On MIMO Fading Channels with Side Information at TX

EE8510 Project

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Introduction

- CSI available at both TX and RX

$$C_{TRSI} = \sup_{p(\mathbf{x}|\mathbf{S}) \in \Omega} I(X;Y|S)$$

- CSI available at RX only

$$C_{RSI} = \sup_{p(\mathbf{x}) \in \Omega} I(X;Y|S)$$

- CSI not available at either side

  Bounds and Many Results available

- CSI available at TX only

Channel Model

- MIMO fading:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1M} \\ s_{21} & s_{22} & \cdots & s_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NM} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix}$$

$x_m, y_n, s_{nm} \in \mathcal{R}$ $\quad$ $z_n \sim \mathcal{N}(0, 1)$

$\mathbf{y} = \mathbf{S}\mathbf{x} + \mathbf{z}$ $\quad$ $E[\|\mathbf{x}\|^2] \leq P$

$\mathbf{S}(i)$ $\quad$ Non-causal: $\{\mathbf{S}(k) | -\infty \leq k \leq \infty\}$

Causal: $\{\mathbf{S}(k) | -\infty \leq k \leq i\}$

Existing Results

Non-causal:

$$C_{nc} = \sup_{p(\mathbf{u}|\mathbf{S}),\ \mathcal{F}:\mathcal{U}\times\mathcal{S}\to\mathcal{X},\ E[\|\mathcal{F}(U,S)\|^2]\le P} \{I(U;Y) - I(U;S)\}$$

$$p(\mathbf{S},\mathbf{u},\mathbf{x},\mathbf{y}) = \begin{cases} p(\mathbf{S})p(\mathbf{u}|\mathbf{S})p(\mathbf{y}|\mathbf{x},\mathbf{S}) & \text{if } \mathbf{x} = \mathcal{F}(\mathbf{u},\mathbf{S}), \\ 0 & \text{otherwise} \end{cases}$$

Causal:

$$C_{c} = \sup_{p(\mathbf{u}),\ \mathcal{F}:\mathcal{U}\times\mathcal{S}\to\mathcal{X},\ E[\|\mathcal{F}(U,S)\|^2]\le P} I(U;Y)$$

$$p(\mathbf{S},\mathbf{u},\mathbf{x},\mathbf{y}) = \begin{cases} p(\mathbf{S})p(\mathbf{u})p(\mathbf{y}|\mathbf{x},\mathbf{S}) & \text{if } \mathbf{x} = \mathcal{F}(\mathbf{u},\mathbf{S}), \\ 0 & \text{otherwise} \end{cases}$$

Lower Bounds

Non-causal:

$$p_L(\mathbf{x}|\mathbf{S}) = \text{arg}\left\{\text{sup}_{p(\mathbf{x}|\mathbf{S})\in\Omega} I(X;Y|S)\right\}$$

$$p_L(\mathbf{u}|\mathbf{x},\mathbf{S}) = \begin{cases} \delta(\mathbf{u} - g_\beta(\mathbf{x},\mathbf{S})) & \text{if}\mathbf{S} \in \mathcal{S}, \\ Q_\beta(\mathbf{u}|\mathbf{S}) & \text{if}\mathbf{S} \in \overline{\mathcal{S}} \end{cases} \qquad \beta \equiv P/N$$

$$\mathcal{S} = \{\mathbf{S}|I(X;Y|\mathbf{S}) \neq 0, X \sim p_L(\mathbf{x}|\mathbf{S})\}$$

$$\overline{\mathcal{S}} = \{\mathbf{S}|I(X;Y|\mathbf{S}) = 0, X \sim p_L(\mathbf{x}|\mathbf{S})\}$$

Causal: $p_L(\mathbf{u}) \sim$ Gaussian

Upper Bounds

Non-causal:

$$C_{nc} \leq C_{TRSI} = \sup_{p(\mathbf{x}|\mathbf{S}) \in \Omega} I(X;Y|S)$$

Causal:

Lemma: $I(U;Y) \leq \sum_{\mathbf{u}} p(\mathbf{u}) D(w_{\mathcal{F}}(\cdot|\mathbf{u}) \| q(\cdot))$

$$D(w_{\mathcal{F}}(\cdot|\mathbf{u}) \| q(\cdot)) = \sum_{\mathbf{y}} \sum_{\mathbf{S}} p(\mathbf{y}|\mathcal{F}(\mathbf{u},\mathbf{S}),\mathbf{S}) p(\mathbf{S}) \ln \frac{\sum_{\mathbf{S}'} p(\mathbf{y}|\mathcal{F}(\mathbf{u},\mathbf{S}'),\mathbf{S}') p(\mathbf{S}')}{q(\mathbf{y})}$$

$$C_c(P) \leq \inf_{\gamma \geq 0} \sup_{\mathcal{F}} \sup_{u} \left\{ D(W_{\mathcal{F}}(\cdot|\mathbf{u}) \| Q(\cdot)) + \gamma \left(P - \int \mathcal{F}^2(u,s) dP(s) \right) \right\}$$

Application Example

Parallel fading channel: degraded MIMO

$$M = N = K$$

$$\mathbf{S} = \text{diag}(s_1, \ldots, s_K)$$

On/Off fading channel:

$$K = 2$$

$$P_r(s_1 = 1) = 1 - P_r(s_1 = 0) = \alpha_1$$

$$P_r(s_2 = 1) = 1 - P_r(s_2 = 0) = \alpha_2$$

Lower Bounds

Non-causal:

$$f_L(\mathbf{x}|\mathbf{S}) = \begin{cases} \delta(x_1)\delta(x_2) & s_1 = 0, s_2 = 0 \\ \frac{1}{\sqrt{2\pi P_1}} \exp(-\frac{x_1^2}{2P_1})\delta(x_2) & s_1 = 1, s_2 = 0 \\ \frac{1}{\sqrt{2\pi P_2}} \exp(-\frac{x_2^2}{2P_2})\delta(x_1) & s_1 = 0, s_2 = 1 \\ \frac{1}{\pi P_3} \exp(-\frac{x_1^2+x_2^2}{P_3}) & s_1 = 1, s_2 = 1 \end{cases}$$

$$P_1 = P_2 = \frac{P}{\alpha_1+\alpha_2} \qquad P_3 = \frac{2P}{\alpha_1+\alpha_2}$$

$$f_L(\mathbf{u}|\mathbf{x},\mathbf{S}) = f_L(u_1|x_1,s_1) f_L(u_2|x_2,s_2)$$

$$f_L(u_1|x_1,s_1) = \begin{cases} \delta(u_1 - x_1) & s_1 = 1 \\ \frac{1}{\sqrt{2\pi N {\psi_1}^2}} \exp(-\frac{u_1^2}{2N{\psi_1}^2}) & s_1 = 0 \end{cases}$$

$$f_L(u_2|x_2,s_2) = \begin{cases} \delta(u_2 - x_2) & s_2 = 1 \\ \frac{1}{\sqrt{2\pi N {\psi_2}^2}} \exp(-\frac{u_2^2}{2N{\psi_2}^2}) & s_2 = 0 \end{cases}$$

Lower Bounds

Non-causal:

$$C_{nc} \geq I(U;Y) - I(U;S)$$
$$= \iint \sum_{\mathbf{S}} p(\mathbf{S})p(\mathbf{u}|\mathbf{S})p(\mathbf{y}|\mathbf{u},\mathbf{S}) \ln \frac{\sum_{\mathbf{S}'} p(\mathbf{S}')p(\mathbf{u}|\mathbf{S}')p(\mathbf{y}|\mathbf{u},\mathbf{S}')}{\int \sum_{\mathbf{S}''} p(\mathbf{S}'')p(\mathbf{u}'|\mathbf{S}'')p(\mathbf{y}|\mathbf{u}',\mathbf{S}'')d\mathbf{u}' p(\mathbf{u}|\mathbf{S})} d\mathbf{y} d\mathbf{u}$$

Causal:

$$p(\mathbf{u}) = f_L(\mathbf{u}) = f_L(u_1)f_L(u_2)$$
$$f_L(u_1) = \frac{1}{\sqrt{2\pi P/(\alpha_1+\alpha_2)}} \exp(-\frac{u_1^2}{2P/(\alpha_1+\alpha_2)})$$
$$f_L(u_2) = \frac{1}{\sqrt{2\pi P/(\alpha_1+\alpha_2)}} \exp(-\frac{u_2^2}{2P/(\alpha_1+\alpha_2)})$$

$$C_c \geq I(U;Y)$$
$$= \iint \sum_{\mathbf{S}} p(\mathbf{S})p(\mathbf{u})p(\mathbf{y}|\mathbf{u},\mathbf{S}) \ln \frac{\sum_{\mathbf{S}'} p(\mathbf{S}')p(\mathbf{y}|\mathbf{u},\mathbf{S}')}{\int \sum_{\mathbf{S}''} p(\mathbf{S}'')p(\mathbf{u}')p(\mathbf{y}|\mathbf{u}',\mathbf{S}'')d\mathbf{u}'} d\mathbf{y} d\mathbf{u}$$

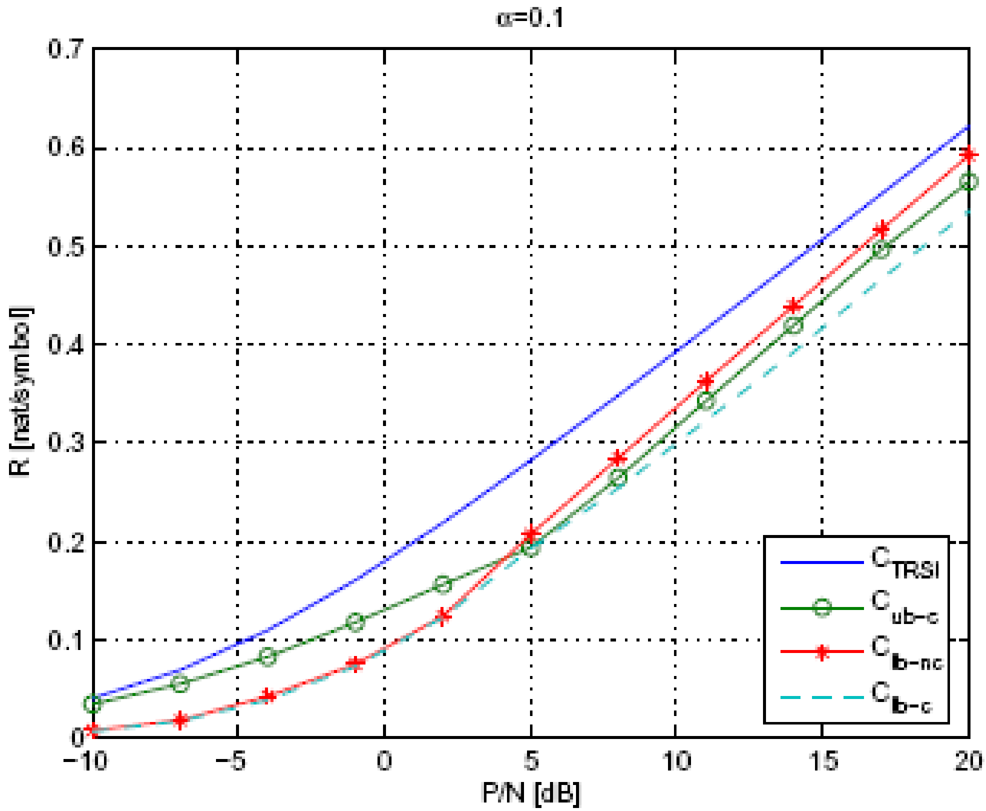
Upper Bounds

Non-causal:

$$C_{nc} \leq C_{TRSI} = \frac{\alpha_1 + \alpha_2}{2} \ln(1 + \frac{P}{N(\alpha_1 + \alpha_2)})$$

Causal:

Numerical Results



Mid-High SNR:

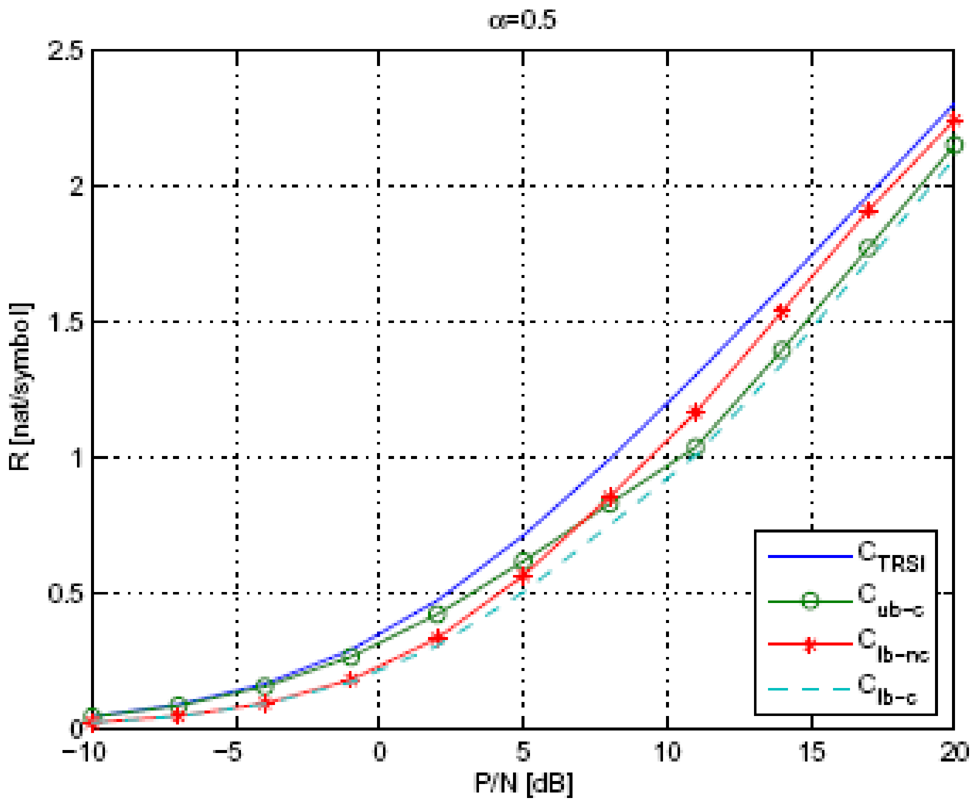
$$C_{nc} > C_c$$

High SNR:

$$C_{nc} \rightarrow C_{TRSI}$$

$$P_r(s_1 = 1) = P_r(s_2 = 1) = 0.1$$

Numerical Results



Mid-High SNR:

$$C_{nc} > C_c$$

High SNR:

$$C_{nc} \to C_{TRSI}$$

$$P_r(s_1 = 1) = P_r(s_2 = 1) = 0.5$$

Some Useful Strategies

Channel Inversion: [Goldsmith 1997]

$$\int_\gamma P(\gamma)p(\gamma)d\gamma \leq P \qquad P(\gamma)/P = \sigma/\gamma$$

$$\int \sigma/\gamma p(\gamma) = 1$$

$$\sigma = 1/E[1/\gamma]$$

$$C(P) = B\log[1+\sigma] = B\log[1+\frac{1}{E[1/\gamma]}]$$

Rayleigh & On/Off fading:

$$E[1/\gamma] = \infty \rightarrow C = 0$$

Some Useful Strategies

Log-DPC: $y_i = s_i x_i + z_i \quad \{e^{k} \mid -\infty \leq k \leq \infty\}$

$E[X_i^2] \leq P \qquad \mathcal{C} \rightarrow \mathcal{R}$ [Shamai 2004]

$P/N \rightarrow \infty \qquad \ln y_i \rightarrow \ln s_i + \ln x_i$

$Y = S + X \quad X = \ln x_i \quad S = \ln s_i$

$U \in [-1, 1]$ desired signal

$X = [U - S]_{[-1,1]}$ $\qquad [-1, 1]$ modulo

$x_i = \exp(X)$

$Y = \ln y_i$

$Y' = [Y]_{[-1,1]} = [X + S] = [(U - S) + S] = [U]$

Conclusion

- Scalar to MIMO: Exponential Complexity
- Tradeoff: Rate & Complexity
- Basic: "$C_{TRSI} \geq C_{nc} \geq C_c$"
- CSI at TX only is also very helpful!

Thank you!