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Diversity-Multiplexing tradeoff in the Rayleigh Fading Relay Channel

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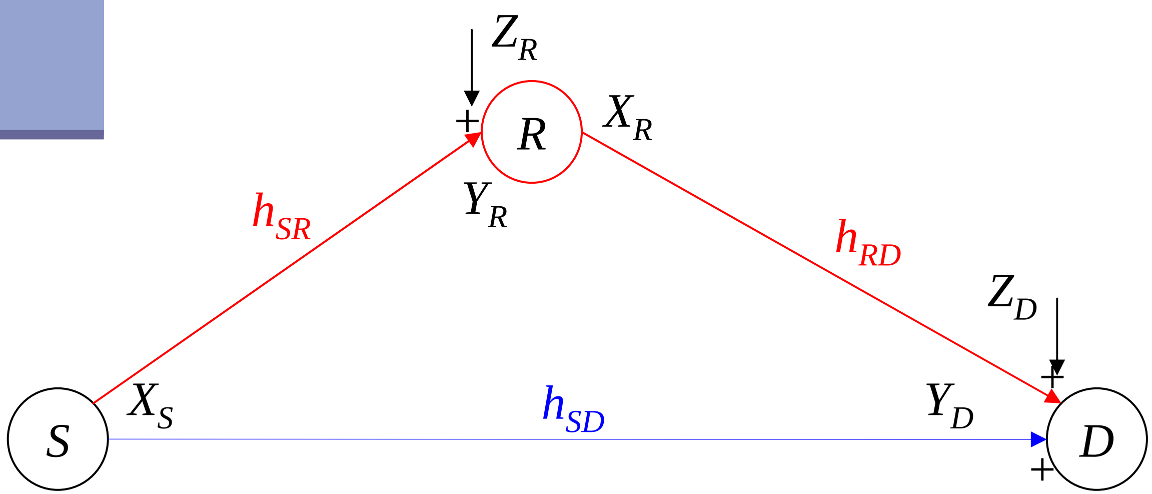
Relay Channel / Diversity-Mux

- Rayleigh fading relay channel

$$Y_D = \sqrt{h_{SD}}\, e^{j\phi_{SD}} X_S + \sqrt{h_{RD}}\, e^{j\phi_{RD}} X_R + Z_D$$
$$Y_R = \sqrt{h_{SR}}\, e^{j\phi_{SR}} X_S + Z_R$$

- SNR: $\gamma = \frac{P_S}{N_D} = \frac{P_S}{N_R} = \frac{P_R}{N_D}.$



- Diversity gain

$$d := -\lim_{\gamma\to\infty} \frac{\log[P_{out}(\gamma)]}{\log\gamma}$$

- Multiplexing gain

$$r := \lim_{\gamma\to\infty} \frac{R(\gamma)}{\log\gamma}$$

Ergodic Capacity

- Capacity depends on channel realization

$$C = C(h_{SD}\ e^{j\phi_{SD}}, h_{SR}\ e^{j\phi_{SR}}, h_{RD}\ e^{j\phi_{RD}})$$

- Ergodic – all channel realizations in one packet

$$\bar{C} = \mathsf{E}[C(h_{SD}\ e^{j\phi_{SD}}, h_{SR}\ e^{j\phi_{SR}}, h_{RD}\ e^{j\phi_{RD}})]$$

- Max-flow min-cut

$$C < \max_{\rho\in[0,1]} \min \left\{ \begin{array}{l} \log\left(1 + \left(h_{SD} + h_{SR} + 2\rho\sqrt{h_{SD}h_{RD}}\ |e^{j(\phi_{SD}+\phi_{RD})}|\right)\gamma\right) \\ \log(1 + (h_{SD} + h_{RD})(1-\rho^2)\gamma) \end{array} \right\}$$

- Average over phases

$$\bar{C} < \mathsf{E}\left[\min\left[\log(1 + (h_{SD} + h_{SR})\gamma), \log(1 + (h_{SD} + h_{RD})\gamma)\right]\right]$$

- Prevents coherent superposition

Outage Capacity / Markov coding

- Prob. that a rate is not achievable

$$P_{out}(R_{out}) = \mathrm{Pr}\{C(h_{SD}\ e^{j\phi_{SD}}, h_{SR}\ e^{j\phi_{SR}}, h_{RD}\ e^{j\phi_{RD}}) < R_{out}\}$$

- Markov coding achievable rate (capacity lower bound)

$$C > I_{MC} = \min\left[\log(1 + h_{SR}\gamma), \log(1 + (h_{SD} + h_{RD})\gamma)\right]$$

- Outage probability upper bound:

$$Pout(R_{out}) < P_{out}^{MC}(R_{out}) \ := \ \mathrm{Pr}\{I_{MC} < R_{out}\}$$

- Channel power outage: $h_{out} = \dfrac{2^{R_{out}} - 1}{\gamma}$
- Markov coding outage: $P_{out}^{MC}(h_{out}) = \mathrm{Pr}\{\min(h_{SR}, h_{SD} + h_{RD}) < h_{out}\}$
- Large SNR behavior: $P_{out}^{MC}(h_{out}) \sim \dfrac{h_{out}}{\bar{h}_{SR}} + \dfrac{h_{out}^2}{2\bar{h}_{SD}\bar{h}_{RD}} \sim \dfrac{h_{out}}{\bar{h}_{SR}}$

  No diversity gain

Adaptive Markov coding (AMC)

- Cooperate only if h_{SR} is good

$$C > I_{AMC} = \begin{cases} \log(1 + h_{SD}\gamma), & h_{SR} < h_{out} \\ \log(1 + (h_{SD} + h_{RD})\gamma), & h_{SR} > h_{out} \end{cases}$$

- AMC outage probability

$$\begin{aligned} P_{out}^{AMC}(h_{out}) = & \quad \Pr\{h_{SD} < h_{out}\} \Pr\{h_{SR} < h_{out}\} \\ & + \Pr\{h_{SD} + h_{RD} < h_{out}\} \Pr\{h_{SR} > h_{out}\} \end{aligned}$$

- Large SNR behavior: $P_{out}^{AMC}(h_{out}) \sim \left[\frac{1}{\bar{h}_{SD}\bar{h}_{SR}} + \frac{1}{2\bar{h}_{SD}\bar{h}_{RD}} \right] h_{out}^2$

- Capacity upper bound: $C < I_{TA} = \log(1 + (h_{SD} + h_{RD})\gamma)$

- Outage prob. lower bound: $P_{out}^{TA}(h_{out}) \sim \left[\frac{h_{out}^2}{2\bar{h}_{SD}\bar{h}_{RD}} \right]$

Diversity-Mux tradeoff

- Upper bound diversity gain

$$d := -\lim_{\gamma \to \infty} \frac{\log[P_{out}(\gamma)]}{\log \gamma} \leq -\lim_{\gamma \to \infty} \frac{\log[P_{out}^{TA}(\gamma)]}{\log \gamma}$$

- Use outage at high SNR behavior

$$d \leq -\lim_{\gamma \to \infty} \frac{\log(h_{out}^2)}{\log \gamma} = -2 \lim_{\gamma \to \infty} \frac{R_{out} - \log(\gamma)}{\log \gamma}$$

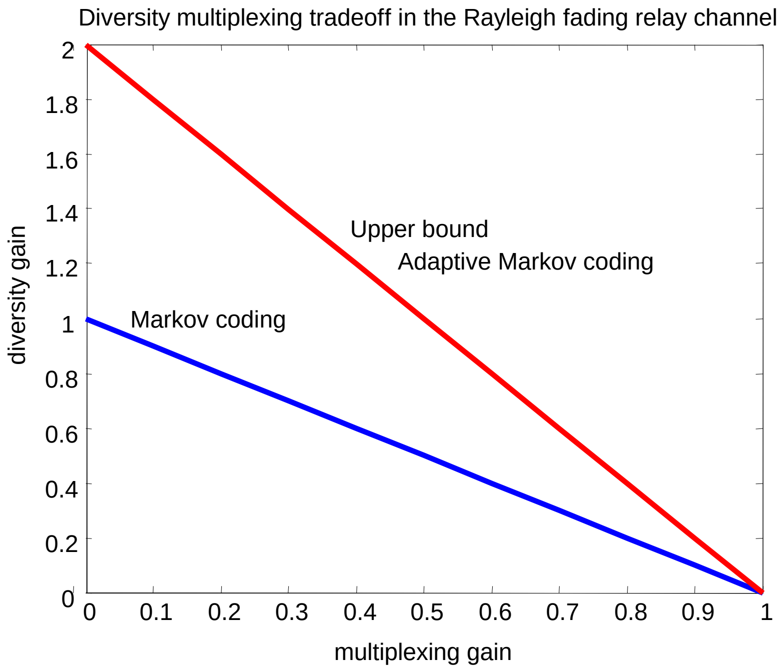
- Recall multiplexing gain definition ($r := \lim_{\gamma \to \infty} \frac{R(\gamma)}{\log \gamma}$)

$$d \leq 2(1 - r)$$

- Divesrity gain not greater than 2

Diversity-Mux tradeoff

- Diversity-rate curve for MC: $d^{MC} = (1 - r)$
- Diversity-rate curve for AMC: $d^{AMC} = 2(1 - r)$



- AMC achieves tradeoff upper-bound
- Best possible tradeoff

$$d^* = 2(1 - r)$$

- Achieved by AMC not by MC

Conclusions

- Studied diversity multiplexing tradeoff in Rayleigh fading relay channel
- Best achievable tradeoff : $d^* = 2(1-r)$
- Achieved by Adaptive version of Markov coding
  - Relays cooperate only if h_{SR} is good enough