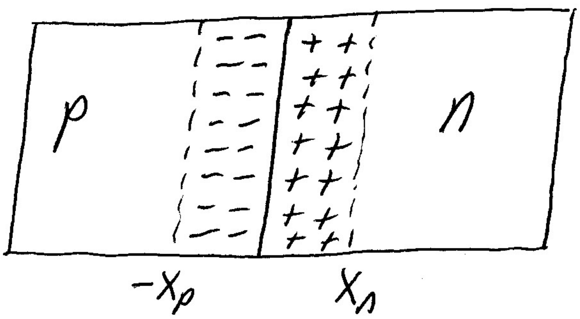
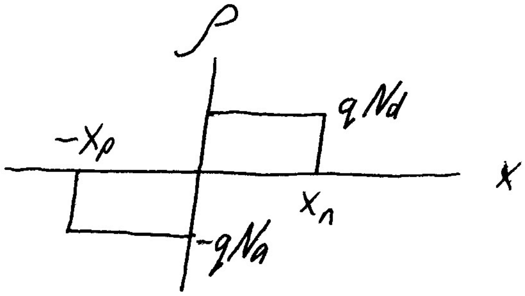
Derivation of Depletion Widths in Depletion Approximation for an abrupt junction.

An uncompensated diode in the depletion approximation will have the following charge distribution:

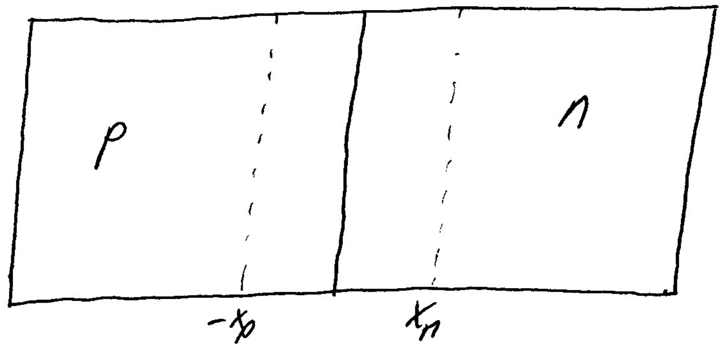


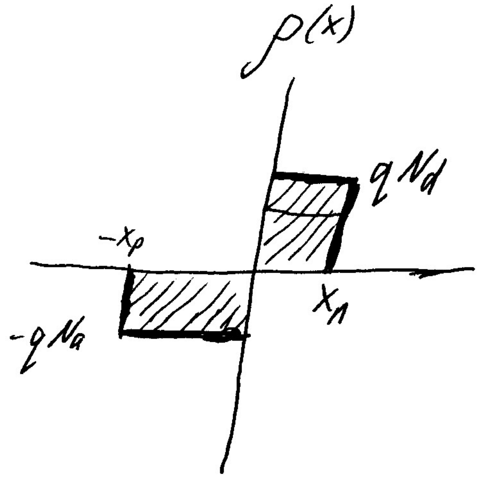


Recall that we are assuming that there is no free charge (electrons and holes) in the depleted region. Thus only the exposed dopant ions remain.

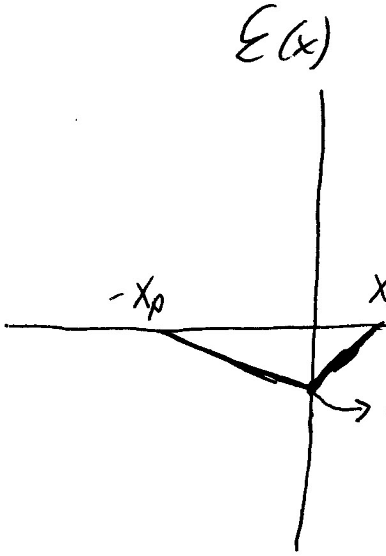
We need to find the widths x_n, x_p, and $W = x_n + x_p$. Fortunately we know N_a, N_d, and $V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)$. ($N_a$ and N_d will be known design parameters usually.)

Diagrams for Depletion Approximation

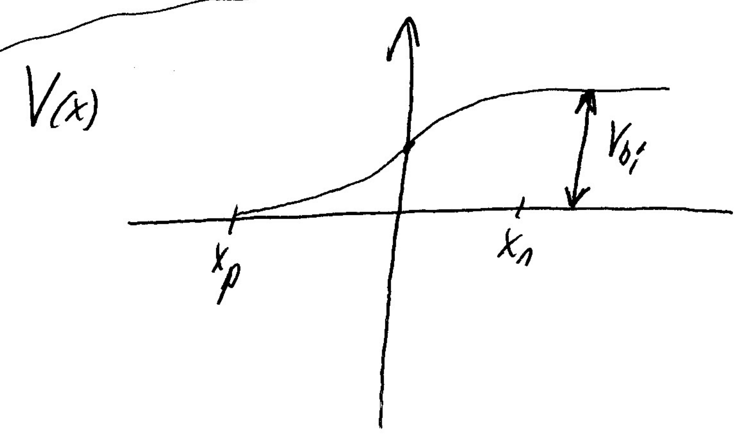




shaded regions
have equal area
$x_n N_d = x_p N_a$



at $x = 0$, $\mathcal{E}(0) = \frac{-qN_d}{\epsilon} x_n = \frac{-qN_a}{\epsilon} x_p$



Thus the basic procedure is as follows:
for a p-n diode

I. Find the Electric Field in the depleted region

$$\mathcal{E}(x) = \frac{1}{\epsilon}\int_{-\infty}^{x} \rho(x)\,dx$$

This is Gauss's Law

II. Find Voltage, $V(x)$ in the depleted region

$$V(x) = -\int_{-\infty}^{x} \mathcal{E}(x)\,dx$$

III. Set $V(x_n) = V_{bi}$ and solve for x_n, x_p and W

So let's try it.

I. $$\rho(x) = \begin{cases} qN_d & \text{for } 0 \le x \le x_n \\ -qN_a & \text{for } -x_p \le x \le 0 \\ 0 & \text{everywhere else} \end{cases}$$

$\therefore$ $\mathcal{E}(x)$ must be calculated in two regions:
$0 \le x \le x_n$ and $-x_p \le x \le 0$

For $-x_p \leq x \leq 0$ (p-depletion)

$$\mathcal{E}(x) = \int_{-x_p}^{x} -\frac{qN_a}{\epsilon}\,dx$$

$$= \frac{qN_a}{\epsilon}x\Big|_{-x_p}^{x} = -\frac{qN_a x}{\epsilon} - -\frac{qN_a}{\epsilon}(-x_p)$$

$$\underline{\mathcal{E}(x) = -\frac{qN_a}{\epsilon}(x_p + x) \quad \text{for } -x_p \leq x \leq 0}$$

For $0 \leq x \leq x_n$ (n-depletion)

$$\mathcal{E}(x) = \int_{-x_p}^{0} -\frac{qN_a}{\epsilon}\,dx + \int_{0}^{x} \frac{qN_d}{\epsilon}\,dx$$

$$\mathcal{E}(x) = -\frac{qN_a}{\epsilon}x_p + \frac{qN_d}{\epsilon}x\Big|_0^x$$

$$\mathcal{E}(x) = -\frac{qN_a x_p}{\epsilon} + \frac{qN_d x}{\epsilon} \quad \text{for } 0 \leq x \leq x_n$$

At this point, we should simplify the above expression. To do this, let's develop an expression for the charge on both sides of the junction. Since the diode as a whole is charge neutral, the charge on each side of the junction must be equal and opposite.

i.e. $\left| -q N_a x_p \underset{\text{area}}{A} \right| = q N_d x_n A$

$$q N_a x_p A = q N_d x_n A$$

$$\underline{N_a x_p = N_d x_n}$$

Thus we can rewrite

$$\mathcal{E}(x) = \frac{-q N_a x_p}{\epsilon} + \frac{q N_d x}{\epsilon} \quad \text{for} \quad 0 \leq x \leq x_n$$

as

$$\mathcal{E}(x) = -\frac{q N_d x_n}{\epsilon} + \frac{q N_d x}{\epsilon}$$

$$\underline{\mathcal{E}(x) = -\frac{q N_d}{\epsilon}(x_n - x) \quad \text{for} \quad 0 \leq x \leq x_n}$$

$$\mathcal{E}(x) = \begin{cases} -\frac{q N_a}{\epsilon}(x_p + x) & \text{for } -x_p \leq x \leq 0 \\ -\frac{q N_d}{\epsilon}(x_n - x) & \text{for } 0 \leq x \leq x_n \end{cases}$$

II.

$$V(x) = -\int_{-\infty}^{x} \mathcal{E}(x)\, dx$$

First take region $-x_p \leq x \leq 0$ (p-depletion)

$$V(x) = -\int_{-x_p}^{x} -\frac{q N_a}{\epsilon}(x_p + x)\,dx \quad \text{for} \quad -x_p \leq x \leq 0$$

$$= \left[\frac{q N_a}{\epsilon} x_p x + \frac{q N_a}{\epsilon}\frac{x^2}{2} \right]_{-x_p}^{x}$$

$$V(x) = \frac{qN_a}{\epsilon} x_p x + \frac{qN_a}{\epsilon} x_p^2 + \frac{qN_a}{2\epsilon} x^2 - \frac{qN_a}{2\epsilon} x_p^2$$

$$V(x) = \frac{qN_a}{\epsilon}\left(x_p x + x_p^2 + \frac{1}{2}x^2 - \frac{1}{2}x_p^2\right)$$

$$V(x) = \frac{qN_a}{2\epsilon}\left(x^2 + 2x_p x + x_p^2\right)$$

$$\underline{V(x) = \frac{qN_a}{2\epsilon}(x + x_p^2) \quad \text{for } -x_p \le x \le 0}$$

Now take region $0 \le x \le x_n$

$$V(x) = -\int_{-\infty}^{x} \mathcal{E}(x)\,dx$$

$$= -\left[\int_{-x_p}^{0} \mathcal{E}(x)\,dx + \int_{0}^{x} \mathcal{E}(x)\,dx\right] \text{ for } 0 \le x \le x_n$$

$$= \frac{qN_a}{2\epsilon} x_p^2 + \int_0^x \frac{qN_d}{\epsilon}(x_n - x)\,dx$$

$$= \frac{qN_a}{2\epsilon} x_p^2 + \left(\frac{qN_d}{\epsilon} x_n x - \frac{qN_d}{\epsilon}\frac{x^2}{2}\right)\Big|_0^x$$

$$V(x) = \frac{qN_a}{2\epsilon} x_p^2 + \frac{qN_d}{\epsilon} x_n x - \frac{qN_d}{\epsilon}\frac{x^2}{2}$$

At $x = x_n$, $V(x_n) = V_{bi}$

$$\therefore V_{bi} = \frac{qN_a}{2\epsilon} x_p^2 + \frac{qN_d}{2\epsilon} x_n^2$$

$$\frac{qN_a}{2\epsilon} x_p^2 = V_{bi} - \frac{qN_d}{2\epsilon} x_n^2 \quad (*)$$

Now use eqn. (*) to rewrite $V(x)$

$$V(x) = V_{bi} - \frac{qN_d}{2\epsilon} x_n^2 + \frac{qN_d}{2\epsilon} 2x_n x - \frac{qN_d}{2\epsilon} x^2$$

$$\underline{V(x) = V_{bi} - \frac{qN_d}{2\epsilon}(x + x_n)^2 \text{ for } 0 \le x \le x_n}$$

III,

$$V(x_n) = V_{bi}$$

$$\frac{qN_d}{2\epsilon} x_n^2 + \frac{qN_a}{2\epsilon} x_p^2 = V_{bi}$$

Also $\quad x_n N_d = x_p N_a$

$$x_p = x_n \frac{N_d}{N_a}$$

$$\frac{qN_d}{2\epsilon} x_n^2 + \frac{qN_a}{2\epsilon} \frac{N_d^2}{N_a^2} x_n^2 = V_{bi}$$

$$x_n^2 = \frac{2\epsilon}{q} V_{bi} \frac{1}{\left[\frac{N_d^2}{N_a} + N_d\right]}$$

$$\boxed{x_n = \left[\frac{2\epsilon}{q} V_{bi} \frac{N_a}{N_d(N_a + N_d)}\right]^{1/2}}$$

similarly for x_p

$$\boxed{x_p = \left[\frac{2\epsilon}{q} V_{bi} \frac{N_d}{N_a(N_a + N_d)}\right]^{1/2}}$$

And the total width of the depletion region is:

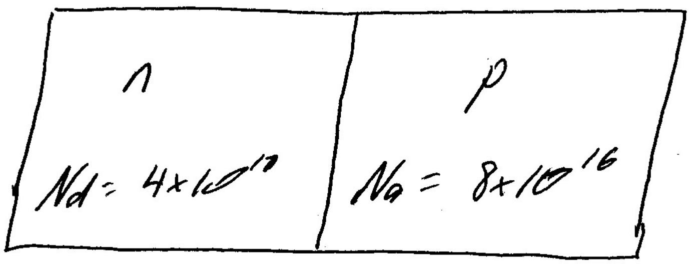
$$W = x_n + x_p$$

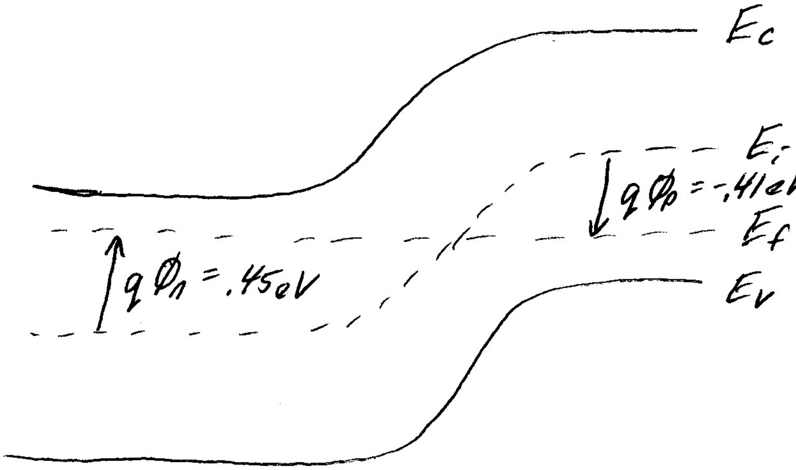
$$\boxed{W = \left[\frac{2\epsilon V_{bi}}{q} \cdot \frac{N_a + N_d}{N_a N_d}\right]^{1/2}}$$

from x_n & x_p we can see that the depletion region extends further into a lightly doped region rather than a more heavily doped one. This is due to the necessity to have an equal amount of depletion charge on both sides of the junction.

Example 1:

For the following np junction, draw the band diagram and find V_{bi}, x_n, x_p, W. Also draw $\rho(x)$, $\mathcal{E}(x)$, and $V(x)$ and find ϕ_n + ϕ_p





$$q\phi_n = E_f - E_i = kT\ln\frac{n}{n_i} = .45eV$$

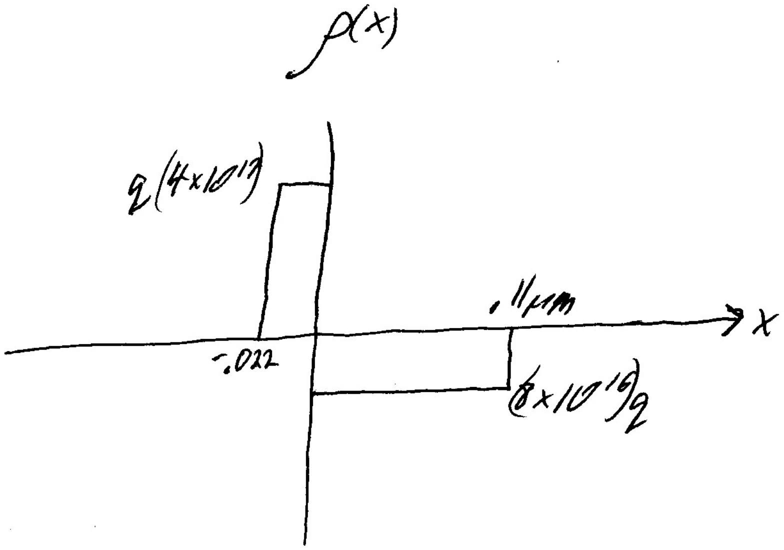
$$q\phi_p = E_f - E_i = -kT\ln\frac{p}{n_i} = -.41eV$$

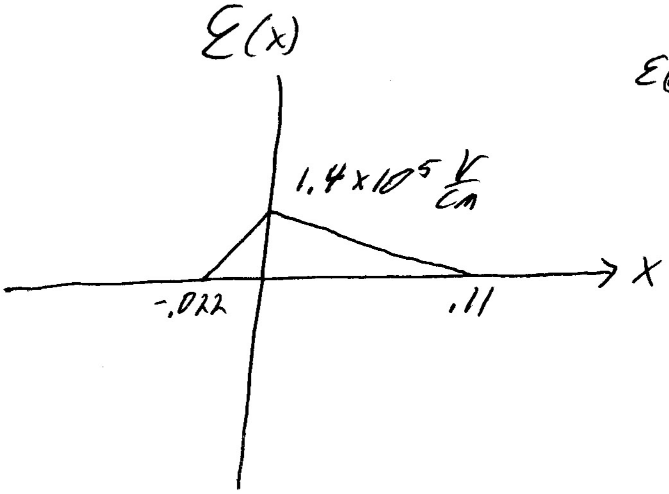
$$\boxed{V_{bi} = |\phi_n| + |\phi_p| = .86\ V}$$

$$\boxed{x_p = \sqrt{\frac{2\epsilon}{q}V_{bi}\frac{N_d}{N_a(N_a+N_d)}} = .11\,\mu m}$$

$$\boxed{x_n = \sqrt{\frac{2\epsilon}{q}V_{bi}\frac{N_a}{N_d(N_a+N_d)}} = .022\,\mu m}$$

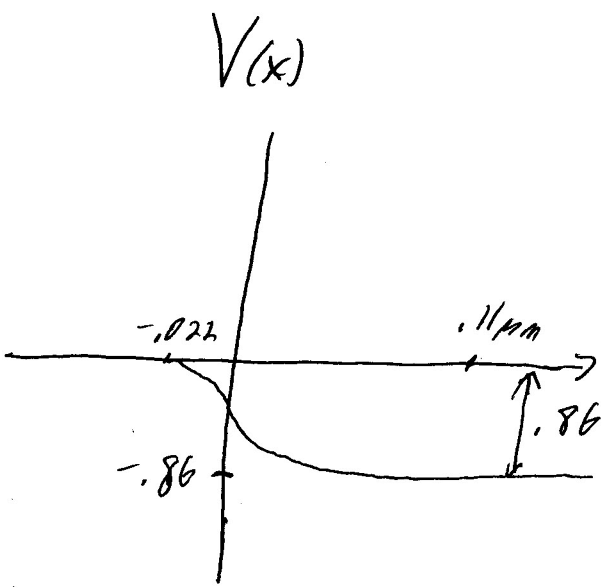
$$\boxed{W = x_p + x_n = .132\,\mu m}$$





$$\mathcal{E}(0) = +\frac{qN_a}{\epsilon}x_p = +\frac{qN_d}{\epsilon}x_n$$

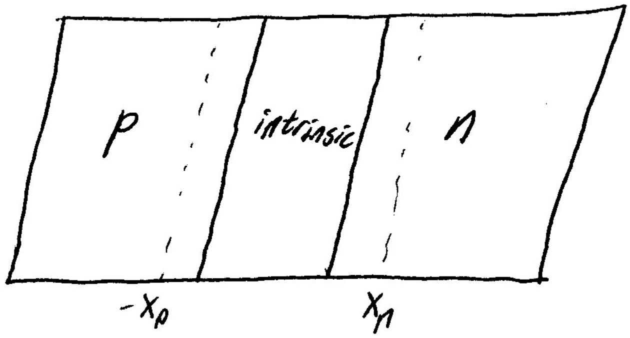
$$\mathcal{E}(x) = \int \frac{\rho(x)}{\epsilon}\,dx$$

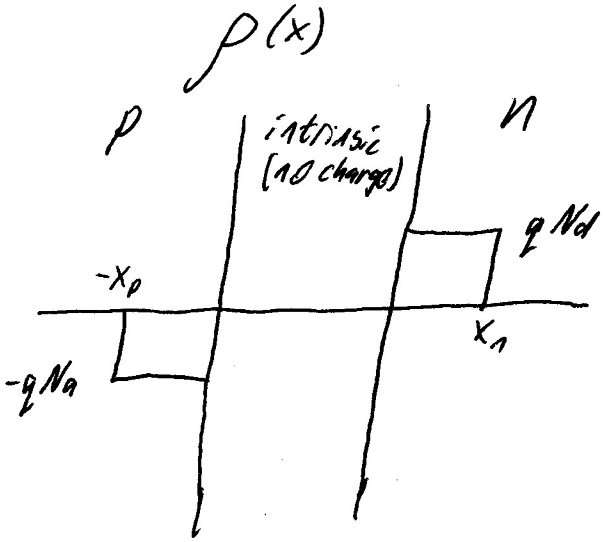


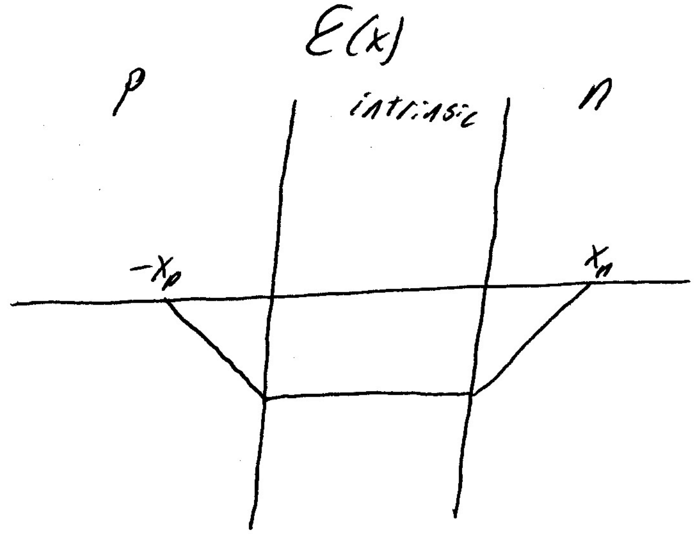
$$V(x) = -\int \mathcal{E}(x)\,dx$$

Example 2

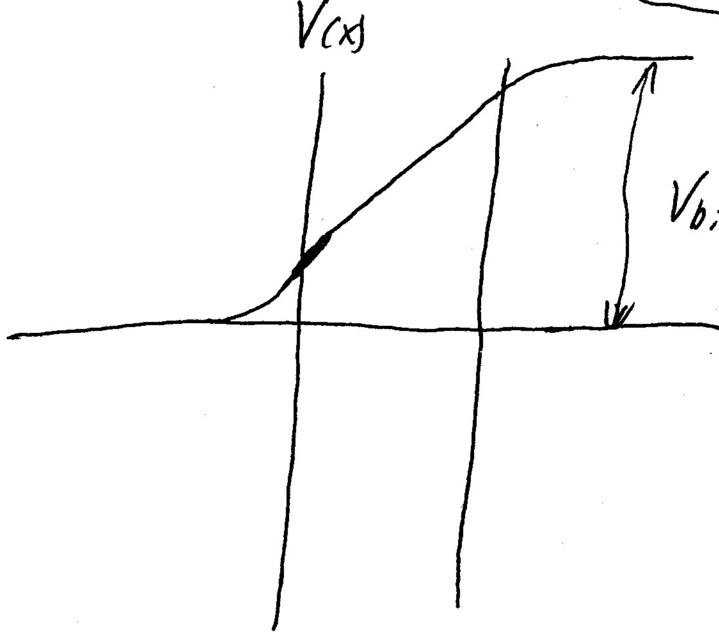
Sketch $\rho(x)$, $\mathcal{E}(x)$, $V(x)$ for the following diode in equilibrium.







$$\mathcal{E}(x) = \int \frac{\rho(x)}{\epsilon}\,dx$$



$$V(x) = -\int \mathcal{E}(x)\,dx$$