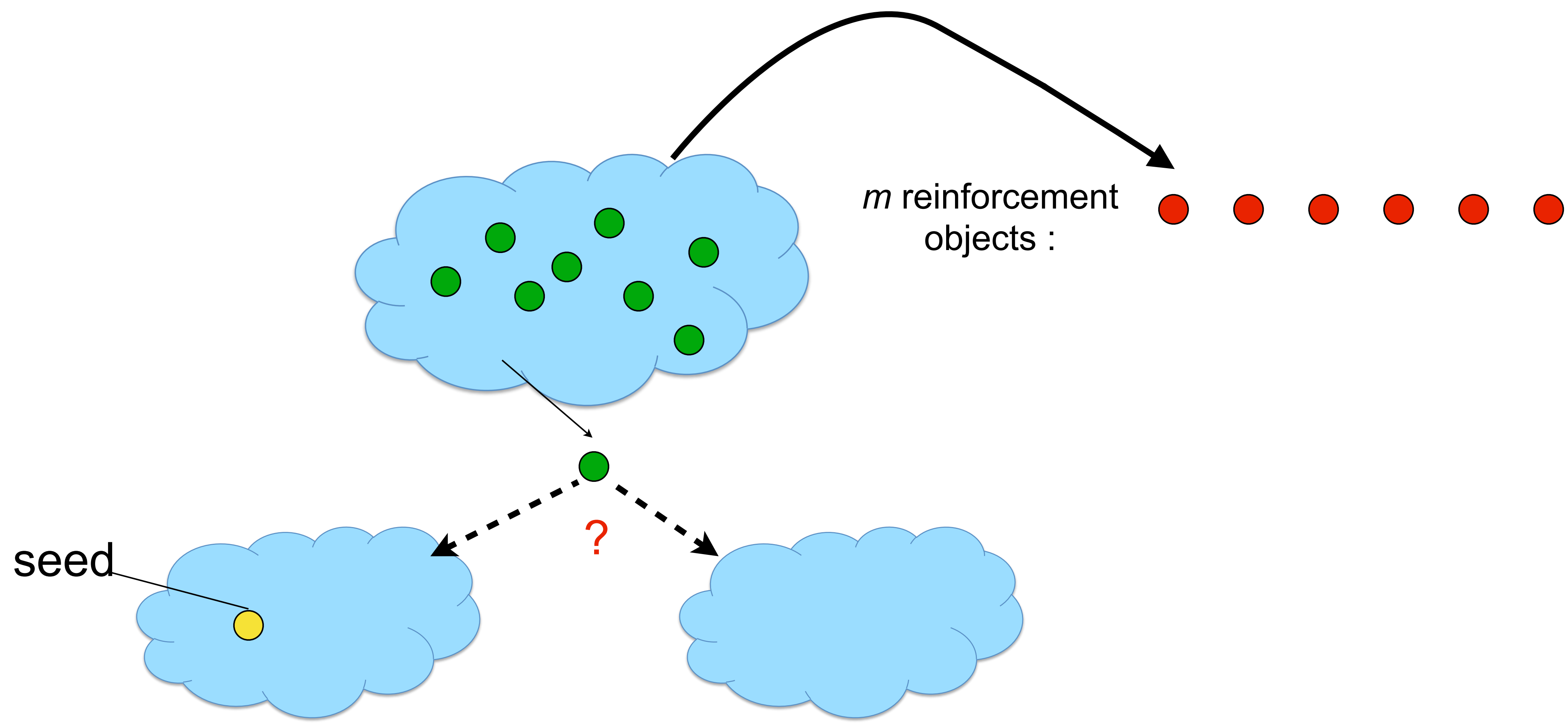
Robust Active Clustering Procedure



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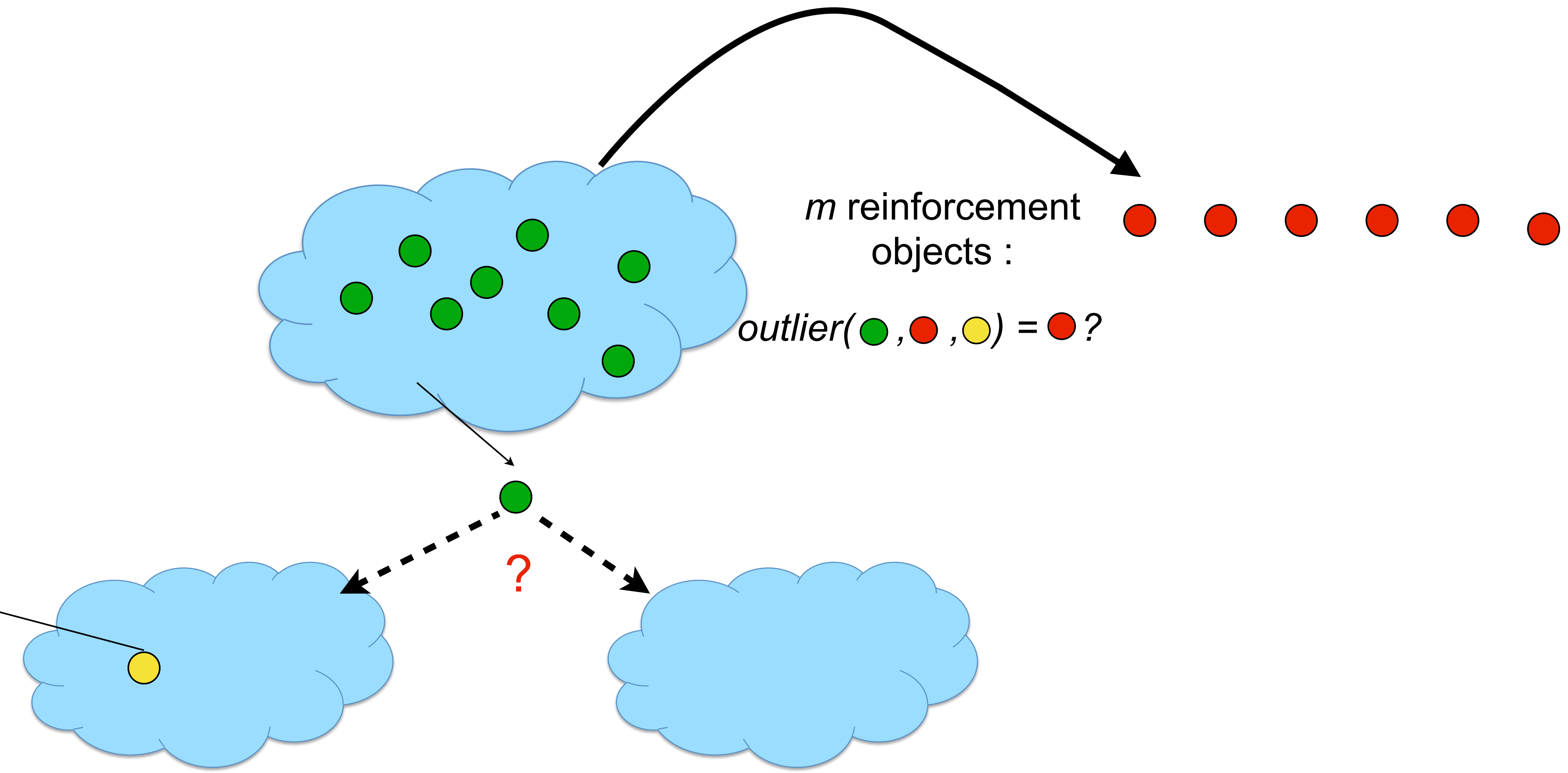

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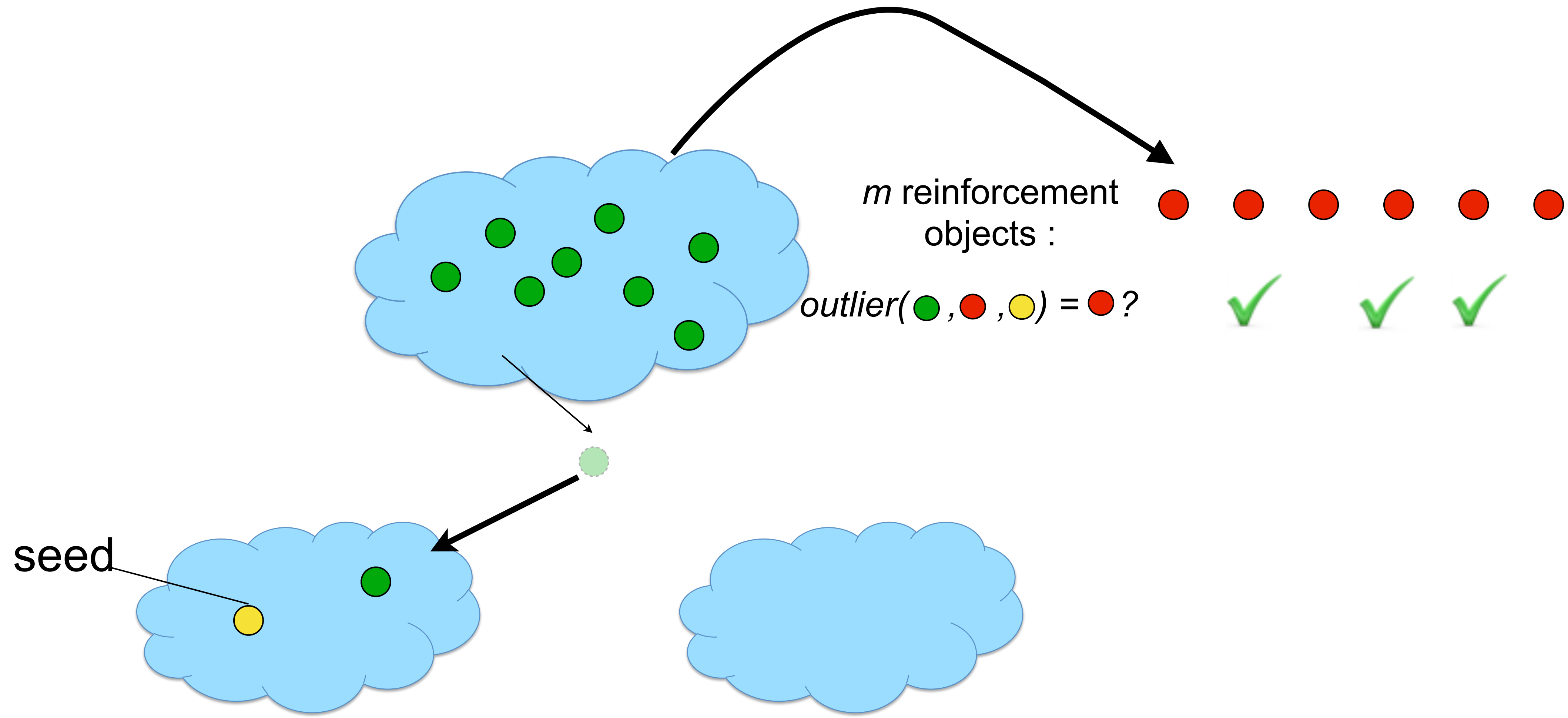

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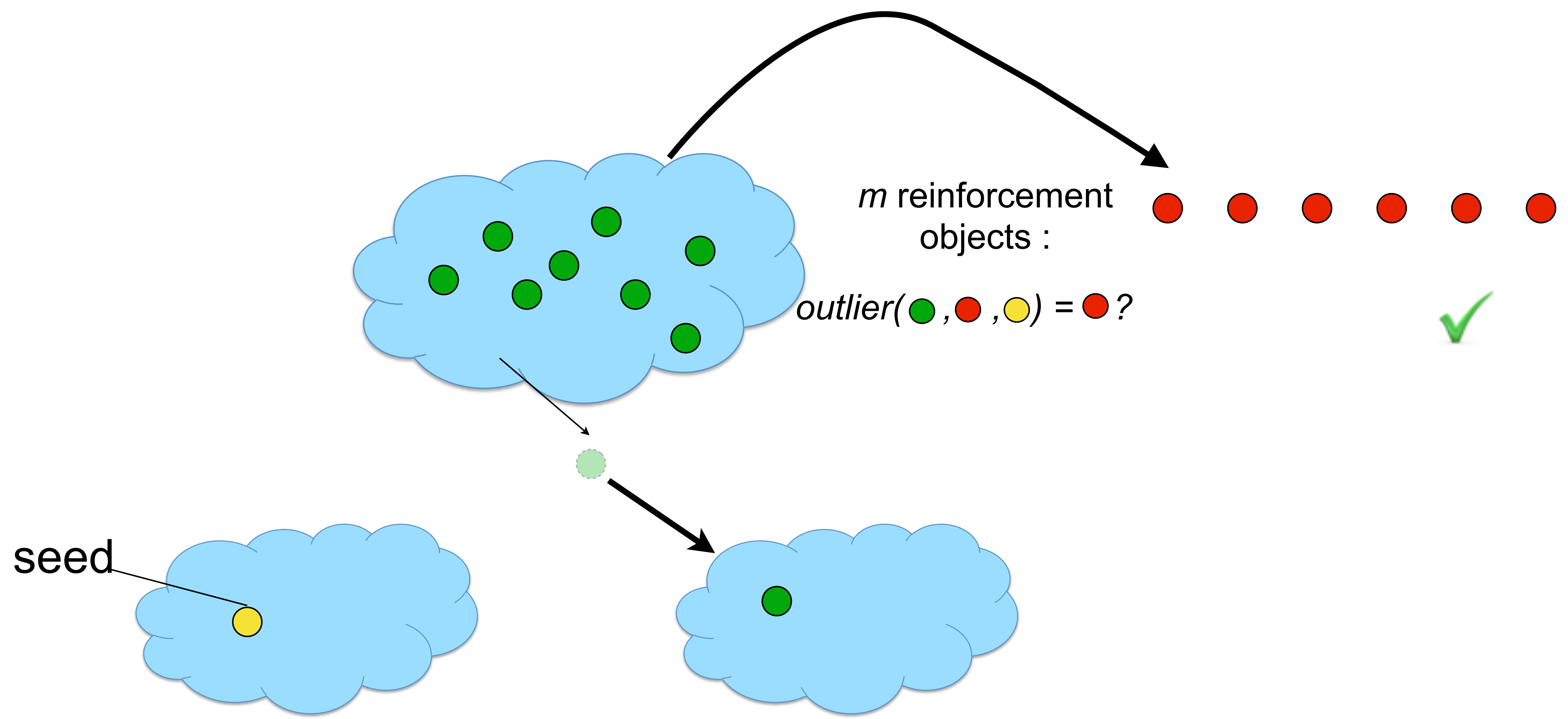

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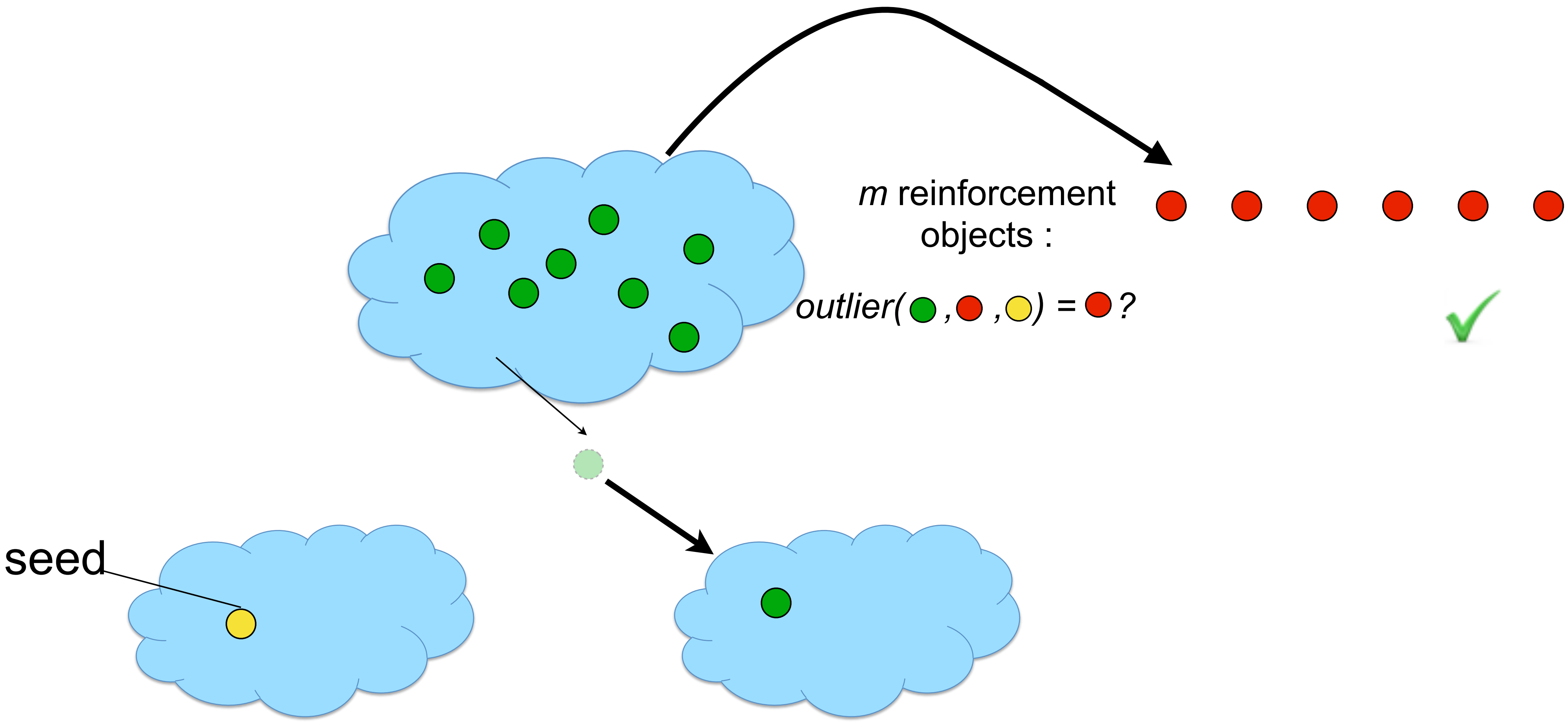

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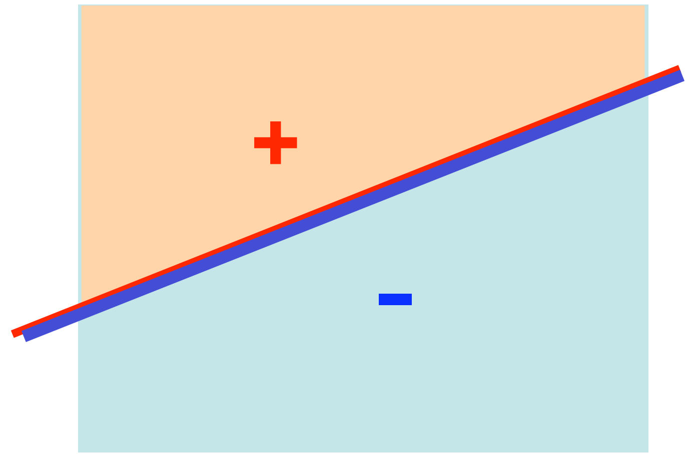
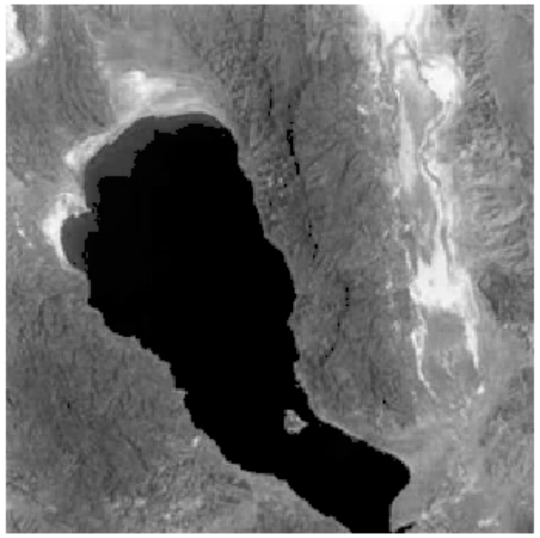
Theorem: This procedure correctly clusters n objects using **O(n $\log^2$ n)** similarities and is robust to a significant fraction of errors.

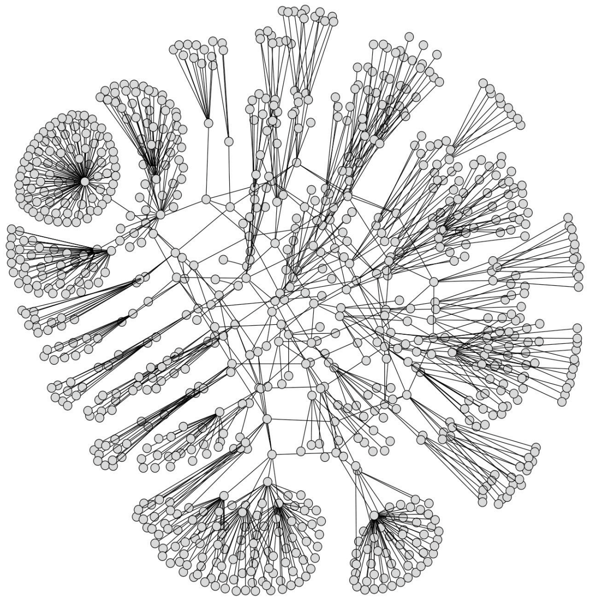
Active Learning Summary



Classification:
NA $\Rightarrow$ sample complexity $n \sim d/\epsilon$
A $\Rightarrow$ sample complexity $n \sim d \log \epsilon^{-1}$



Remote Sensing:
NA $\Rightarrow$ error $\sim O(n^{-1/2})$
A $\Rightarrow$ error $\sim O(n^{-2})$



Network Mapping:
NA $\Rightarrow O(n^2)$ probes
A $\Rightarrow O(n \log n)$ probes

Related Work (an incomplete list)

Active learning
Kulkarni, Mitter, &Tsitsiklis (1993), Cohn, Atlas & Ladner (1994), P. Hall & I. Molchanov (2003), Willett, Castro & Nowak (2005), Dasgupta (2004,2005), Balcan, Beygelzimer & Langford (2006), Kääriäinen (2006), Hanneke (2007), Dasgupta, Hsu, Monteleoni (2007), Castro & Nowak (2008), Beygelzimer, Dasgupta & Langford (2009), Hanneke (2011)

Minimax Analysis of Statistical Learning
Marron (1983), Yatrocos (1985), Barron (1991), Korostelev & Tsybakov (1993), Mammen & Tsybakov (1999), Tsybakov (2004), Scott & Nowak (2006)

Binary Search and Learning by Queries
Rivest, Meyer, & Kleitman (1980), Hegedüs (1995), Hellerstein et al (2006), Karp & Kleinberg (2007)

Channel Coding with Feedback (just the classics)
Horstein (1963), Schalkwijk & Kailath (1966), Burnashev & Zigangirov (1974), Burnashev (1976)