Beamforming with Finite Rate Feedback for LOS MIMO Downlink Channels

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***Abstract*— We consider a MIMO line of sight (LOS) broadcast channel where each user has perfect knowledge of its own channel and feeds back 'quantized' channel information to the transmitter with finite number of bits. The rate loss incured due to imperfect channel information at the transmitter is quantified in terms of the number of feedback bits, assuming that low-complexity zero-forcing beamforming is used by the transmitter. Moreover, in order to maintain a constant rate loss relative to a perfect CSIT system, it is shown that the number of feedback bits per user should grow linearly with the system SNR in dB but only logarithmitcally with the number of transmit antennas. Simulation results comparing zero-forcing, MMSE and conventional beamforming with greedy user selection and finite rate feedback are also presented.**

I. Introduction

We consider a downlink channel where a transmitter with M antennas simultaneously serves multiple users, each with a single antenna. Because of the complexity of capacity-achieving techniques (dirty paper coding), we consider simpler suboptimum techniques. Zero-forcing (ZF) beamforming is a suboptimum technique implemented with conventional single-user coding that exhibits optimum capacity scaling as the number of users grows without bound [1]. During a given transmission interval, under ideal channel phase knowledge at the transmitter, up to M users can be served, each by a separate beam such that each user receives zero interference from beams serving the other users.

In this paper we study the performance of ZF beamforming under limited channel feedback in line of sight (LOS) channels. LOS channels occur in practice when the transmitter is located well above its surrounding scatterers. This would occur, for example, in fixed wireless systems when base station transmit antennas are mounted on a tower which is significantly higher than surrounding buildings or trees. Note that the receivers could be located lower, even below the height of its surrounding scatterers. LOS channels also serve as a good approximation for channels with very low angle spread, for example those typically found in suburban and rural cellular environments. Assuming the receivers are sufficiently far from the transmitter so the received signal can be modeled as a planar wavefront, the channel phases of a LOS channel are totally correlated and can be characterized by the signal's direction of arrival relative to the position of the transmitter array elements.

In practical frequency division duplexed (FDD) systems, channel knowledge at the transmitter is obtained from channel estimates that are fed back from the users over limited rate channels. Previous work on ZF beamforming with limited feedback focused on independent and identically distributed (IID) Rayleigh channels [2], i.e., rich scattering environments. It was shown that in order to maintain a constant rate loss with respect to the ideal case (perfect CSIT), the number of feedback bits must be scaled linearly with both the system SNR in dB and M the number of antennas. As we show in this paper, for LOS channels feedback must also be scaled linearly with SNR, albeit at a slower rate, but because of the antenna correlation, the scaling relationship with respect to the number of antennas is only $\log M$. It is quite intuitive that the feedback requirements should be much less stringent in LOS environments as compared to rich scattering environments since pure LOS channels are completely characterized by direction of arrival, and our analysis confirms this intuition and accurately quantifies the feedback requirements.

It is important to note that performing ZF beamforming allows the system to realize spatial multiplexing benefits (i.e., simultaneous transmission of multiple data streams) even though each receiver has only a single antenna. This is true for either LOS or rich scattering environments, and is perhaps the key benefit of so-called multi-user MIMO techniques. In rich scattering, spatial multiplexing can altenatively be realized using point-to-point MIMO techniques if receivers (mobiles) are equipped with multiple antennas. However, this alternative is not possible in LOS channels: parallel channels cannot be formed due to the lack (or very low degree) of scattering, and thus *multi-user MIMO is in effect the only possible method to realize spatial multiplexing in LOS environments.*

II. System Model

We consider a single transmitter and K user system where each user (mobile) has a single antenna and the transmitter (base station) has M antennas. The broadcast channel is described as:

$$y_i = \mathbf{h}_i^H \mathbf{x} + n_i, \quad i = 1, \dots, K \tag{1}$$

where $y_i \in \mathbb{C}$ is the received signal at the i^{th} user, $\mathbf{h}_i \in \mathbb{C}^M$ the channel vector from the transmitter to the i^{th} user, $\mathbf{x} \in \mathbb{C}^M$ the transmitted signal and $n_i \in \mathbb{C},\ i = 1, \dots, K$ are independent and complex Gaussian noise with unit variance. The input must satisfy a transmit power constraint such that $E[||\mathbf{x}||^2] \leq P$. We also assume that Zero-forcing (ZF) beamforming is employed and that the number of users is given as $K = M$ exactly. Simulation results for greedy user selection and other beamforming schemes are presented later.

We assume a pure line-of-sight (LOS) channel model with a one dimensional antenna array at the transmitter with antenna spacing d:

$$\mathbf{h}_i = \begin{bmatrix} 1 \\ e^{-j\frac{2\pi d}{\lambda}\sin(\theta_i)} \\ \vdots \\ e^{-j(M-1)\frac{2\pi d}{\lambda}\sin(\theta_i)} \end{bmatrix} \tag{2}$$

where λ is the communication signal wavelength and θ_i is the angle of departure measured with respect to the antenna array boresight direction. For simplicity the users are all assumed to be at an equal distance from the transmitter.

Each of the receivers is assumed to have perfect and instantaneous knowledge of its own channel vector, which in LOS channels corresponds to knowledge of θ_i (or $\sin(\theta_i)$) at user i. The channel vector is quantized at each receiver and fed back to the transmitter over a zero delay, error free, finite rate channel. We analyze the case where each user quantizes its $\sin(\theta_i)$ uniformly (with rounding) using B bits and feeds this back to the transmitter i.e. $\hat{w}_i = \mathrm{q}(\sin(\theta_i))$ where:

$$|\sin(\theta_i) - \hat{w}_i| = |\delta| \leq \frac{r}{2^{B+1}} \tag{3}$$

where r is the difference between the maximum and minimum values over which $\sin(\theta_i)$ is quantized. $\hat{w}_i$ is used to construct the quantized channel vector $\hat{\mathbf{h}}_i$ at the transmitter:

$$\hat{\mathbf{h}}_i = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 \\ e^{-j\frac{2\pi d}{\lambda}\hat{w}_i} \\ \vdots \\ e^{-j(M-1)\frac{2\pi d}{\lambda}\hat{w}_i} \end{bmatrix} \tag{4}$$

This corresponds to a codebook $\mathcal{C}$ of 2^B M-dimensional complex vectors, the mth component of the bth codevector ($m = 1, \dots, M$ and $b = 1, \dots, 2^B$) being:

$$\frac{1}{\sqrt{M}} e^{-j2\pi(m-1)\frac{b-\frac{1}{2}}{2^B}} \tag{5}$$

when $d = \frac{\lambda}{2}$. These vectors efficiently span the subspace of LOS vectors and are similar to those used in beamforming proposals found in LTE standards [3].

For linear beamforming the transmitted signal is chosen as:

$$\mathbf{x} = \sqrt{\frac{P}{M}} \sum_{i=1}^{M} \mathbf{v}_i s_i \tag{6}$$

where $s_i \in \mathbb{C}$ is the data symbol destined for user i (of unit variance) and $\mathbf{v}_i \in \mathbb{C}^M$ is the unit beamforming vector. Following the Zero-Forcing procedure $\mathbf{v}_i$ is selected such that $\hat{\mathbf{h}}_k^H \mathbf{v}_i = 0\ \forall k = 1 \dots K, k \neq i$ so that the interference terms are zero. This procedure can be written as:

$$\mathbf{V} = \hat{\mathbf{H}}(\hat{\mathbf{H}}^H \hat{\mathbf{H}})^{\dagger} \tag{7}$$

where $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \dots, \hat{\mathbf{h}}_K]$ and the normalized columns of $\mathbf{V}$ are used as the beamforming vectors.

III. Throughput Analysis

In case of perfect shannel state information at the transmitter (CSIT), the transmitter is able to suppress all interference terms giving a per user throughput of:

$$R_{CSI}(P) = \mathbb{E}\left[\ \log_2\left(1 + \frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_{ZF_i}|^2\right)\right] \tag{8}$$

where $\mathbf{v}_{ZF_i}$ is selected such that $\mathbf{h}_k^H \mathbf{v}_{ZF_i} = 0\ \forall k = 1 \dots K, k \neq i$.

For finite rate feedback of B bits per user, the per user throughput is given as (i is any user from 1 to $K = M$):

$$R_{FB}(P) = \mathbb{E}\left[\ \log_2\left(1 + SINR_i\right)\right] \tag{9}$$

where

$$SINR_i = \frac{\frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_i|^2}{1 + \frac{P}{M}\sum_{k=1,k\neq i}^{K} |\mathbf{h}_i^H \mathbf{v}_k|^2} \tag{10}$$

This gives:

$$R_{FB}(P) = \mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}\sum_{k=1}^{K}|\mathbf{h}_i^H\mathbf{v}_k|^2\right)\right] \\ -\mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}\sum_{k=1,k\neq i}^{K}|\mathbf{h}_i^H\mathbf{v}_k|^2\right)\right] \quad (11)$$

The rate loss per user incurred due to finite rate feedback with respect to perfect CSIT at the transmitter can thus be written as:

$$\begin{aligned}
\Delta R(P) &= R_{CSI}(P) - R_{FB}(P) \\
&= \mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}|\mathbf{h}_i^H\mathbf{v}_{ZF_i}|^2\right)\right] \\
&\quad -\mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}\sum_{k=1}^{K}|\mathbf{h}_i^H\mathbf{v}_k|^2\right)\right] \\
&\quad +\mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}\sum_{k=1,k\neq i}^{K}|\mathbf{h}_i^H\mathbf{v}_k|^2\right)\right] \\
&\stackrel{\text{(a)}}{\leq} \mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}|\mathbf{h}_i^H\mathbf{v}_{ZF_i}|^2\right)\right] \\
&\quad -\mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}|\mathbf{h}_i^H\mathbf{v}_i|^2\right)\right] \\
&\quad +\mathbb{E}\left[\ \log_2\left(1+\frac{P}{M}\sum_{k=1,k\neq i}^{K}|\mathbf{h}_i^H\mathbf{v}_k|^2\right)\right] \\
&\stackrel{\text{(b)}}{\approx} \mathbb{E}\left[\log_2\left(1+\frac{P}{M}\sum_{k=1,k\neq i}^{K}|\mathbf{h}_i^H\mathbf{v}_k|^2\right)\right] \quad (12)
\end{aligned}$$

where (a) follows by neglecting the interference terms w.r.t. the signal component, and (b) follows from the fact that when the quantizations are very good, the angles formed between $\mathbf{h}_i, \mathbf{v}_{ZF_i}$ and $\mathbf{h}_i, \mathbf{v}_i$ are nearly the same.

In order to bound the term in (12), it is desirable to bound the angle between $\mathbf{h}_i, \mathbf{v}_k$ for $k \neq i$. Taking advantage of the fact that $\hat{\mathbf{h}}_i$ and $\mathbf{v}_k$ are orthogonal (by ZF):

$$\begin{aligned}
||\mathbf{h}_i||^2 = M &\geq \frac{|\mathbf{h}_i^H\hat{\mathbf{h}}_i|^2}{||\hat{\mathbf{h}}_i||^2} + |\mathbf{h}_i^H\mathbf{v}_k|^2 \\
\Rightarrow |\mathbf{h}_i^H\mathbf{v}_k|^2 &\leq M(1-\cos^2\angle\mathbf{h}_i,\hat{\mathbf{h}}_i) \quad (13)
\end{aligned}$$

where

$$\begin{aligned}
\cos^2\angle\mathbf{h}_i,\hat{\mathbf{h}}_i &= \frac{1}{M^2}\left|\sum_{n=0}^{M-1} e^{jn\frac{2\pi d}{\lambda}\ (sin(\theta_i)-\hat{w}_i)}\right|^2 \\
&\stackrel{\text{(c)}}{=} \frac{1}{M^2}\left|\frac{\sin\left(\frac{M\pi d}{\lambda}\delta\right)}{\sin\left(\frac{\pi d}{\lambda}\delta\right)}e^{j\frac{(M-1)\pi d}{\lambda}\delta}\right|^2 \\
&= \frac{1}{M^2}\frac{\sin^2\left(\frac{M\pi d}{\lambda}\delta\right)}{\sin^2\left(\frac{\pi d}{\lambda}\delta\right)} \quad (14) \\
&\stackrel{\text{(d)}}{\approx} 1-\frac{(M^2-1)}{3}\pi^2\frac{d^2}{\lambda^2}\delta^2 \\
&\geq 1-\frac{(M^2-1)}{12}\pi^2\frac{d^2}{\lambda^2}r^2 2^{-2B} \quad (15)
\end{aligned}$$

where (c) follows by evaluating the sum of the series of exponentials, (d) is obtained by a second order Taylor's expansion about $\delta = 0$, and (15) follows from (3). This bound also suggests that quantizing $\sin(\theta_i)$ such that $|\delta|$ is minimized is approximately equivalent to choosing a vector from the codebook $\mathcal{C}$ (see (5)) such that the angle between the vector and the actual channel is minimized.

Using (15) and (13) in (12), the rate loss can be approximately bounded by a function of the number of feedback bits per user:

$$\Delta R(P) \lessapprox \log_2(1+C(M,d,\lambda,r)\ P\ 2^{-2B}\ ) \quad (16)$$

where $C(M,d,\lambda,r) = \frac{(M-1)(M^2-1)\pi^2d^2r^2}{12\lambda^2}$ The rate loss appears to be an increasing function of the SNR P and (16) suggests that increasing B as P increases will bound the rate loss by a constant. In order to maintain a rate loss $\Delta R(P)$ of no more than $\log_2(b)$ (bps/Hz) per user, the sufficient number of bits can be (approximately) found by inverting expression (16) and solving for B.

$$B \approx \frac{P_{dB}}{6} + \frac{1}{2}\log_2 C(M,d,\lambda,r) - \frac{1}{2}\log_2(b-1) \quad (17)$$

To maintain an SNR loss of at most 3dB between the rates under finite rate feedback and perfect CSIT, set $b = 2$:

$$B \approx \frac{P_{dB}}{6} + \frac{1}{2}\log_2\left[\frac{(M-1)(M^2-1)\pi^2}{12}\right] \quad (18)$$

In (18) d is selected to have a $\frac{\lambda}{2}$ spacing between antenna elements and θ_i ranges from 0 to 2π i.e. $\sin(\theta_i)$ ranges from -1 to 1 and $r = 2$. Interestingly, the number of bits must be scaled with the system SNR but the slope of this linear dependence is independent of the number of antenna elements M. This is in contrast to the scaling law for IID Rayleigh fading systems [2], where the number of bits is scaled linearly with M. For a 3 dB gap in such environments:

$$B_{\text{Rayleigh}} = \frac{M-1}{3}\ P_{\text{dB}}. \tag{19}$$

Note that the slope with SNR is considerbly higher and that there is a linear dependence on M.

IV. Simulation Results

A. Feedback without User Selection

Simulations for a system with $M = 4$ transmit antennas and $\frac{\lambda}{2}$ antenna placement with θ_i distributed uniformly in $[0, 2\pi]$ are shown in the Figure 1. The θ_i's are assumed to take a value for a certain time block, independent of previous values or each other and all users are distributed uniformly on a circle of fixed radius about the transmitter. The number of bits per user is determined by (18) to maintain an SNR gap of 3 dB. The number of bits required is found to be quite small (only 7 bits even at 30 dB). However, if there are only $K = M = 4$ users in the system, the probability of the channels of two or more users having the same quantization is quite large. At high SNR however (due to large B), it is possible to beamform to all of the $K = M$ users with high probability. Although not practical, these simulations serve the purpose of illustrating that a constant rate gap is maintained.

The SNR gap is found to be at most 1.15 dB instead of 3 dB as (16) is a conservative bound. Nevertheless, a constant gap is indeed maintained between the perfect CSIT rate and the rate with CSI feedback by scaling the bits as per (18). Simulations suggest that using one less bit than that suggested by (18) would be more practical. Moreover, the number of bits required varies only as $\mathrm{O}(\log M)$ from (18) and varies no more than a bit from $M = 5$ to $M = 20$.

B. Feedback Performance with Greedy User Selection

Figures 2-4 compare pure Zero-Forcing, MMSE (or regularized Zero-Forcing [4]), and 'matched' or conventional beamforming (i.e. PU^{2}RC with codebook $\mathcal{C}$ [3], [5]). The three schemes select beamforming vectors according to:

$$\text{ZF} \qquad \mathbf{V} = \hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{\dagger} \tag{20}$$

$$\text{MMSE} \qquad \mathbf{V} = \hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \frac{\tilde{K}}{P}\mathbf{I}_{\tilde{K}})^{\dagger} \tag{21}$$

$$\text{PU}^2\text{RC} \qquad \mathbf{v}_{\pi(i)} = \hat{\mathbf{h}}_{\pi(i)}, \tag{22}$$

where $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_{\pi(1)}, \hat{\mathbf{h}}_{\pi(2)}, \ldots, \hat{\mathbf{h}}_{\pi(\tilde{K})}]$. Here, among the $K = 20$ users in the system, $\tilde{K}$ users forming the set $\Pi = \{\pi(1), \pi(2), \ldots, \pi(\tilde{K})\}$ are selected using a greedy algorithm [6] (with quantized information in the finite rate feedback case). $\mathbf{V} = [\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2, \ldots, \tilde{\mathbf{v}}_{\tilde{K}}]$ and the beamformers for the $\tilde{K}$ users are $\mathbf{v}_i = \frac{\tilde{\mathbf{v}}_i}{||\tilde{\mathbf{v}}_i||}$, $i = 1, \ldots, \tilde{K}$. This procedure is described in Algorithm 1.

Algorithm 1 Greedy User Selection

1: $\Pi = \phi$ ▷ Initialize set of users
2: **for** $k = 1$ to K **do**
3: Find user u that maximizes $R(\Pi \cup u)$
4: **if** $R(\Pi \cup u) < R(\Pi)$ **or** $|\Pi| = M$ **then**
5: **break**
6: **else**
7: $\Pi = \Pi \cup u$
8: **end if**
9: **end for**

In the algorithm, $R(\Pi)$ is the computed rate given a set of users Π and their corresponding channel vectors i.e. by summing (11) for each user in Π. The channel vectors may be the actual channel (2) or the quantized channel (4) and the beamformers are selected according to (20), (21) or (22).

Figure 2 and Figure 3 compare perfect CSIT and limited feedback with fixed number of bits ($B = 3, 5$) and different M. Keeping the number of bits fixed is seen to result in an increasing gap between the rate curves, resulting in failure to capture full multiplexing gain. The performance of ZF and MMSE with user selection is found to be nearly the same, with MMSE performing marginally better at lower SNR values. With $B = 3$, the SNR gap between perfect CSIT and limited feedback for ZF and MMSE is found to be less than 3 dB upto a system SNR of 10 dB. For $B = 5$, this is extended to 22 dB. Note that 5 feedback bits allows performance very close to perfect CSIT for a very broad range of SNR's; this is quite different than the iid fading case, where feedback of 10 or more bits per user would be required for similar performance [2].

Figure 4 includes bit scaling as per (18), with a minimum of one bit used for feedback. The gap at low SNR in Figure (4) is due to the fact that only a single bit of feedback is used at low SNR. Again, the performance of ZF and MMSE is similar. The SNR offset between ZF with perfect CSIT and limited feedback is bounded by about 1.75 dB. The sum rate with conventional beamforming and limited feedback does not scale with SNR as the system becomes interference limited.

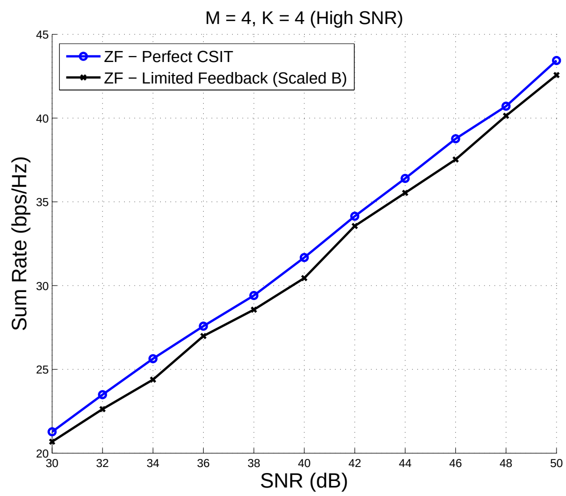

Fig. 1. M = 4 and K = 4

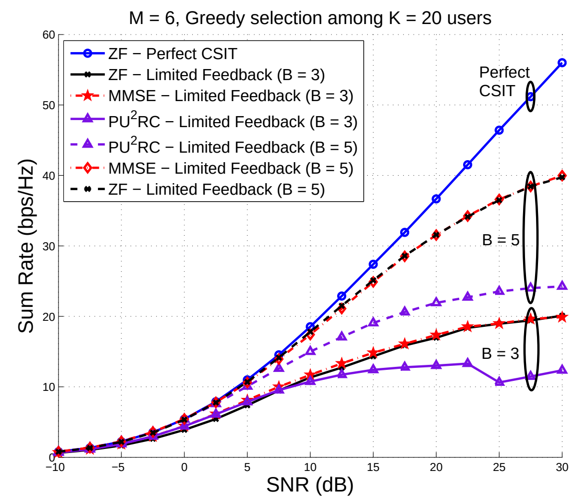

Fig. 3. M = 6 and User Selection (20 Users) with B = 3, 5

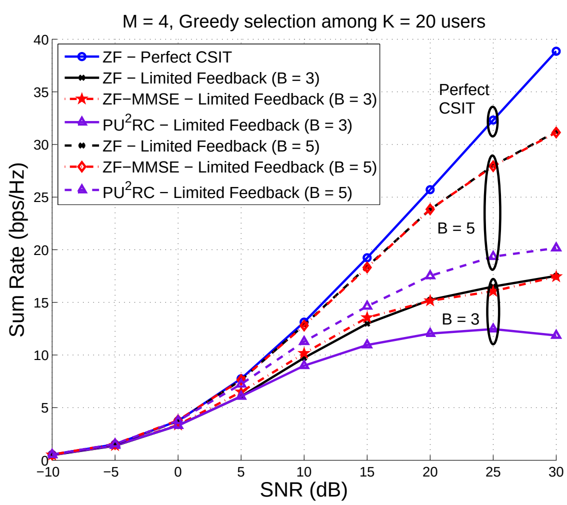

Fig. 2. M = 4 and User Selection (20 Users) with B = 3, 5

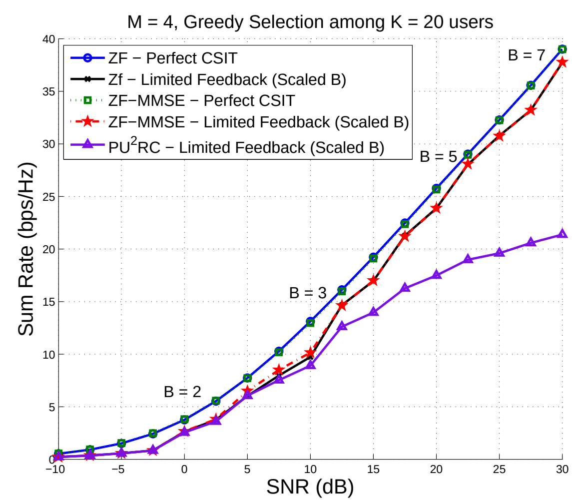

Fig. 4. M = 4 and User Selection (20 Users) with scaled B

V. Conclusion

We studied the performance of zero-forcing beamforming with limited feedback for the special case of LOS channels. In such an environment, multi-user beamforming appears to be the only method that allows for spatial multiplexing because the low level of scattering makes point-to-point MIMO techniques ineffective. Our results show that only a small number of channel feedback bits are needed to perform near-ideal beamforming. Given that LOS channels change quite slowly and the throughput advantage of multi-user MIMO, beamforming appears to be a very compelling transmission strategy for such environments.

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