Analysis of Fixed Outage Transmission Schemes: A Finer Look at the Full Multiplexing Point

Peng Wu and Nihar Jindal
Department of Electrical and Computer Engineering
University of Minnesota
Email: pengwu, nihar@umn.edu

***Abstract*— This paper studies the performance of transmission schemes that have rate that increases with average SNR while maintaining a fixed outage probability. This is in contrast to the classical Zheng-Tse diversity-multiplexing tradeoff (DMT) that focuses on increasing rate and decreasing outage probability. Three different systems are explored: antenna diversity systems, time/frequency diversity systems, and automatic repeat request (ARQ) systems. In order to accurately study performance in the fixed outage setting, it is necessary to go beyond the coarse, asymptotic multiplexing gain metric. In the case of antenna diversity and time/frequency diversity, an affine approximation to high SNR outage capacity (i.e., multiplexing gain plus a power/rate offset) accurately describes performance and illustrates the very significant benefits of diversity. ARQ is also seen to provide a significant performance advantage, but even an affine approximation to outage capacity is sometimes unable to capture this advantage and outage capacity must be directly studied in the non-asymptotic regime.**

I. Introduction

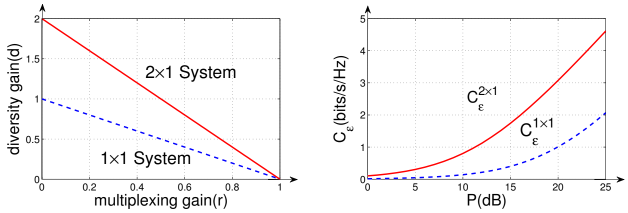
In many time-varying communication systems, the receiver has accurate *instantaneous* channel state information (CSI), generally acquired from received pilot symbols, while the transmitter only knows the channel statistics (e.g., average received SNR and the fading distribution) but has no instantaneous CSI. Performance in such a setting is generally dictated by fading and the relevant performance metric is known to be the *outage probability*, which is the probability that the instantaneous mutual information is smaller than the transmission rate, because this quantity reasonably approximates the probability of decoder (frame) error if a strong channel code is used [1]. Such systems have traditionally been studied by considering outage probability versus average SNR for a *fixed* transmission rate, leading to measures such as diversity order (generally defined as the slope of the outage vs. SNR curve in log-scale).

In modern communication systems, however, transmission rate is generally adjusted according to the average SNR (via adaptive modulation and coding) and thus systems need to be studied at various rates and SNR levels. The seminal work of Zheng and Tse [2] took precisely this viewpoint in introducing the diversity-multiplexing tradeoff (DMT). Loosely speaking, the DMT framework considers the performance of a *family* of codes indexed by average SNR such that the coding rate increases as $r \log_2 \mathrm{SNR}$, and the outage/error probability of the code decreases approximately as SNR^{-d}. The quantity r is the *multiplexing gain* while d is the *diversity order* (of the family of codes, not of a particular code). The DMT region is the set of (r, d) pairs achievable by *any* family of codes, and can be simply quantified in terms of the number of transmit and receive antennas, N_t and N_r respectively, and the receiver strategy for MIMO systems.

Over the past few years the DMT framework has become the benchmark for comparing different transmission strategies for different systems (MIMO, cooperative transmission, multiple access channel). Although the DMT framework has been incredibly useful in this role by providing a meaningful and tractable metric to compare different schemes that simultaneously achieve *increasing rate* and *decreasing outage probability*, one very important paradigm not sufficiently captured by the DMT are codes that achieve *increasing rate* and *fixed outage probability*.

Families of codes that achieve a fixed rather than decreasing outage are important because they are used in a number of important wireless systems, most prominently in the cellular domain. In this setting, as the average SNR of a user increases (i.e., as a user moves closer to the base station), it is desirable to use the additional SNR to increase rate but not to decrease outage probability (i.e., packet error rate); indeed, many systems continually adjust rate precisely to maintain a target error probability (e.g., 10^{-2}). In a voice system, for example, the voice decoder may be able to provide sufficient quality if no more than 1% of packets are received incorrectly and thus there is no benefit to decrease outage below 1%. Therefore, serving each user at the largest rate that maintains 1% outage minimizes per-user resource consumption (i.e., time/frequency slots) and thereby maximizes system capacity.

In order to accurately study fixed-outage schemes, it is necessary to directly study the manner in which outage capacity scales with SNR. We denote outage capacity as $R(P, \epsilon)$, where this quantity is the rate that achieves an outage probability of ϵ at an average SNR of P. In the context of the DMT, fixed outage systems (for any $\epsilon > 0$) achieve zero diversity ($d = 0$) and thus can achieve the maximum multiplexing gain. In other words, the DMT tells us that $R(P, \epsilon) \approx r_{\max} \log_2 P$ for any $\epsilon > 0$, where $r_{\max}$ is the maximum multiplexing gain, but cannot provide a more precise characterization than multiplexing gain (or pre-log factor). In many scenarios of interest, such a characterization is not sufficient to meaningfully characterize performance. Prasad and Varanasi were perhaps the first to recognize the need to go beyond the multiplexing gain [3]



(a) DMT Regions (b) $R(P, \epsilon)$ for $\epsilon = 0.01$

Fig. 1. Full multiplexing performance in the diversity-multiplexing tradeoff

[4], and our work follows along these lines.

To further illustrate the need to directly study outage capacity, let us consider a simple example. For a 1×1 system the maximum diversity order is $d^*(r) = 1 - r$, while for a 2×1 ($N_t = 2$, $N_r = 1$) system $d^*(r) = 2(1 - r)$ [2]. The DMT regions for both systems are shown in Fig. 1 (a). Because $\min(N_t, N_r) = 1$ both regions share the same full multiplexing point ($r = 1, d = 0$), a DMT-based comparison would indicate that the systems are equivalent in a fixed outage setting. However, the plot of $R(P, \epsilon)$ for $\epsilon = 0.01$ in Fig. 1 (b) shows that there is a huge power gap (≈ 8.5 dB) between the two systems; it clearly is not sufficient to consider only the multiplexing gain, which is the slope of the $R(P, \epsilon)$ curve.

Motivated by this example, one step in the right direction is to consider *affine* rather than *linear* approximations to outage capacity (at high SNR):

$$R(P, \epsilon) = r_{\max} \log_2 P + O(1), \tag{1}$$

where the non-vanishing $O(1)$ term, which depends on ϵ and the system configuration (i.e., N_t and N_r), is capable of capturing power/rate offsets such as that seen in Fig. 1 (b). This affine approximation, first proposed in [5], has been extremely useful in analysis of the ergodic capacity of CDMA systems [5] and MIMO systems [6] [7]. More recently, the affine approximation has also been employed to study the outage capacity of MIMO systems at asymptotically high SNR [4]. In [4], an expression for the constant term in (1) is given in terms of the statistics of the channel matrix (more precisely, in terms of the distribution of the determinant of $\mathbf{H}\mathbf{H}^H$ where $\mathbf{H}$ is the channel matrix).

A. *Contribution and Organization*

In this paper, we first consider the case of antenna diversity (SIMO or MISO; Section III) and show that fixed outage capacity can be exactly specified in terms of the inverse of the fading CDF. Although this result can be viewed as a special case of [4] (see also [9, Section 5.4]), it is useful to consider this base case to more precisely illustrate the importance of fixed-outage analysis (thus this section should be treated as background material). Next we consider systems with time and/or frequency diversity (Section IV), which are modeled as block-fading channels. Finally, we consider the performance of systems using hybrid ARQ (automatic repeat request) for incremental redundancy as well as Chase combining.

II. System Model

We consider a block-fading channel, denoted by $\mathbf{H}$, which is randomly drawn according to a known probability distribution (e.g., spatially white Rayleigh fading) and then fixed for the duration of a codeword. Furthermore, the receiver is assumed to have perfect channel state information (CSI) but the transmitter has no instantaneous knowledge of the channel realization and is only aware of the probability distribution. The received signal $\mathbf{y}$ is given by:

$$\mathbf{y} = \sqrt{P}\mathbf{H}\mathbf{x} + \mathbf{z}, \tag{2}$$

where the input $\mathbf{x}$ is constrained to have unit norm, $\mathbf{z}$ is the complex Gaussian noise and P represents the (average) received SNR. We consider cases where the input is Gaussian and further specify its structure where needed.

The outage probability is the probability that the *instantaneous mutual information* is smaller than the transmission rate R:

$$P_{out}(R, P) = \mathcal{P}[I(X; Y|P) < R], \tag{3}$$

and the outage capacity $R(P, \epsilon)$ is defined as the maximum rate that achieves the desired outage probability:

$$R(P, \epsilon) \triangleq \sup_{P_{out}(R,P) \le \epsilon} R. \tag{4}$$

Note that this quantity is essentially the same as the ϵ-capacity defined by Verd$\acute{u}$ and Han [8][1], denoted by C_ϵ in the following parts.

III. Antenna Diversity

We begin by examining antenna diversity, which is one of the commonly employed diversity techniques. If the transmitter has $N_t > 1$ antennas while $N_r = 1$, the mutual information for a spatially white Gaussian input (components of $\mathbf{x}$ are iid Gaussian with variance $\frac{1}{N_t}$) is $\log_2\left(1 + ||\mathbf{H}||^2 \frac{P}{N_t}\right)$ and therefore the outage probability is given by:

$$P_{out}(R, P) = \mathcal{P}\left[\log_2\left(1 + ||\mathbf{H}||^2 \frac{P}{N_t}\right) < R\right]. \tag{5}$$

By setting this quantity to ϵ and solving for R, we get:

$$C_\epsilon^{N_t \times 1}(P) \quad = \quad \log_2\left(1 + F^{-1}_{||\mathbf{H}||^2}(\epsilon)\frac{P}{N_t}\right), \tag{6}$$

where $F^{-1}_{||\mathbf{H}||^2}(\cdot)$ is the inverse of the CDF of random variable $||\mathbf{H}||^2$. In iid Rayleigh fading the components of $\mathbf{H}$ are iid

[1]In some cases we compute outage probability assuming the input $\mathbf{x}$ is Gaussian and spatially white, while the precise definition of ϵ-capacity requires an explicit optimization over the input distribution. This choice of input is easily shown to be optimal when $N_t = 1$, but is not necessarily optimal for $N_t > 1$.

$\mathcal{CN}(0,1)$ and thus $||\mathbf{H}||^2$ is chi-square with $2N_t$ degrees of freedom and has the following CDF:

$$F_{||\mathbf{H}||^2}(x) \quad = \quad 1 - e^{-x}\sum_{k=1}^{N_t}\frac{x^{k-1}}{(k-1)!}. \qquad (7)$$

If $N_t = 1$ the channel gain is exponential and the inverse CDF can be written in closed form to yield:

$$C_\epsilon^{1\times 1}(P) \quad = \quad \log_2\left(1 + \ln\left(\frac{1}{1-\epsilon}\right)P\right), \qquad (8)$$

while for $N_t > 1$ the inverse CDF needs to be computed numerically.

It can be convenient to relate the outage capacity to the AWGN capacity at SNR P: $C_{AWGN}(P) = \log_2(1+P)$:

$$C_\epsilon(P) = C_{AWGN}(\Gamma_\epsilon P) = \log_2(1+\Gamma_\epsilon P) \qquad (9)$$

where the gap to capacity is:

$$\Gamma_\epsilon^{N_t\times 1} = \frac{F^{-1}_{||\mathbf{H}||^2}(\epsilon)}{N_t}. \qquad (10)$$

For small ϵ we can approximate the CDF of $||\mathbf{H}||^2$ as $F_{||\mathbf{H}||^2}(x) \approx \frac{x^{N_t}}{N_t!}$ and therefore the gap can be approximated as:

$$\Gamma_\epsilon^{N_t\times 1} \approx \epsilon^{\frac{1}{N_t}}\frac{(N_t!)^{\frac{1}{N_t}}}{N_t}. \qquad (11)$$

In the case of receive diversity ($N_t = 1, N_r > 1$) the achieved mutual information is $\log_2\left(1+||\mathbf{H}||^2P\right)$, because using optimal maximum-ratio combining prevents the power loss experienced with transmit diversity, and therefore:

$$C_\epsilon^{1\times N_r}(P) \quad = \quad \log_2\left(1 + F^{-1}_{||\mathbf{H}||^2}(\epsilon)P\right), \qquad (12)$$

where $||\mathbf{H}||^2$ is chi-square with $2N_r$ degrees of freedom. As expected, there is a $\log_{10}(N_t)$ dB power gap between $C_\epsilon^{1\times N_r}(P)$ and $C_\epsilon^{N_t\times 1}(P)$.

In Fig. 2 the outage capacity of 2×1, 1×2, and 1×1 systems are plotted for $\epsilon = 0.01$. The capacity gap for the 1×1 system is approximately 20 dB ($\Gamma_\epsilon = -\ln(1-\epsilon) \approx \epsilon$), while it is about 11.5 dB for the 2×1 system ($\Gamma_\epsilon \approx \sqrt{\frac{\epsilon}{2}}$). Fixed outage analysis very clearly illustrates the advantage of antenna diversity.

IV. Time / Frequency Diversity

Another very common method of realizing diversity is through time or frequency, i.e., by coding across multiple coherence times/bands. If a codeword spans L coherence bands (in time and/or frequency), the outage probability is given by [9, Equation 5.83]:

$$P_{out}(R,P) = \mathcal{P}\left[\frac{1}{L}\sum_{i=1}^{L}\log_2(1+P|h_i|^2) < R\right] \qquad (13)$$

where h_i is the channel gain over the i-th band, each h_i is unit variance complex Gaussian (Rayleigh fading), and $h_1,\ldots,h_L$ are assumed to be iid. It is important to note that this outage probability expression approximates the performance of a strong channel code that is interleaved across the L bands, and not that of a sub-optimal repetition code.

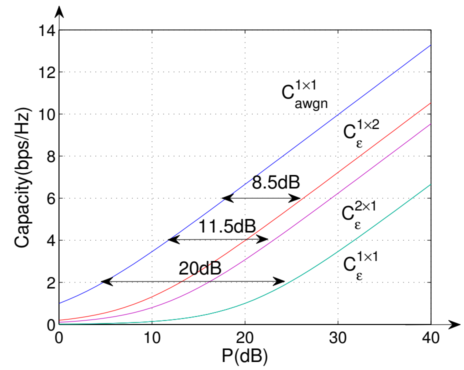


Fig. 2. C_{awgn} and C_ϵ (bps/Hz) vs P (dB), $\epsilon = 0.01$

For notational convenience we define the function $G_L(R)$ to be equal to the outage expression in (13). In terms of this function

$$C_\epsilon(P) = G_L^{-1}(\epsilon). \qquad (14)$$

Although we cannot reach a closed form for $G_L^{-1}(\epsilon)$, this quantity can be computed numerically by noting that $C_\epsilon(P)$ is equal to R that satisfies:

$$\epsilon = \int\int\cdots\int_{\frac{1}{L}\sum_{i=1}^{L}\log_2(1+x_i)<R}\frac{1}{P^L}e^{-\frac{x_1+x_2+\cdots\ x_L}{P}} dx_1dx_2\cdots dx_L. \qquad (15)$$

It can be shown that the affine approximation to outage capacity is quite useful in the high SNR analysis. Besides, a simple application of Jensen's inequality shows that the mutual information achieved with L-order time/frequency diversity is smaller than that achieved in a $L\times 1$ system:

$$\frac{1}{L}\sum_{i=1}^{L}\log_2(1+P|h_i|^2) \le \log_2\left(1+\frac{P}{L}\sum_{i=1}^{L}|h_i|^2\right). \qquad (16)$$

As a result, the outage probability is larger for time/frequency diversity and therefore the outage capacity of a $L\times 1$ system is no smaller than the outage capacity of an L-order time/frequency diversity system. In Fig. 3, outage capacity is shown for $L = 1,\ 2$ and 3 along with the outage capacity of 2×1 and 3×1 systems for $\epsilon = 0.01$. The time/frequency diversity curve is smaller than the corresponding antenna diversity system, but the difference is relatively small for low and moderate SNR. However, there is a nontrivial gap at high SNR that can be explained by the concavity of the log function.

V. ARQ

ARQ protocols can significantly improve performance by allowing for retransmission of packets or transmission of additional parity symbols when an initial transmission is unsuccessful. We are particularly interested in the performance of *hybrid ARQ* (H-ARQ) protocols that allow for decoding on the basis of multiple received packets. Upon reception of a packet,

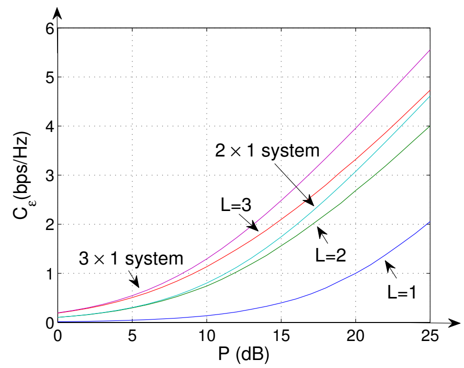


Fig. 3. Outage capacity of the time/frequency diversity system and the transmit diversity system, $\epsilon = 0.01$

the receiver attempts to decode and feeds back a one-bit ACK/NACK message (often based on the success or failure of a CRC check). If an ACK is received the transmitter moves on to the next message, while a NACK results in retransmission of the same packet (Chase combining) or transmission of additional parity symbols (incremental redundancy). There generally is a limit to the number of ARQ rounds for the same message, denoted by L, and an outage occurs whenever the message cannot be successfully decoded after L ARQ rounds.

If each message contains b bits and each transmitted packet is T symbols long, then the initial rate of transmission is $R \triangleq \frac{b}{T}$. If random variable X is used to denote the number of ARQ rounds used for a particular message (clearly $X \leq L$), then the long-term *transmitted rate* is [10]:

$$\eta = \frac{R}{E[X]}. \tag{17}$$

To see why this is the case, note that the number of packet transmissions used to attempt to transmit N messages is $\sum_{i=1}^{N} X_i$, where X_i is the number of ARQ rounds used for the i-th message. Thus, the average transmission rate (in bits/channel symbol) is:

$$\frac{Nb}{T\sum_{i=1}^{N} X_i} = \frac{R}{\frac{1}{N}\sum_{i=1}^{N} X_i}, \tag{18}$$

and this quantity converges (by the law of large numbers) to $\frac{R}{E[X]}$ as $N \to \infty$.

A. Incremental Redundancy

We first investigate incremental redundancy techniques, in which the transmitter sends additional parity bits (rather than retransmitting the same packet) whenever a NACK is received. In this setting it has been shown that the total mutual information is the *sum* of the mutual information received in each ARQ round, and that decoding is possible once the *accumulated* mutual information is larger than the number of information bits [10]. In other words, the number of ARQ rounds X is the smallest number l such that:

$$\sum_{i=1}^{l} \log_2(1 + P|h_i|^2) > R. \tag{19}$$

The constraint caps this quantity by L, and an outage occurs whenever the mutual information after L rounds is smaller than R:

$$P_{out}(R) = \mathcal{P}\left[\sum_{i=1}^{L} \log_2(1 + P|h_i|^2) < R\right]. \tag{20}$$

For simplicity we consider single antenna systems ($N_t = N_r = 1$), and use h_i to denote the channel during the i-th ARQ round. In the following sections we consider the case where the channel is iid across ARQ rounds. Similar to the definition in Section IV, here we use $G_{IR,i}(R)$ to denote the probability that the sum of mutual information is less than the first round rate R after i rounds. Then, $R = G_{IR,L}^{-1}(\epsilon)$. Go back to the definition of η, we have[2]

$$\eta_{IR} = C_\epsilon^{IR,L} = \frac{G_{IR,L}^{-1}(\epsilon)}{1 + \sum_{i=1}^{L-1} G_{IR,i}(G_{IR,L}^{-1}(\epsilon))} \tag{21}$$

It is useful to compare performance to a system without ARQ that always codes over the L available slots (whereas ARQ allows for early completion), which precisely corresponds to L-order time/freq diversity (Section IV). After properly normalizing rates, we get:

$$\frac{C_\epsilon^{IR,L}}{C_\epsilon^{nARQ}} = \frac{L}{E[X]} \tag{22}$$

where C_ϵ^{nARQ} is the outage capacity of a corresponding no ARQ protocol. Since $L \geq E[X]$, then

$$C_\epsilon^{IR,L} \geq C_\epsilon^{nARQ} \tag{23}$$

Actually, the quantity $\frac{L}{E[X]}$ determines the advantage of ARQ, and it is not difficult to show the following limit:

$$\lim_{P\to\infty} E[X] = L \tag{24}$$

Note that R is set such that the mutual information accumulated over L ARQ rounds is very likely (i.e., with probability equal to $1 - \epsilon$) to be larger than R. In order for ARQ to terminate after a single round, the received mutual information in a single round needs to be L times larger than that expected across L rounds. At high SNR this occurs with very low probability because the received mutual information received in a single round is well approximated as $\log_2 P + \log_2 |h_i|^2$; the probability that this quantity is larger than R, which is of order $L \log_2 P$, is extremely rare because $\log_2 P >> \log_2 |h_i|^2$ for large P. Indeed, it can further be shown that the rate advantage of ARQ also vanishes at asymptotically high SNR:

$$\lim_{P\to\infty} [C_\epsilon^{IR,L}(P) - C_\epsilon^{nARQ}(P)] = 0 \tag{25}$$

In other words, the high SNR affine approximation is the same regardless of whether ARQ is used[3]. On the other

[2]Strictly speaking, outage capacity is different from the long-term transmitted rate. But for convenience here we still use outage capacity to denote the fixed-outage transmitted rate.

[3]Somewhat counter-intuitively, the performance of ARQ at very high SNR can be improved by selecting the initial rate R strictly smaller than $G_{IR,L}^{-1}(\epsilon)$ (which means the outage probability is strictly smaller than the constraint). Although space constraints preclude further discussion here, this point is investigated in detail in [11].

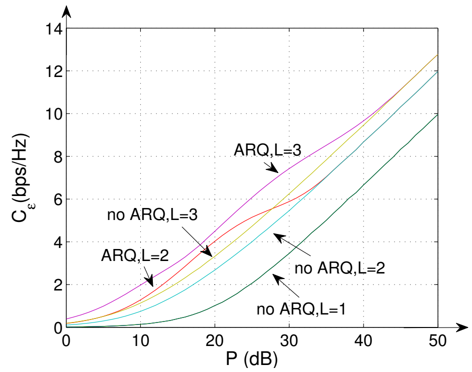


Fig. 4. Outage capacity for IR strategy and no ARQ strategy in the iid Rayleigh fading channel, $\epsilon = 0.01$

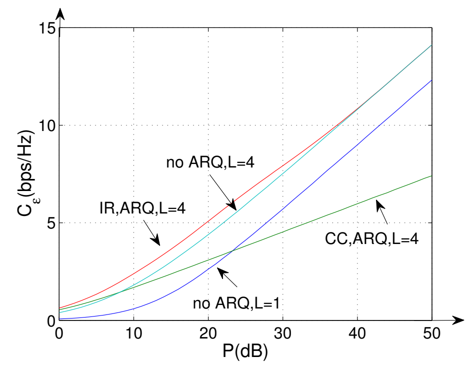


Fig. 5. Outage capacity for IR strategy, CC strategy and no ARQ strategy in the iid Rayleigh fading channel, $\epsilon = 0.05$

hand, the number of expected ARQ rounds is less than L at asymptotically low SNR.

Based on these asymptotic results one might conclude that ARQ provides a benefit only at relatively low SNR's. However, numerical results indicate that the high SNR asymptotics kick in only for very large SNR's. Indeed, ARQ does achieve a significant advantage for a relatively large range of SNR's. In Fig. 4 the outage capacity is shown for $\epsilon = 0.01$ and $L = 1, 2$ and 3. Note that 2 rounds of ARQ provide a significant power advantage relative to no ARQ up to approximately 30 dB, while the advantage lasts past 40 dB for $L = 3$. Asymptotic measures such as multiplexing gain and rate/power offset are clearly misleading in this context.

B. Chase Combining

If Chase combining is used, the transmitter retransmits the same packet whenever a NACK is received and the receiver performs maximal-ratio-combining (MRC) on all received packets. As a result, SNR rather the mutual information is accumulated over ARQ rounds and the outage probability is given by:

$$P_{out}(R) = \mathcal{P}\left[\log_2(1 + P\sum_{i=1}^{L}|h_i|^2) < R\right] \quad (26)$$

Note that this strategy essentially allows a repetition code to be used up to L times. A straightforward derivation shows the outage capacity of CC in the iid Rayleigh fading channel is:

$$C_\epsilon^{CC,L}(P) = \frac{R}{L - e^{-\frac{2^R-1}{P}}\sum_{k=1}^{L-1}(L-k)\frac{(2^R-1)^{k-1}}{P^{k-1}(k-1)!}} \quad (27)$$

where R has to be obtained from $G_{CC,L}^{-1}(\epsilon)$ numerically. Chase combining indeed provides some advantage at low and moderate SNR, but performs poorly at high SNR because of the sub-optimality of the repetition codes. In Fig. 5 we compare the performance of IR, CC and no ARQ strategy for $L = 4$ and $\epsilon = 0.05$. We see that CC performs reasonably at low SNR but trails off at high SNR.

VI. Conclusion

In this paper we have studied open-loop communication systems under the assumption that the rate is adjusted such that a fixed outage probability is maintained. The over-arching takeaways of this work are two-fold. First, we have argued that schemes that increase rate but have a *fixed* rather than decreasing outage probability may be more practically relevant than the increasing rate/decreasing outage schemes addressed by the diversity-multiplexing tradeoff. Second, we have shown that asymptotic measures should be used very carefully in analysis of fixed-outage systems. Multiplexing gain is certainly too coarse in this context, while high SNR rate/power offsets are sometimes meaningful (antenna diversity, time/frequency diversity) but can also be misleading in other settings (e.g., ARQ systems) due to their asymptotic nature.

References

[1] G. Caire, G. Taricco and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1468-1489, July 1999.

[2] L. Zheng and D. Tse, "Diversity and multiplexing:a fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073-1096, May 2003.

[3] N. Prasad and M. K. Varansi, "Outage capacities of space-time architecures," *IEEE Inf. Theory Workshop*, pp. 402-407, Oct. 2004.

[4] N. Prasad and M. K. Varanasi, "MIMO outage capacity in the high SNR regime," *IEEE Int. Symp. Inf. Theory*, pp 656-660, Sept. 2005.

[5] S. Shamai and S. Verdú, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Trans. Inf. Theory*, vol. 47, no. 5, pp. 1302-1327, May 2001.

[6] A. Lozano, A. M. Tulino, and S. Verdú, "High-SNR power offset in multiantenna communication," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4134-4151, Dec. 2005.

[7] J. Lee and N. Jindal,"High SNR analysis for MIMO broadcast channels: dirty paper coding vs. linear precoding," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4787-4792, Dec. 2007.

[8] S. Verdú and T. S. Han, "A General formula for channel capacity," *IEEE Trans. Inf. Theory*, vol. 40, no. 4, pp. 1147-1157, Jul. 1994.

[9] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.

[10] G. Caire and D. Tuninetti, "The throughput of hybrid-ARQ protocols for the Gaussian collision channel," *IEEE Trans. Inf. Theory*, vol.47, no.4, pp. 1971-1988, Jul. 2001.

[11] P. Wu and N. Jindal, "Analysis of fixed outage transmission schemes: a finer look at the full multiplexing point" journal version in preparation.