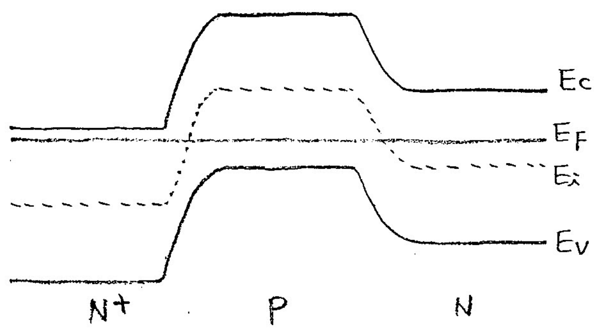
HW#5 SOLUTION

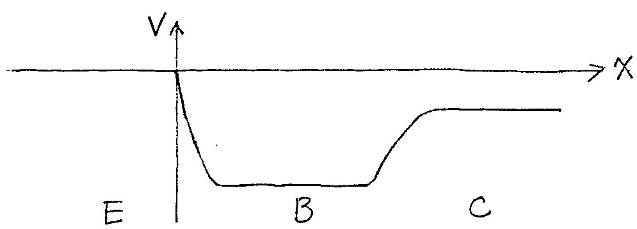
1) prob. 10.4.

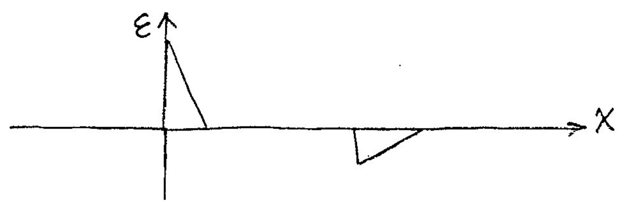
a) For the given doping concentrations, one computes $E_F - E_i = 0.477\,eV$, $-0.358\,eV$, and $0.298\,eV$ respectively in the emitter, base, and collector.

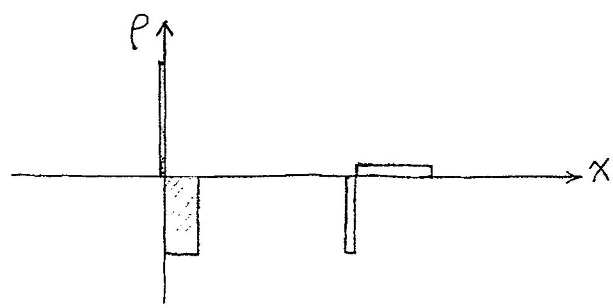
Also, with $N_{DE} \gg N_{AB}$, the E-B depletion width will lie almost exclusively in the base. likewise, the majority of the C-B depletion width will lie in the collector. The diagram produced by BJT_Eband program is displayed below.



b)







c) $\Delta V_{CE} = \frac{1}{q}\left[(E_F - E_i)_{collector} - (E_F - E_i)_{emitter}\right]$

$$= \frac{KT}{q}\left[\ln(N_{DC}/n_i) - \ln(N_{DE}/n_i)\right]$$

$$= \frac{KT}{q}\ln(N_{DC}/N_{DE}) = -0.179\ V$$

d) Analogous to Eq 10.3 in the text,

$$W = W_B - x_{pEB} - x_{pCB}$$

where, $x_{pEB} \simeq \left[\frac{2K_s\epsilon_0}{q N_{AB}} V_{biEB}\right]^{1/2} = 3.30\times10^{-5}\ cm$

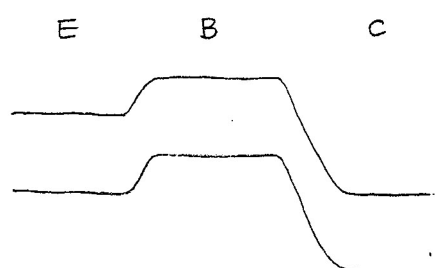
$$x_{pCB} = \left[\frac{2K_s\epsilon_0}{qN_{AB}}\cdot\frac{N_{DC}}{N_{DC}+N_{AB}}\cdot V_{biCB}\right]^{1/2} = 8.82\times10^{-6}\ cm$$

$\therefore$ $W = 2\times10^{-4} - 3.30\times10^{-5} - 8.82\times10^{-6} = 1.58\times10^{-4}\ cm.$

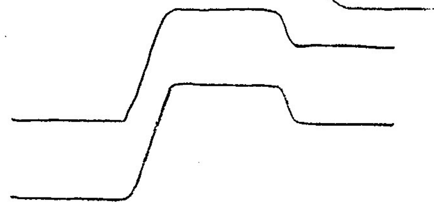
e)
$$\mathcal{E}_{max}(E\text{-}B) = \frac{qN_{AB}}{K_s\epsilon_0}x_{pEB} = 5.06\times10^{4}\ V/cm$$

$$|\mathcal{E}|_{max}(C\text{-}B) = \frac{qN_{AB}}{K_s\epsilon_0}x_{pCB} = 1.35\times10^{4}\ V/cm$$

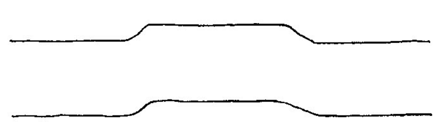
d') i) active bias mode:



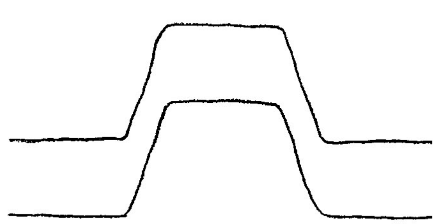
ii) inverted bias mode:



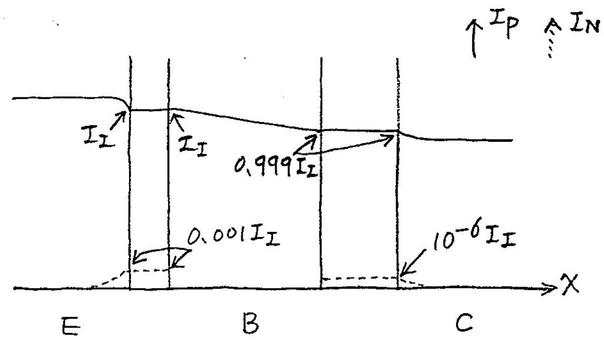
iii) saturation mode:



iv) Cut off mode:



2) prob. 11.2



a) $\gamma = \dfrac{I_{Ep}}{I_{Ep}+I_{En}} = \dfrac{I_I}{1.001\, I_I} = 0.9990$

b) $\alpha_T = \dfrac{I_{CP}}{I_{EP}} = \dfrac{0.999\, I_I}{I_I} = 0.9990$

c) $\alpha_{dc} = \gamma \alpha_T = 0.9980$

$\beta_{dc} = \dfrac{\alpha_{dc}}{1-\alpha_{dc}} = 499$

d) method 1. $I_E = I_{Ep} + I_{En} = 1.001\, I_I$

$I_C = I_{Cp} + I_{Cn} = 0.999\, I_I + 10^{-6}\, I_I$

$I_B = I_E - I_C = 1.999 \times 10^{-3} \cdot I_I$

method 2. $I_{B1} = I_{En} = 0.001\, I_I$

$I_{B2} = I_{Ep} - I_{Cn} = 0.001\, I_I$

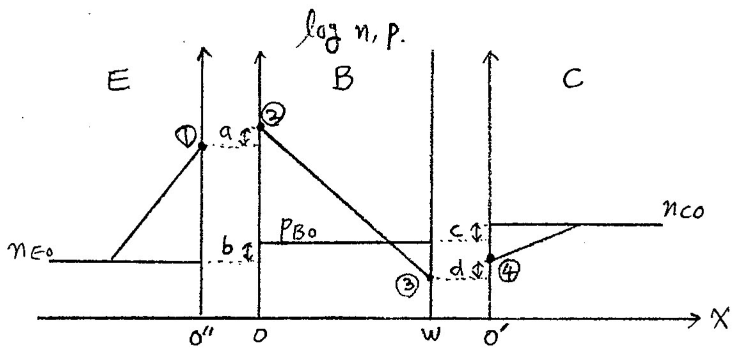
$I_{B3} = -I_{Cn} = -(10^{-6})\, I_I$

$I_B = I_{B1} + I_{B2} + I_{B3} = 1.999 \times 10^{-3} \cdot I_I$

e) Yes. All currents on the figure are constant across the depletion regions. This implies (see subsection 6.1.2) that recombination-generation is negligible in these regions.

3)

i) point A is in active region.



for point ① $\Delta n_E(0'') = n_{E0}\left(e^{qV_{EB}/kT} - 1\right)$

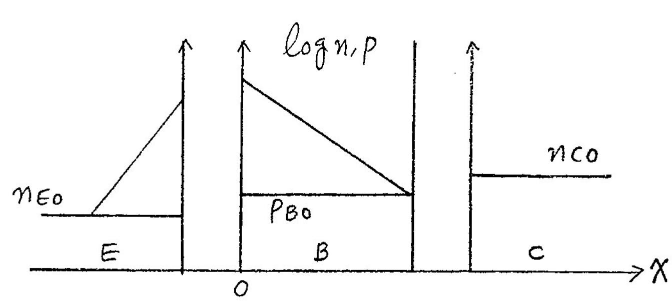
" ② $\Delta p_B(0) = p_{B0}\left(e^{qV_{EB}/kT} - 1\right)$

" ③ $\Delta p_B(W) = p_{B0}\left(e^{qV_{CB}/kT} - 1\right)$

" ④ $\Delta n_C(0') = n_{C0}\left(e^{qV_{CB}/kT} - 1\right)$

∴ we see $a=b$, $c=d$.

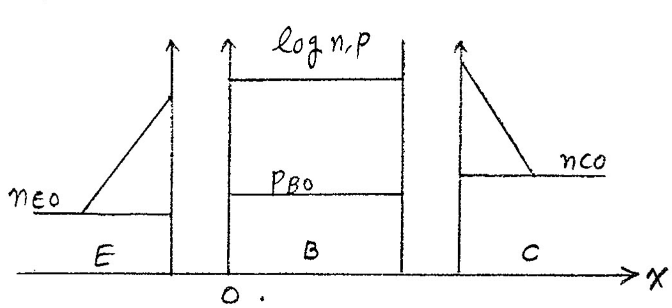
ii) point B is in onset saturation region.



· V_{CB} have zero value.

So, $n_C(x) = n_{C0}$.

iii) point C is in strong saturation region.



· $I_C = qAD_B \dfrac{dp_B(x)}{dx}$ have to be zero.

So, $p_B(x)$ is constant.

4)

a) From eq. 10.3

$$W = W_B - x_{pEB} - x_{pCB}$$

where, $N_{DE} = 10^{18}\ cm^{-3}$, $N_{AB} = 10^{16}\ cm^{-3}$, $N_{DC} = 10^{15}\ cm^{-3}$.

$W_B = 1.0 \times 10^{-4}\ cm$.

$$\left.\begin{aligned}(E_F - E_i)_E &= kT \ln \frac{N_{DE}}{n_i} = 0.477\ eV \\ (E_F - E_i)_B &= -kT \ln \frac{N_{AB}}{n_i} = -0.358\ eV \\ (E_F - E_i)_C &= kT \ln \frac{N_{DC}}{n_i} = 0.298\ eV\end{aligned}\right\} \Rightarrow \begin{aligned} V_{biEB} &= 0.835\ V \\ V_{biCB} &= 0.656\ V\end{aligned}$$

$$x_{pEB} \simeq \left(\frac{2K_s\epsilon_0}{qN_{AB}} V_{biEB}\right)^{1/2} = 3.30 \times 10^{-5}\ cm.$$

$$x_{pCB} = \left(\frac{2K_s\epsilon_0}{qN_{AB}} \cdot \frac{N_{DC}}{N_{DC}+N_{AB}} \cdot V_{biCB}\right)^{1/2} = 8.82 \times 10^{-6}\ cm.$$

$$\therefore W = 1 \times 10^{-4} - 0.33 \times 10^{-4} - 0.0882 \times 10^{-4} = 0.58 \times 10^{-4}\ cm.$$

b) Since $e^{qV_{EB}/kT} \gg 1$ and $e^{qV_{CB}/kT} \ll 1$, we can write the following: (NPN)

$$I_{En} = q n_i^2 A \cdot \frac{D_B}{W N_B} \cdot e^{qV_{EB}/kT}$$

$$I_{Ep} = q n_i^2 A \cdot \frac{D_E}{L_E N_E} \cdot e^{qV_{EB}/kT}$$

$$I_{Cn} = q n_i^2 A \left(\frac{D_B}{W N_B} - \frac{W}{2\tau_B \cdot N_B}\right) \cdot e^{qV_{EB}/kT}$$

$$I_{Cp} = q n_i^2 A \cdot \left(\frac{D_C}{L_C N_C}\right)$$

where, $D_B = 0.026 \times 1248 = 32.45$

$D_E = 0.026 \times 115 = 2.99$

$D_C = 0.026 \times 458 = 11.91$

$L_E = \sqrt{D_E \tau_E} = 1.7 \times 10^{-3}\ cm$, assume $\tau_E = \tau_C = \tau_B = 1\mu s$

$L_C = \sqrt{D_C \tau_C} = 3.45 \times 10^{-3}\ cm$

$A = 0.01\ cm^2$

$W = 1 \times 10^{-4} - 2.55 \times 10^{-5} - 2.44 \times 10^{-5} = 5.01 \times 10^{-5}\ cm.$

$\therefore\ I_{En} = 2.3298 \times 10^{-3}$ (A)

$I_{Ep} = 6.32 \times 10^{-8}$ (A)

$I_{Cn} = 2.3296 \times 10^{-3}$ (A)

$I_{Cp} = 5.52 \times 10^{-11}$ (A)

c) Including Base recombination,

$I_B = I_{B\,ideal} + I_{B\,recomb.}$

$$= q n_i^2 A \left(\frac{D_E}{L_E N_E} + \frac{W}{2\tau_B} \cdot \frac{1}{N_B} \right) \left(e^{qV_{EB}/kT} - 1 \right)$$

$$+ q n_i^2 A \left(\frac{D_C}{L_C N_C} + \frac{W}{2\tau_B N_B} \right) \left(e^{qV_{CB}/kT} - 1 \right)$$

$I_C = I_{C\,ideal} - I_{B\,recomb} \simeq I_{C\,ideal}.$

$I_E = I_{E\,ideal}.$

$$\gamma \text{ (emitter injection efficiency)} = \frac{I_{Ep}}{I_{Ep} + I_{En}} = 0.99997$$

$$\alpha_T \text{ (base transport factor)} = \frac{I_{Cp}}{I_{Ep}} = 0.99991$$

$$\alpha_{dc} = \gamma \cdot \alpha_T = 0.99988$$

$$\beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} = 8.33 \times 10^3$$

d) for reducing the length of emitter to $0.3\,\mu m$,

I_{Ep} is same. but

I_{En} is increase because.

$$I_{En} = q n_i^2 A \cdot \frac{D_E}{L_E N_E} e^{qV_{EB}/kT}, \qquad W_E \ll L_E, \text{ so}$$

L_E have to be replaced by W_E.

So, $I_{En} \uparrow \Rightarrow \gamma \downarrow \Rightarrow \beta \downarrow.$

It would increase I_{En} and decrease β.

5)

a) $$I_C = q n_i^2 A\left(\frac{D_B}{W \cdot N_B}\right)\left(e^{qV_{EB}/kT}-1\right)$$

$$- q n_i^2 A\left(\frac{D_C}{L_C N_C} + \frac{D_B}{W N_B}\right)\left(e^{qV_{CB}}-1\right)$$

where $e^{qV_{EB}} \gg 1$, $e^{qV_{CB}} \ll 1$

$$\simeq q n_i^2 A \cdot \frac{D_B}{W \cdot N_B} \cdot e^{qV_{EB}/kT}$$

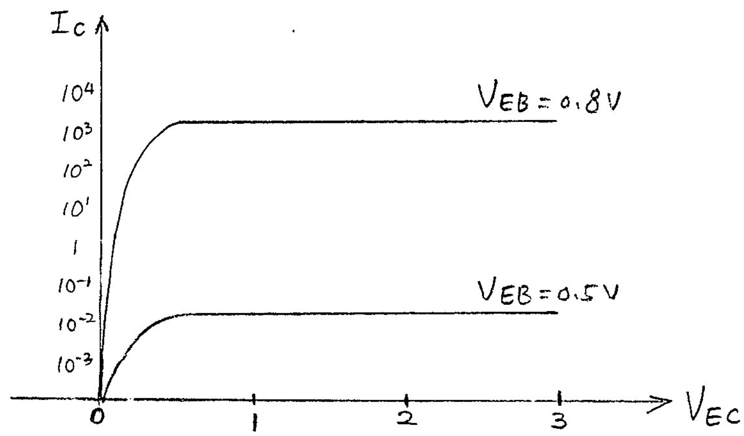
where, $D_B = 0.026 \times 378 = 9.83$ $cm^2 s^{-1}$

$\therefore$ when $V_{EB} = 0.5V$,

$I_C = 2.3 \times 10^{-2}$ (A)

when $V_{EB} = 0.8V$,

$I_C = 2.2 \times 10^3$ (A)



b) $$I_B = q n_i^2 A\left(\frac{D_E}{L_E N_E} + \frac{W}{2\tau_B} \cdot \frac{1}{N_B}\right) \cdot e^{qV_{EB}/kT}$$ for active region,,

where $A = 1$ cm^2

$D_E = 0.026 \times 300 = 7.8$

$L_E = \sqrt{D_E \tau_E} = 0.88 \times 10^{-3}$ cm.

$\tau_B = 5 \times 10^{-7}$

$W = 0.32 \times 10^{-4}$ cm.

$N_B = 5 \times 10^{16}$ cm^{-3}

$N_E = 8 \times 10^{17}$ cm^{-3}

$\therefore$ when $V_{EB} = 0.5V$, $I_B = 4.22 \times 10^{-7}$ (A)

when $V_{EB} = 0.8V$, $I_B = 4.32 \times 10^{-2}$ (A)

c) From eq. 11.49,

$$I_C \simeq q n_i^2 A \frac{D_B}{W \cdot N_B} e^{qV_{EB}/kT}$$

where $D_B = 9.83$
$V_{EB} = 0.8$
$N_B = 5 \times 10^{16}$
$W = 0.6 \times 10^{-4} - x_{nEB} - x_{nCB}$

$$= 0.6 \times 10^{-4} - \left(\frac{2K_S\epsilon_0}{q \cdot N_{DB}} \cdot \frac{N_{AE}}{N_{AE}+N_{DB}} \cdot V_{biEB} \right)^{1/2} - \left(\frac{2K_S \cdot \epsilon_0}{q \cdot N_{DB}} \cdot \frac{N_{AC}}{N_{AC}+N_{DB}} (V_{biCB} - V_{CB}) \right)^{1/2}$$

$$= 0.56 \times 10^{-4} - 1.6 \times 10^{-5} \times (0.76 - V_{CB})^{1/2}$$

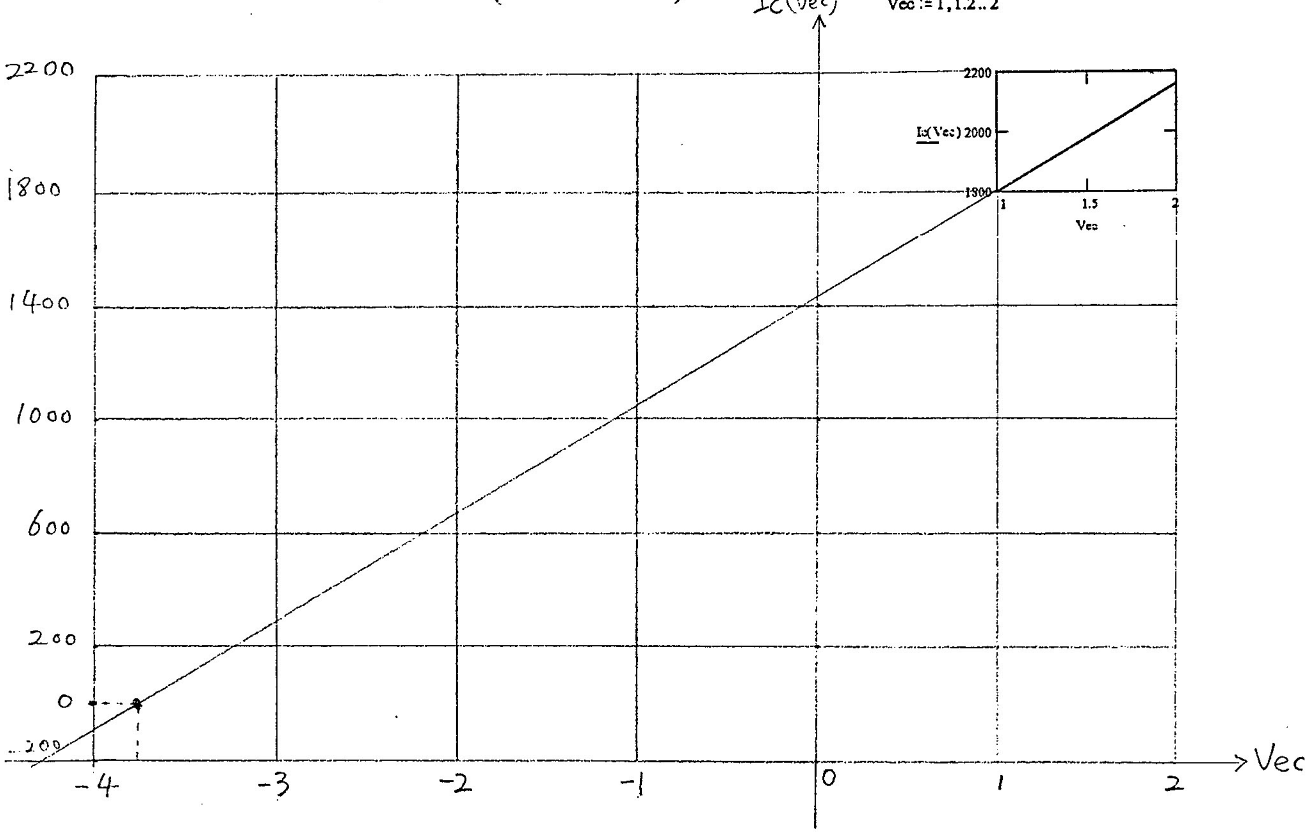
where $V_{CB} = V_{EB} - V_{EC} = 0.8 - V_{EC}$

$$I_C = 16 \times \frac{9.83}{5 \times 10^{16}} \times e^{30.77} \times \frac{1}{0.56 \times 10^{-4} - 1.6 \times 10^{-5} \times (-0.04 + V_{EC})^{1/2}}$$

$$= \frac{7.26 \times 10^{-2}}{0.56 \times 10^{-4} - 1.6 \times 10^{-5} (-0.04 + V_{EC})^{1/2}}$$

$$Ic(Vec) := \frac{7.26 \cdot 10^{-2}}{.56 \cdot 10^{-4} - 1.6 \cdot 10^{-5} \cdot \sqrt{-.04 + Vec}}$$

Vec := 1, 1.2 .. 2



∴ Early Voltage $\simeq -3.8$ V

d)

$$W = W_B - x_{nEB} - x_{nCB}$$

$$= 0.6\times10^{-4} - \left[\frac{2K_s\epsilon_0}{q}\cdot\frac{N_E}{N_B(N_E+N_B)}\cdot(V_{biEB}-V_{EB})\right]^{1/2}$$

$$- \left[\frac{2K_s\epsilon_0}{q}\cdot\frac{N_C}{N_B(N_C+N_B)}\cdot(V_{biCB}-V_{CB})\right]^{1/2}$$

where $V_{biEB} = 0.026 \ln \frac{N_E N_B}{n_i^2} = 0.874$

$V_{biCB} = 0.026 \ln \frac{N_C N_B}{n_i^2} = 0.76$

$V_{EB} = 0.5\,V$

$$W = 0.6\times10^{-4} - 0.96\times10^{-5} - \left[0.43\times10^{-10}(0.76-V_{CB})\right]^{1/2}$$

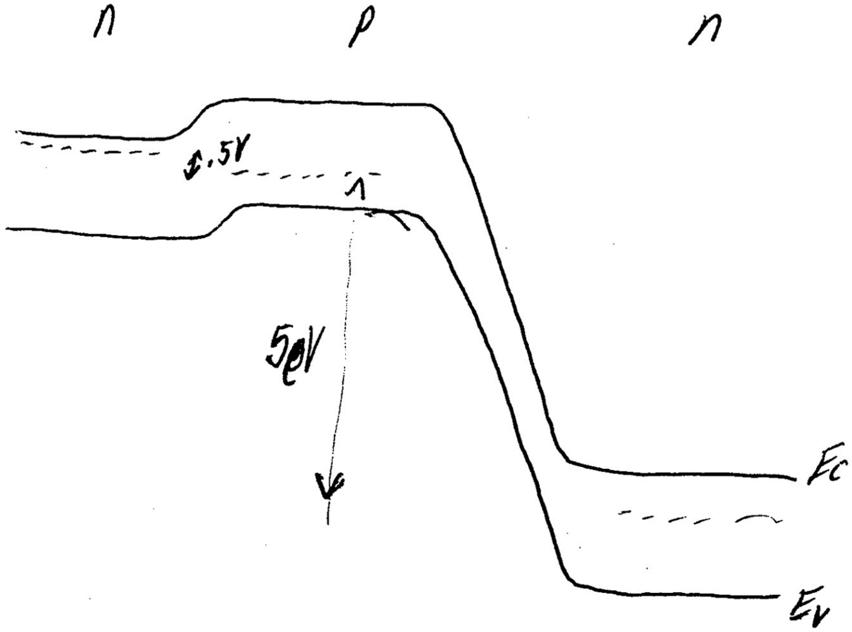
the condition for punch through is $W=0$, so,

$$0 = 5.04\times10^{-5} - 0.65\times10^{-5}\sqrt{0.76-V_{CB}}$$

$\therefore\ V_{CB} \simeq -59.36$ (V)

Problem 1, midterm exam #2, spring 2007

6. a) Forward Active Bias



Emitter $N_d = 10^{17}$ or 10^{16}

$$E_c - E_f = -kT \ln \frac{N_d}{N_c} = .21eV \text{ for } N_d = 10^{16}$$
$$= .15eV \text{ " } N_d = 10^{17}$$

Base $N_a = 10^{15} cm^{-3}$

$$E_v - E_f = -kT \ln \frac{N_a}{N_v} = .24eV$$

Collector $N_d = 10^{14}$

$$E_c - E_f = -kT \ln \frac{N_d}{N_c} = .33eV$$

b) $I_x \Leftrightarrow I_E$ for npn

$I_z \Leftrightarrow I_c$

This is forward active so we can assume $e^{-qV_x/kT} \gg 1 \gg e^{qV_z/kT}$

$$I_E = I_x = q n_i^2 A \left(\frac{D_{E_1}}{2W_{E_1} N_{E_1}} + \frac{D_{E_2}}{2W_{E_2} N_{E_2}} + \frac{D_B}{W_B N_B} \right)$$

$2mm^2$

$\sim 3\mu m$

$$\times e^{-qV_x/kT}$$

$$I_C = q n_i^2 A \left[\left(\frac{D_B}{W_B N_B} \right) - \left(\frac{W_B}{2\tau_B} \frac{1}{N_B} \right) \right] e^{-qV_x/kT}$$

$$N_B = 10^{15} cm^{-3} \quad A = 2mm^2$$

$$W_B = 3\mu m - x_{pE} - x_{pC}$$

$$= 3\mu m - \sqrt{\frac{2\epsilon}{q}(V_{bi_E} - .5)\frac{N_d}{N_a}\frac{1}{(N_a + N_d)}}$$

$$- \sqrt{\frac{2\epsilon}{q}(V_{bi_C} + 5V)\frac{N_d}{N_a}\frac{1}{(N_a + N_d)}}$$

$$V_{bi_E} \approx .65V \text{ or } .71V \left(\frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} \right)$$

$$V_{bi_C} \approx .53V$$

$$W_B = 3\mu m - .5\mu m - .81\mu m$$

$$W_B \approx 1.7\mu m$$

$$D_B = \frac{kT}{q}\mu_{n_B}(10^{15}) = (.026)(1345)$$

$$D_B = 35 \frac{cm^2}{s}$$

$$I_Z (= I_C) = q n_i^2 A \left(\frac{D_B}{W_B N_B} - \underbrace{\frac{W}{2\tau_B}\frac{1}{N_B}}_{\text{very small compared to } \frac{D}{WN}} \right) e^{-qV_x/kT}$$

$$\boxed{I_Z \approx 18 mA}$$

c.)

$$\boxed{\gamma = \frac{\dfrac{D_B}{W_B N_B}}{\dfrac{D_{E_1}}{2W_{E_1}N_{E_1}} + \dfrac{D_{E_2}}{2W_{E_2}N_{E_2}} + \dfrac{D_B}{W_B N_B}}}$$

$$\frac{D_B}{W_B N_B} \approx 2.1 \times 10^{-10}$$

$$W_{E_1} \sim W_{E_2} \sim 3\mu m$$

(Note that widest $x_{nE} < .05\mu m$)
($< 2\%$ of W_E)

$$D_{E_1} = \frac{kT}{q}\mu_p(10^{17}) = (.026)(331) = 8.6 \frac{cm^2}{s}$$

$$D_{E_2} = \frac{kT}{q}\mu_p(10^{16}) = (.026)(437) = 11.4 \frac{cm^2}{s}$$

$$\frac{D_{E_1}}{2W_{E_1}N_{E_1}} = 1.4 \times 10^{-13} \qquad \frac{D_{E_2}}{2W_{E_2}N_{E_2}} = 1.9 \times 10^{-12}$$

$$\boxed{\gamma = .99}$$

$$\boxed{\alpha_T = 1 - \frac{W_B^2}{2L_B^2}}$$

$$L_B = \sqrt{D_B \tau_B}$$
$$= \sqrt{35 \tfrac{cm^2}{s}(.1\times10^{-6}s)}$$
$$\underline{L_B \approx 19\mu m}$$

$$\alpha_T = 1 - \frac{1}{2}\left(\frac{1.7}{19}\right)^2$$

$$\boxed{\alpha_T = .996}$$

$$\boxed{\beta = \frac{\gamma\alpha_T}{1-\gamma\alpha_T}} \qquad \boxed{\beta \approx 71}$$

d.) $I_{CC} = I_2 = 18mA$

$$I_{BB} = \frac{18mA}{20} \quad I_{BB} = .9mA$$

$$t_r = \tau_B \ln\left[\frac{1}{1-\left(\frac{18}{.9}\,\frac{1}{71}\right)}\right]$$

$\tau_B / t_t = \beta$

could also use $t_t = \frac{W^2}{2D_B}$

$$t_r = .1\mu s(.33)$$

$$\boxed{t_r = 33ns}$$

e.)

$$I_C = \frac{M\alpha_{dc}}{1 - M\alpha_{dc}} I_B$$

Now $\frac{\alpha_{dc}}{1-\alpha_{dc}} = 71$

$\therefore$ Let "significant" mean 10%
(Other reasonable values ok.)

Find $\frac{M\alpha_{dc}}{1-M\alpha_{dc}} \approx 78$ $\alpha_{dc} = .986$

$$(M\alpha_{dc}) = 78 - 78M\alpha_{dc}$$

$$79M\alpha_{dc} = 78$$

$$M\alpha_{dc} = \frac{78}{79}$$

$$M = 1.001$$

$$M = \frac{1}{1 - \left(\frac{V_{CE}}{V_{CBO}}\right)^m}$$ let $m = 6$

$$V_{CBO} = \frac{\epsilon}{2q} \frac{N_a + N_d}{N_a N_d} \mathcal{E}_{cr}^2$$

$$\underline{V_{CBO} = 5700 \text{ V}}$$

$$1-\left(\frac{V_{CE}}{V_{CBO}}\right)^6 = \frac{1}{M}$$

$$1-\frac{1}{M} = \left(\frac{V_{CE}}{V_{CBO}}\right)^6$$

$$V_{CE} = V_{CBO}\left(1-\frac{1}{M}\right)^{1/6} = 5700\left(1-\frac{1}{1.001}\right)^{1/6}$$

$$\underline{V_{CE} \sim V_Z \sim 1800V}$$

Not a significant contribution to base current until this value is reached (~1800V) but significant beyond that.