

# Theory and experimental verification of Kramers-Kronig self-phasing in coherently combined fiber lasers

James R. Leger<sup>1</sup>, Hung-Sheng Chiang<sup>1</sup>, Johan Nilsson<sup>2</sup>, Jayanta Sahu<sup>2</sup>

<sup>1</sup>Dept. of Electrical and Computer Engineering, University of Minnesota, Minneapolis, Minnesota 55455

<sup>2</sup>Optoelectronics Research Centre, University of Southampton, Highfield, Southampton, Hampshire, SO17 1BJ, United Kingdom  
leger@umn.edu

**Abstract:** The recently observed self-phasing of coherently coupled fibers due to Kramers-Kronig effects is theoretically described using a simple model. Direct measurements of the Kramers-Kronig effect and Henry's alpha parameter are reported.

**OCIS codes:** 140.3298, 140.3410

Self-phasing phenomena have been observed in several passively coupled laser systems, including semiconductor laser arrays [1] and fiber lasers [2]. Self-phasing can result from wavelength tuning, thermal effects, and optical nonlinear effects such as Kramers-Kronig (K-K) phase pulling and the optical Kerr effect. Recently, experiments have been reported that isolate the influence of K-K self-phasing by eliminating all other phasing mechanisms [3]. These other effects are eliminated by using a custom dual-core Yb-doped fiber laser where the optical path lengths of the two cores are precisely matched. The final path length difference in the three-meter long fiber laser was measured to be 23  $\mu\text{m}$ . This precise matching of path lengths eliminates the effect of wavelength tuning as a phase adjustment mechanism. The two cores were separated by a distance of 20  $\mu\text{m}$ , where the separation distance is specifically chosen to remove the effect of evanescent coupling while keeping the two cores in the same environment. This effectively eliminates differential path length changes due to thermal effects. Stress rods were added to the fiber to ensure polarization maintaining performance so that a single spatial supermode could be studied. The perpendicularly cleaved fiber facet was cladding-pumped by a 975 nm semiconductor laser, and served as one end of the resonator. The opposite end of the fiber was angle cleaved to eliminate reflections, and a Dammann grating was used to couple light from the two fiber cores together. Differential phase errors were then applied to the cores to study lasing response to path length errors. A probe laser was used to measure the differential phase of the two cores directly as a function of path length error. A precise differential path length error was applied to the cores by translating the Dammann grating, and the total phase (from both the applied path length error and the induced K-K phase) was accurately measured by observing the interference between the two cores. The system layout is shown in fig. 1a.

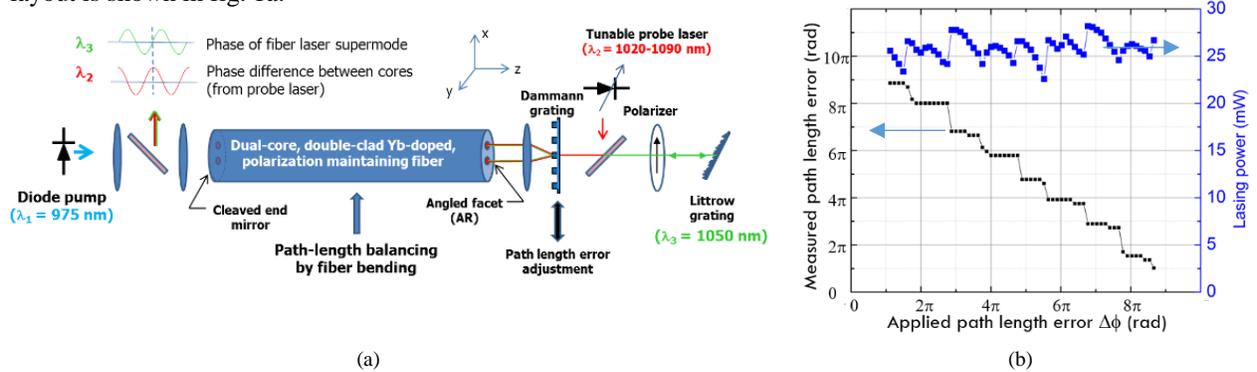


Fig. 1. (a) Experimental set-up; (b) Measurements of laser output power and total path length errors as a function of applied path length error.

We have calculated the response of the system to phase errors and have shown that the attenuation in the cavity is given by

$$L = \frac{64}{\pi^4} R_{oc} a^4 \left[ \frac{1 + \cos(2\phi_T)}{2} \right] \quad (1)$$

where  $L$  is the coupled cavity attenuation,  $R_{oc}$  is the output coupler reflectivity,  $a$  is the coupling loss into each core (assumed to be equal here for simplicity), and  $\phi_T$  is the total phase shift given by the sum of the applied path length

error and the phase shift induced by K-K effects. It is clear from this expression that the optimum cavity phases occur at integer multiples of  $\pi$ . Figure 1b shows experimental measurements of these total cavity phases as a function of the continuous applied path length error introduced by shifting the Dammann grating. Clearly the induced K-K phase is compensating for the applied path length error, giving rise to a stair-case like curve whose total phase jumps from one integer value of  $\pi$  to the next as the applied path length phase is introduced. The coherently combined laser power coupled into a single on-axis mode was also measured on the right side of the Dammann grating and is also shown in fig. 1b. The self-phasing effect is responsible for the efficient lasing and relatively constant output powers of the system in the presence of all path length errors.

We have developed a simple model that partially explains the observed effects. The relationship between the K-K induced phase shift  $\Delta\phi$  and the intensity gain change  $\Delta g$  is given by  $\Delta\phi = \left(\frac{\alpha}{2}\right)\Delta g$ , where  $\alpha$  is Henry's alpha parameter [4]. We have measured this relationship directly in our fibers, and show the data in fig. 2a. A qualitative understanding of the self-phasing can then be understood by plotting the required gain to overcome the loss in eq. 1 (blue periodic curve) simultaneously with the K-K relationship above (red linear curve). This is shown in fig. 2b, where the points of intersection show possible solutions to this nonlinear system. The intersection points containing opposite slopes are stable and those containing slopes of the same sign are unstable. A change in the applied phase is represented by a shift of the line in the horizontal direction (shown as a change from the solid red line to the dashed red line). Since the system will lase in the stable state that requires the minimum gain, it is clear from this plot that the total phase jumps from one low-loss phase state to the next, always maintaining a total phase that is close to an integer multiple of  $\pi$ .

We note that the above simple explanation does not account for the differential phase between the two fiber cores. An extended theory that more accurately accounts for this differential phasing and other details will be presented in the talk.

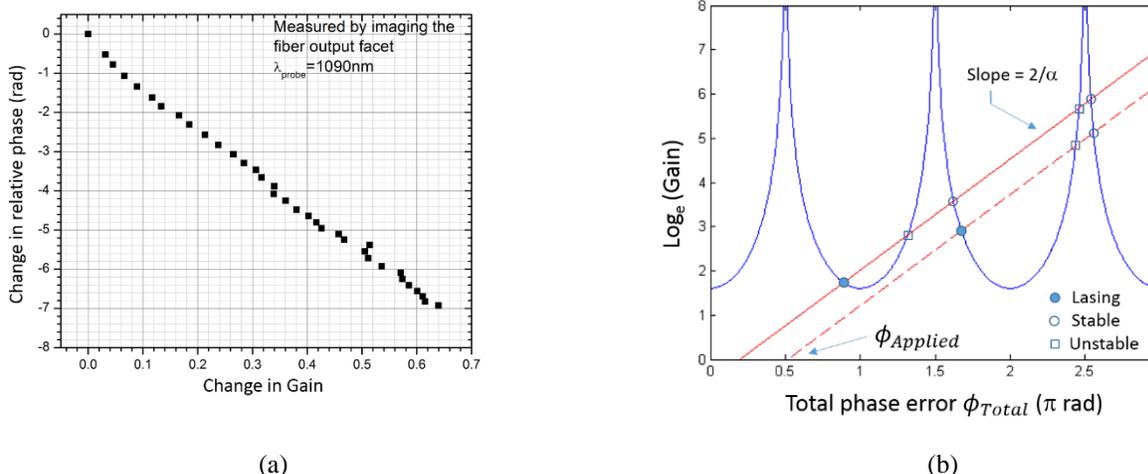


Fig. 2. a) Measurement of K-K induced phase shift as a function of gain; b) Simultaneous equations relating the required cavity gain to overcome cavity loss as a function of total phase error and the induced K-K phase shift as a function of gain.

## References

- [1] E. Philipp-Rutz, "Spatially coherent beam formation and mode locking of an array of solid-state lasers," *IEEE J. Quantum Electron.*, **14**, 112 (1978)
- [2] A. Shirakawa, T. Saitou, T. Sekiguchi, and K. Ueda, "Coherent addition of fiber lasers by use of a fiber coupler," *Opt. Express* **10**, 1167 (2002).
- [3] Hung-Sheng Chiang, James R. Leger, Johan Nilsson, and Jayanta Sahu, "Direct observation of Kramers-Kronig self-phasing in coherently combined fiber lasers," *Opt. Lett.* **38**, 4014 (2013).
- [4] C. H. Henry, "Theory of the linewidth of semiconductor lasers", *IEEE J. Quantum Electron.* **18**, 259 (1982).