EE 8510 Project Report:
Achievable Rates of Transmit-Reference Ultra-Wideband Radio with PPM

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Abstract

In this paper, we study the achievable rates of practical ultra-wideband (UWB) systems using pulse position modulation (PPM) and transmitted-reference (TR) transceivers. TR obviates the need for complex channel estimation, which is particularly challenging in the context of UWB. Based on an upper bound we derived for the error probability with random coding, we establish that for SNR values of practical interest, PPM-UWB with TR can achieve rates on the order of $C(\infty) = P/N_0$ (nats/second).

1 Introduction

Ideal for providing short-range high-rate wireless connectivity in a personal area network (PAN), ultra-wideband (UWB) technology (a.k.a. impulse radio (IR)) relies on ultra-narrow pulses (at nanosecond scale) to convey information, and has received a lot of attention recently. However, there are still major design challenges to overcome. For instance, timing synchronization with pulse level accuracy is difficult due to the fact that the transmitted pulse duration is very small. What is more, the channel typically consists of hundreds of multi-path returns, which renders channel estimation prohibitively costly. For this reason, the RAKE receiver, which is typically adopted to collect the multi-path energy, is not as efficient in the context of UWB.

To overcome these difficulties, transmitted-reference (TR) transceivers relying on non-coherent detection have received revived interest for UWB systems (see [4] and [8]). TR entails two pulses per symbol period: the first is unmodulated, while the second one is information bearing and delayed relative to the first by an amount exceeding the channel's delay spread. This way, the first pulse can serve as a template at the receiver side to demodulate the message carried by the second one. Clearly, the costly channel estimation required in RAKE reception is bypassed by TR.

Motivated by the work of Souilmi and Knopp (see [9]), where the achievable rates of UWB using pulse position modulation (PPM) and energy detection are examined, we will investigate here the achievable rates of UWB radios using PPM and TR transceivers. The rest of this report is organized as follows: Section II describes the system, while Section III deals with the derivation of detection error probability and the calculation of achievable rates. In Section IV, numerical results of the achievable rates are provided and compared against the AWGN channel capacity. Finally, conclusions are drawn in Section V.

2 Modeling

2.1 Channel Model

The multi-path fading channel is modelled as:

$$h(t,\tau) = \sum_{l=0}^{L-1} a_l(t)\delta(\tau - \tau_l(t)), \tag{1}$$

where L is the number of paths, $a_l(t)$ is the gain of path l at time t, and $\tau_l(t)$ is the corresponding delay. Without loss of generality, we assume $\tau_0 = 0 < \tau_1 < ... < \tau_{L-1}$ with τ_{L-1} denoting the channel delay spread. In this work, we are interested in the scenario where the channel has a coherent period T_c which is much larger than τ_{L-1}, i.e., $T_c \gg \tau_{L-1}$. We consider a block fading channel, i.e., $\{a_l(t), \tau_l(t)\}$ remain constant over each T_c-period and are independent across coherence periods.

In the UWB regime, extremely large bandwidth ($\geq$ 1GHz) enables the receiver to resolve a large number of paths. If the channel has a high diversity order (i.e. in dense multi-path fading environments), the aggregate channel gain $\sum_{l=0}^{L-1} a_l^2(t)$ varies slowly compared to $a_l(t)$ and $\tau_l(t)$. We can thus assume for all practical purposes that the total channel gain is constant and, without loss of generality, can be normalized to 1; i.e., $\sum_{l=0}^{L-1} a_l^2(t) = 1$ (see [9]).

2.2 Transmitter Structure

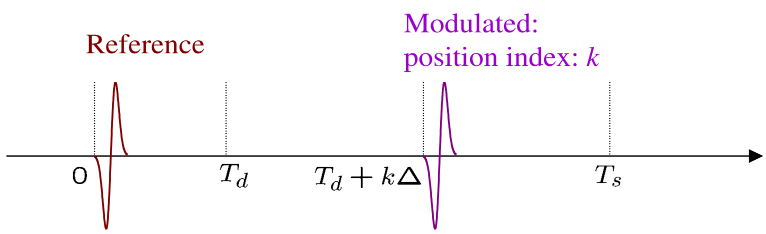

Figure 1: PPM with TR.

Flash-signaling has been shown to enjoy first order optimality (capacity achieving) in the wide-band regime even when the receiver does not have channel knowledge [12]. As a practical means of implementing flash-signaling, we adopt base-band PPM to transmit information bits. In particular, the transmitted waveform per channel use is (see also Fig. 1),

$$x(t;k) = \sqrt{PT_s}\left[\alpha p(t) + \beta p(t - T_d - k\Delta)\right], \quad k \in [0, m-1], \quad t \in [0, T_s], \tag{2}$$

where P is the transmission power; $p(t)$ is the normalized monocycle with duration $T_p \approx 1/W$ and W is the UWB bandwidth; and α, β are positive scalars satisfying $\alpha^2 + \beta^2 = 1$, which will be optimized later. Delay T_d is chosen such that $T_d \geq \tau_{L-1} + T_p$ and the symbol period T_s is chosen such that $T_s = 2T_d + (m-1)\Delta < T_c$, which avoids inter-symbol interference (ISI).

In order to transmit M messages, we generate a codebook at random: $\mathcal{C} = \{C_1, ..., C_M\}$. Each codeword C_w is a length-N sequence $C_w = [C_{w,1}, ..., C_{w,N}]$ with $C_{w,n}$ specifying the transmitted waveform during the n^{th} channel use when message w is sent. Codewords' entries $\{C_{w,n}\}_{n=1,...,N;w=1,...,M}$ are independently generated according to the uniform distribution over $[0, m-1]$. The aggregate transmitted waveform for message w is thus,

$$u_w(t) = \sum_{n=1}^{N} x(t - (n-1)T_s; C_{w,n}), \quad w \in [1, ..., M], \quad t \in [0, NT_s]. \tag{3}$$

2.3 Receiver Structure

Assuming that message $w = 1$ has been sent, after propagation through the channel in (1), the received signal is then:

$$r(t) = h(t) \star u_1(t) + z(t), \tag{4}$$

where "$\star$" stands for linear convolution and $z(t)$ denotes AWGN with double-sided power spectrum density $N_0/2$. When the system has bandwidth W, the temporal resolution is approximately $1/W \approx T_p$. Let T_d be chosen such that $T_d = K_d T_p$; then, within each coherence period, we can have an equivalent channel [c.f. (1)]:

$$h(\tau) = \sum_{q=0}^{K_d-1} \tilde{a}_q \delta(\tau - qT_p), \tag{5}$$

where $\tilde{a}_q = \sum_{l=0}^{L-1} a_l \mathbf{1}_{\tau_l \in [qT_p,(q+1)T_p)}$ with $\mathbf{1}_{\{.\}}$ denoting the indicator function. With the equivalent channel in (5), when t lies in $[0, T_s]$, we have:

$$\begin{aligned} &r(t + (n-1)T_s) = x(t; C_{1,n}) \star h(t) + z(t + (n-1)T_s) \\ &= \sqrt{PT_s}\left[\alpha \sum_{q=0}^{K_d-1} \tilde{a}_q^{(n)} p(t - qT_p) + \beta \sum_{q=0}^{K_d-1} \tilde{a}_q^{(n)} p(t - T_d - C_{1,n}\Delta - qT_p)\right] + z(t + (n-1)T_s). \end{aligned} \tag{6}$$

Letting $T_s = K_s T_p$ and $\Delta = K_\Delta T_p$, we can project the received signal onto the set of bases $\{p(t - iT_p)\}_{i=0}^{K_s-1}$ to obtain:

$$r_{n,i} := \int p(t - iT_p) r(t + (n-1)T_s) dt, \quad i = 0, ..., K_s - 1. \tag{7}$$

Upon defining $\tilde{\mathbf{a}}^{(n)} := [\tilde{a}_0^{(n)}, ..., \tilde{a}_{K_d-1}^{(n)}]^T$ and $\mathbf{r}_n = [r_{n,1}, ..., r_{n,K_s-1}]^T$, we find:

$$\mathbf{r}_n = [\sqrt{PT_s}\alpha \tilde{\mathbf{a}}^{(n)T}, \overbrace{0, ..., 0}^{C_{1,n}K_\Delta}, \sqrt{PT_s}\beta \tilde{\mathbf{a}}^{(n)T}, 0, ..., 0]^T + \mathbf{z}_n, \tag{8}$$

where $\mathbf{z}_n$ is zero mean Gaussian with covariance matrix $(N_0/2)\mathbf{I}$. In order to detect the transmitted message, the receiver formulates the following decision statistic per message w:

$$d(w) = \frac{1}{N}\sum_{n=1}^{N} d_{w,n} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{r}_n(1 : K_d)^T \mathbf{r}_n(K_d + C_{w,n}K_\Delta : 2K_d + C_{w,n}K_\Delta), \tag{9}$$

where $\mathbf{u}(l : q) := [u(l), ..., u(q)]^T$. When $w = 1$, we find:

$$\begin{aligned} d_{1,n} &= \mathbf{r}_n(1 : K_d)^T \mathbf{r}_n(K_d + C_{1,n}K_\Delta : 2K_d + C_{1,n}K_\Delta) \\ &= \sum_{q=0}^{K_d-1} \left[\sqrt{PT_s}\alpha \tilde{a}_q^{(n)} + z_n(q+1)\right]\left[\sqrt{PT_s}\beta \tilde{a}_q^{(n)} + z_n(K_d + C_{1,n}K_\Delta + q + 1)\right]. \end{aligned} \tag{10}$$

As N grows large, $d(1)$ converges to $\mathsf{E}[d_{1,n}] = \mathsf{E}[\sum_{q=0}^{K_d-1} PT_s\alpha\beta\tilde{a}_q^{(n)2}] = PT_s\alpha\beta$, where we have used the assumption that the total channel gain is constant and has been normalized to have unit gain.

When $w \neq 1$, we have

$$d_{w,n} = \mathbf{r}_n(1:K_d)^T \mathbf{r}_n(K_d + C_{w,n}K_\Delta : 2K_d + C_{w,n}K_\Delta) = \sum_{q=0}^{K_d-1} \left[\sqrt{PT_s}\alpha\tilde{a}_q^{(n)} + z_n(q+1)\right] \times \left[\sqrt{PT_s}\beta\tilde{a}_{(C_{w,n}-C_{1,n})K_\Delta+q}^{(n)} + z_n(K_d + C_{w,n}K_\Delta + q + 1)\right]. \quad (11)$$

Clearly, if $C_{w,n} \neq C_{1,n}$, we have $\mathsf{E}[d_{w,n}] = 0$ because $\{\tilde{a}_q^{(n)}\}_{q=0}^{K_d-1}$ are independent to each other and have zero mean.

Based on the decision variables $d(1), ..., d(M)$, we claim message $\hat{w}$ is sent if $d(\hat{w}) \geq \rho$ and $\forall w \neq \hat{w}$, $d(\hat{w}) < \rho$, where $\rho := PT_s\alpha\beta(1-\epsilon)$ is a certain threshold, and $\epsilon > 0$ can be made arbitrarily close to zero. Now, let us analyze the decoding error probability. It is easy to verify the following expression for $P_e^{(N)}$:

$$P_e^{(N)} = \Pr\left(d(1) < \rho \bigcup \cup_{w=2}^{M} d(w) \geq \rho\right). \quad (12)$$

In the next section, we will upper bound $P_e^{(N)}$ and find the rate that is achievable in the sense that $P_e^{(N)}$ goes to zero as N, the number of channel uses, goes to infinity.

3 Achievable Rates

From the expression of $P_e^{(N)}$ in (12), we can readily have the following union bound:

$$P_e^{(N)} \leq \Pr(d(1) < \rho) + \sum_{w=2}^{M} \Pr(d(w) \geq \rho). \quad (13)$$

Based on the law of large numbers, we know that $\lim_{N\to\infty} d(1) = PT_s\alpha\beta$. Thus, we have

$$\lim_{N\to\infty} \Pr(d(1) < \rho) = \lim_{N\to\infty} \Pr(d(1) < PT_s\alpha\beta(1-\epsilon)) = 0. \quad (14)$$

When $w \neq 1$, in order to characterize $\Pr(d(w) \geq \rho)$, we resort to the Chernoff bound, i.e.,

$$\Pr(d(w) \geq \rho) = \Pr\left(\sum_{n=1}^{N} d_{w,n} \geq N\rho\right) \leq e^{-tN\rho}\mathsf{E}\left[e^{t\sum_{n=1}^{N} d_{w,n}}\right] = e^{-tN\rho}\prod_{n=1}^{N}\mathsf{E}\left[e^{td_{w,n}}\right], \quad \forall t > 0, \quad (15)$$

where in obtaining the last equality, we have used the fact that $\{d_{w,n}\}_{n=1}^{N}$ are independent to each other. Now, our task is to find the moment generating function of $d_{w,n}$ as in (11). From [10, Chapt. 6], we can obtain the following result:

$$\mathsf{E}\left[e^{td_{w,n}} \middle| \{\tilde{a}_q^{(n)}\}_{q=0}^{K_d-1}\right] = \frac{1}{\left(1 - \frac{N_0^2}{4}t^2\right)^{K_d/2}} \times e^{\mathbf{\Xi}}, \quad (16)$$

where $\mathbf{\Xi}$ is defined as follows:

$$\mathbf{\Xi} := \exp\left\{\frac{\frac{N_0}{2}t^2 PT_s\alpha\beta\sum_{q=0}^{K_d-1}\left(\tilde{a}_q^{(n)2} + \tilde{a}_{q+(C_{w,n}-C_{1,n})K_\Delta}^{(n)2}\right) + 2tPT_s\alpha\beta\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)}\tilde{a}_{q+(C_{w,n}-C_{1,n})K_\Delta}^{(n)}}{2\left(1 - \frac{N_0^2}{4}t^2\right)}\right\}.$$

If during the n^{th} channel use, codewords C_1 and C_w dollide, i.e., $|(C_{w,n} - C_{1,n})K_\Delta| < K_d$, then we can upper bound (16) as:

$$\mathsf{E}\left[e^{td_{w,n}}\middle|\{\tilde{a}_q^{(n)}\}_{q=0}^{K_d-1}\right] \leq \frac{1}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta 2\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2} + 2tPT_s\alpha\beta\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\}.$$

Using the assumption that $\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2} = 1$, we have

$$\mathsf{E}\left[e^{td_{w,n}}\middle|\{\tilde{a}_q^{(n)}\}\right] \leq \frac{1}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}\exp\left\{\frac{\frac{N_0}{2}t^2PT_s2\alpha\beta + 2tPT_s\alpha\beta}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\} := \phi_{collision}. \quad (17)$$

If there is no collision over the n^{th} channel use between codewords C_1 and C_w, i.e., $|(C_{w,n} - C_{1,n})K_\Delta| \geq K_d$, then we have

$$\begin{aligned}\mathsf{E}\left[e^{td_{w,n}}\middle|\{\tilde{a}_q^{(n)}\}\right] &= \frac{1}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\}\\ &= \frac{1}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\} := \phi_{clear}. \quad (18)\end{aligned}$$

With c denoting the number of collisions between codewords C_1 and C_w, we obtain [c.f. (15), (17), and (18)]:

$$\Pr(d(w) \geq \rho) \leq e^{-tN\rho}\prod_{n=1}^{N}\mathsf{E}\left[e^{td_{w,n}}\right] \leq e^{-tN\rho}\phi_{collision}^c\phi_{clear}^{N-c}. \quad (19)$$

Now, in order to eliminate the effect of a particular codebook generation, we average the probability $\Pr(d(w) \geq \rho)$ over all codebook realizations and arrive at:

$$\begin{aligned}\mathsf{E}_{\mathcal{C}}\left[\Pr(d(w) \geq \rho)\right] &\leq \mathsf{E}_{\mathcal{C}}\left[e^{-tN\rho}\phi_{collision}^c\phi_{clear}^{N-c}\right]\\ &= \sum_{c=0}^{N}\binom{N}{c}\mu^c(1-\mu)^{N-c}e^{-tN\rho}\phi_{collision}^c\phi_{clear}^{N-c}\\ &= e^{-tN\rho}\left(\mu\phi_{collision} + (1-\mu)\phi_{clear}\right)^N, \quad \forall t > 0, \quad (20)\end{aligned}$$

where $\mu := \Pr(|C_{w,n} - C_{1,n}|K_\Delta < K_d)$. Substituting (17) and (18) into (20) and recalling (13), we have:

$$\begin{aligned}\mathsf{E}_{\mathcal{C}}\left[P_e^{(N)}\right] &\leq \mathsf{E}_{\mathcal{C}}\left[\Pr(d(1) < \rho)\right] + \sum_{w=2}^{M}\mathsf{E}_{\mathcal{C}}\left[\Pr(d(w) \geq \rho)\right]\\ &\leq \mathsf{E}_{\mathcal{C}}\left[\Pr(d(1) < \rho)\right] + M\mathsf{E}_{\mathcal{C}}\left[\Pr(d(w) \geq \rho)\right]\\ &\leq \mathsf{E}_{\mathcal{C}}\left[\Pr(d(1) < \rho)\right] + \min_{t>0}\exp\Bigg\{-N\Bigg[-\frac{\ln M}{N} + t\rho + \frac{K_d}{2}\ln\left(1-\frac{N_0^2}{4}t^2\right)\\ &\quad -\ln\left(\mu\exp\left\{\frac{\left[\frac{N_0}{2}t^2+t\right]PT_s2\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\} + (1-\mu)\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right)\Bigg]\Bigg\}. \quad (21)\end{aligned}$$

One can clearly see from (21) that in order for the error probability to vanish as N goes to infinity, we only need the following condition:

$$\begin{aligned}\frac{\ln M}{N} < \max_{t>0} \; & t\rho + \frac{K_d}{2}\ln\left(1-\frac{N_0^2}{4}t^2\right) \\ & -\ln\left(\mu\exp\left\{\frac{\left[\frac{N_0}{2}t^2+t\right]PT_s2\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}+(1-\mu)\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right). \qquad (22)\end{aligned}$$

Considering the fact that each channel use lasts $T_s = 2T_d + (m-1)\Delta$ seconds, we obtain the rate that is achievable in nats/second unit as follows:

$$\begin{aligned}R := \frac{\ln M}{NT_s} < R_0 := \frac{1}{T_s}\max_{t>0} \; & tPT_s\alpha\beta(1-\epsilon) + \frac{K_d}{2}\ln\left(1-\frac{N_0^2}{4}t^2\right) \\ & -\ln\left(\mu\exp\left\{\frac{\left[\frac{N_0}{2}t^2+t\right]PT_s2\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}+(1-\mu)\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right). \qquad (23)\end{aligned}$$

Adopting practical UWB system parameters, R_0 in (23) will be numerically evaluated and compared to AWGN channel capacity in the next section.

4 Numerical Results

AWGN channel capacity with bandwidth W is given by $C(W) = W\ln(1 + P_R/(N_0W))$ with P_R denoting the average received power. As $W \to \infty$, we have $C(W) \to C(\infty) := P_R/N_0$. Interestingly, provided with infinite bandwidth, frequency shift keying (FSK) has been shown to be able to achieve $C(\infty)$ with just non-coherent reception even in the presence of multi-path fading (see [6], [3], and [11]). In this section, we will examine the achievable rates in a practical TR-PPM based UWB system and compare them with $C(\infty)$.

4.1 Achievable Rates of an 1GHz UWB System

In 2002, FCC released a spectral mask for UWB transmissions (see [1]), where a maximum of 7.5GHz of bandwidth (3.1−10.6GHz) and maximal transmitted power spectral density of −41.3dbm/MHz are specified. For an 1GHz UWB system, the maximum transmission power would be $P_T = -12$dBm. At room temperature, i.e., $T_0 = 300$K, the noise spectral density is $N_0 = \mathsf{k}T_0\cdot\mathcal{F}\cdot\mathcal{L} = -102.83$dBm/MHz, where k is Boltzmann's constant $\mathsf{k} = 1.38\times10^{-23}$J/K, the noise figure is $\mathcal{F} = 6$dB, and a link margin $\mathcal{L} = 5$dB is assumed. According to measurements in [2], 80dB path loss is expected at a 10m Tx-Rx separation, which corresponds to a received-power-to-noise ratio $P_R/N_0 = 70$dB. Thus, 70dB can be thought of as a high-SNR benchmark for practical UWB systems.

The RMS delay spread of a typical UWB channel is on the order of 20ns for indoor environments [5]. Selecting $\Delta = T_d = 20$ns in (2), for different values of the modulation size m, rate R_0 in (23) for $W = 1$GHz is plotted in Fig. 2 ($\alpha = \beta = \sqrt{2}/2$ in (23)), from which we deduce that the achievable rates are indeed on the order $C(\infty)$ within the practical SNR range.

4.2 Bandwidth Scaling

4.2.1 Free Space Propagation

When the channel has 0 delay spread, which corresponds to free space propagation, we can choose $T_d = T_p$ and $\Delta = T_p$. With bandwidths 1GHz and 10GHz, the achievable rates are plotted in Fig.

3. It is clearly evidenced from the figure that larger bandwidth will result in larger achievable rates.

4.2.2 Multi-path Fading Channel

As in Section 4.1, we still choose $T_d = \Delta = 20$ns. The resulting achievable rates are plotted in Fig. 4, from where we verify that larger bandwidth suffers from rate loss. This is the case because the non-coherent receiver collects more noise when the bandwidth increases. Interestingly, similar behavior has been observed in [11] and [7], where the non-coherent capacity of spread-spectrum white-noise-like signaling over a multi-path fading channel is shown to approach zero as bandwidth increases.

5 Conclusions and Future Work

In this paper, the achievable rates of practical UWB systems with PPM and TR are studied. It is shown that for SNR values of practical interest, PPM-UWB with TR can achieve rates on the order of $C(\infty) = P/N_0$ (nats/second).

Future work will explore a tighter bound for the moment generating function of $d_{w,n}$ when there is a collision. The bound provided in (17) turns out to be very loose when $C_{w,n} \neq C_{1,n}$ and $|C_{w,n} - C_{1,n}|K_\Delta < K_d$, which will happen if we choose $\Delta < T_d$ in (2).

References

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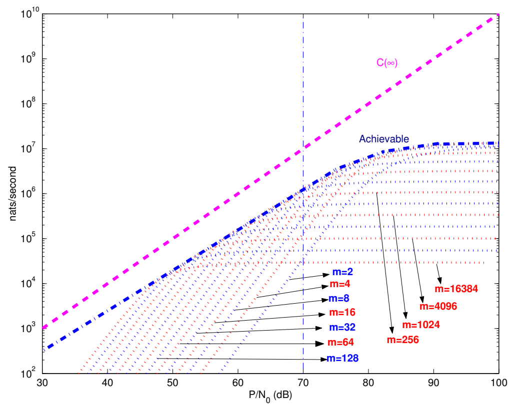

Figure 2: Achievable rates of an 1GHz practical UWB system.

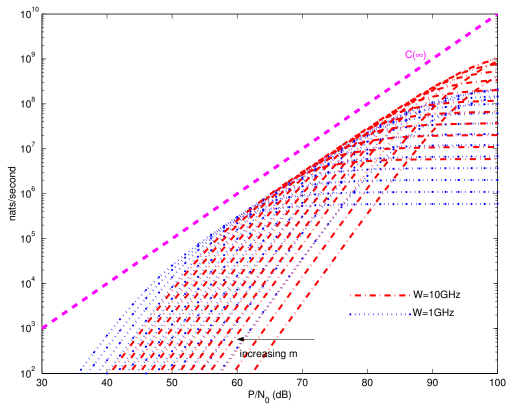

Figure 3: Effect of bandwidth on achievable rates: free space propagation.

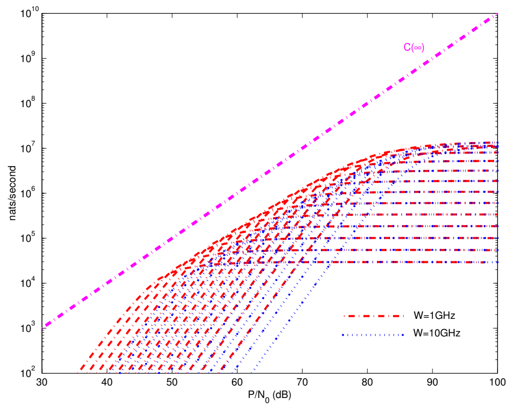

Figure 4: Effect of bandwidth on achievable rates: multi-path fading channel.