

# Transient Stability

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## 1 Introduction

In this document a simple problem has been analysed in order to appreciate the transient stability in Power system. This is a simplest possible problem that can be solved analytically.

## 2 Problem

In Fig. (1) a generator (with reactance  $X_d = 0.25$  pu) is connected to bus (4). The terminal voltage of the generator is regulated at 1.2 pu. This generator is connected to bus (1) through a step up power transformer (leakage reactance  $X_{tr} = 0.2$  pu). Bus (1) is connected to an infinite bus (2) through two identical parallel transmission lines (reactance perunit length is  $0.3174 \Omega$  per Km and it is 100 Km long). The base voltage and MVA of this system is 345 KV and 1500 MVA respectively. Initially the generator was supplying 1500 MW. A three phase symmetrical fault occurs at a distance of 20 km from bus (1) on one of the transmission lines. This fault is cleared after some time. Note that in this problem the change in the magnitude of the back emf  $E$  and  $X_d$  during fault due to the saturation effect (due to the rise in armature current) is neglected. The following parameters are used for the swing equation:  $H = 3.5$  pu,  $\omega_{syn} = 2\pi 60$  and damping coefficient  $D = 7.5$ .

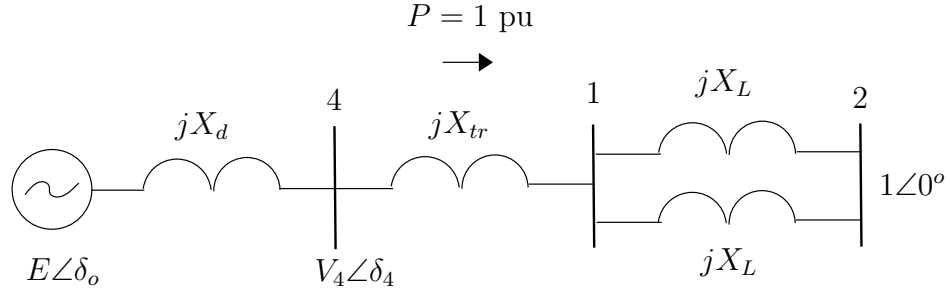


Figure 1: Single line diagram

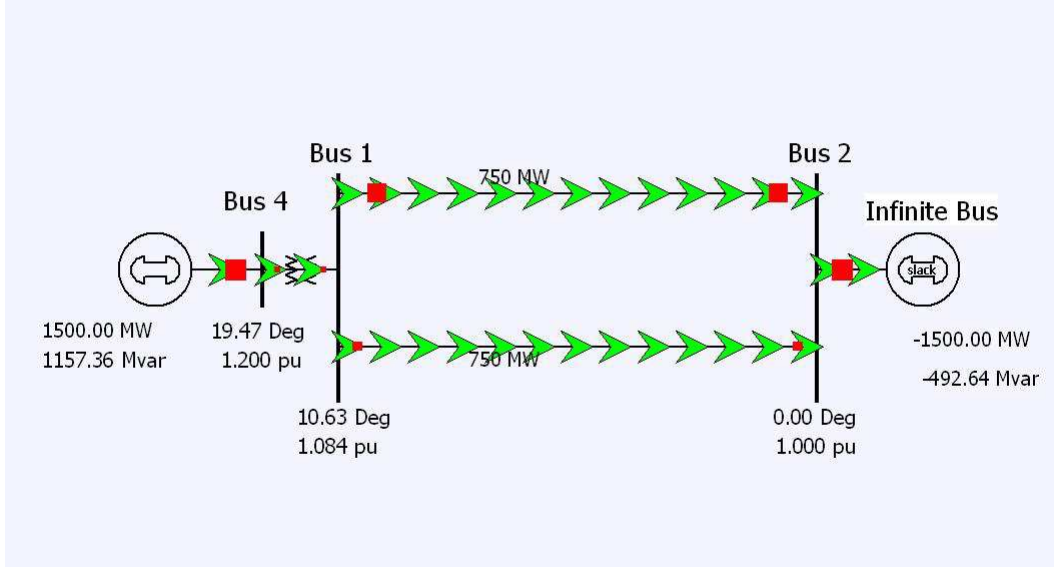


Figure 2: Powerflow in PowerWorld

### 3 Solution

#### 3.1 Pre Fault Calculations

First we need to compute the electrical power flowing out of the generator as a function of  $\delta$  (the phase angle of the back emf of the generator). In order to do that, we need to compute the magnitude of the back emf  $E$ . The per unit reactance of each of the transmission line turns out to be  $X_L = j0.4$ . First let us compute the phase angle of the voltage at bus 4. From (1),  $\delta_4$  is equal to  $19.47^\circ$ . Note that  $V_4 = 1.2$  pu. Now the back emf of the generator can be computed using (2). The back emf is  $1.37 \angle 28.18^\circ$ . The electrical power flowing before the fault as a function of  $\delta$  is given in (3).  $P_{prepk}$  is 2.12.

$$P = \frac{V_4 \sin \delta_4}{\left(X_{tr} + \frac{X_L}{2}\right)} \quad (1)$$

$$E \angle \delta = V_4 \angle \delta_4 + \frac{(V_4 \sin \delta_4 - 1)}{j \left(X_{tr} + \frac{X_L}{2}\right)} j X_d \quad (2)$$

$$P_e(\delta) = \frac{E}{\left(X_d + X_{tr} + \frac{X_L}{2}\right)} \sin \delta = P_{prepk} \sin \delta \quad (3)$$

The same system has been simulated in PowerWorld, Fig. (2), and the results match with the analytical computations.

#### 3.2 Fault

During fault it is assumed that the magnitude of the back emf  $E$  does not change. Fig. (3) gives the equivalent single line diagram of the system during the fault. Where  $X_{th} = \frac{\alpha}{(1+\alpha)} X_L$ ,  $V_{th} = \frac{\alpha}{(1+\alpha)}$

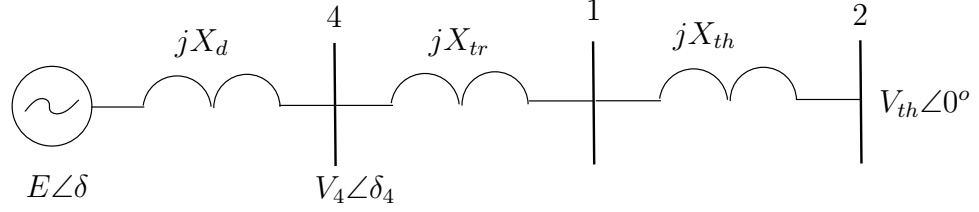


Figure 3: Single line diagram during fault

and  $\alpha = 0.2$ . The electrical power during the fault as a function of  $\delta$  (the generator back emf phase angle) is given by (4).

$$P_e(\delta) = \frac{EV_{th}}{(X_d + X_{tr} + X_{th})} \sin \delta = P_{fault_{pk}} \sin \delta \quad (4)$$

During fault the mechanical input power is greater than the generated electrical power as shown in Fig. (4). So  $\delta$  keeps on increasing according to the swing equation (5). Neglecting the damping in the swing equation *Equal angle criteria* provides a method to compute the maximum or the critical value of  $\delta$ . In order to remain in stability the fault needs to be cleared before  $\delta$  reaches its critical value. In MATLAB the swing equation is numerically solved and the evolution of  $\delta$  with time during fault has been plotted in Fig. (5). Solving (6) we can get the value of the critical angle ( $\delta_{critical}$ ) and then using the plot in Fig. (5) we can get the critical clearing time. Which turns out to be 0.242 secs in this particular case.

$$\left( \frac{2H}{\omega_{syn}} \right) \frac{d^2\delta}{dt^2} + D\delta = P_m - P_e(\delta) \quad (5)$$

$$\int_{\delta_o}^{\delta_{critical}} (P_m - P_{fault_{pk}} \sin \delta) d\delta = \int_{\delta_{critical}}^{\pi - \delta_o} (P_{pre_{pk}} \sin \delta - P_m) d\delta \quad (6)$$

This system has been simulated both in MATLAB and PowerWorld. In both the simulations fault occurs at 0.2 secs and it is cleared at 0.4 secs. If the damping term is removed from the swing equation,  $\delta$  oscillates around its equilibrium point and never settles down. As the fault in this case is cleared before the critical time with damping the system comes back to its original prefault state, Fig. (6)(7) and (8).

In a second case the fault is cleared just after the critical clearing time i.e. at 0.45 secs and  $\delta$  is observed to be diverging, Fig(9) and Fig. (10).

## 4 Conclusion

In this document a simplest possible transient stability problem has been analysed. The introduction of an infinite bus in a two bus case may seem to be impractical. But in order to appreciate the problem of transient stability in a simple case like this, inclusion of an infinite bus is necessary. This problem clearly illustrates the idea of critical clearing time and its importance in power system stability. It is to be noted that in a multi-machine situation no analytical approach (like that of *Equal angle criteria*) exists to compute the critical clearing time. In that case numerical simulation

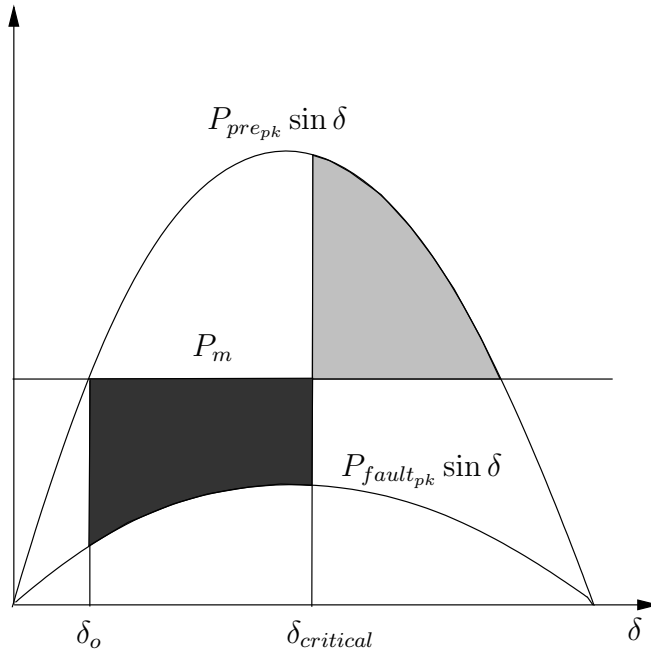


Figure 4: Power angle curves

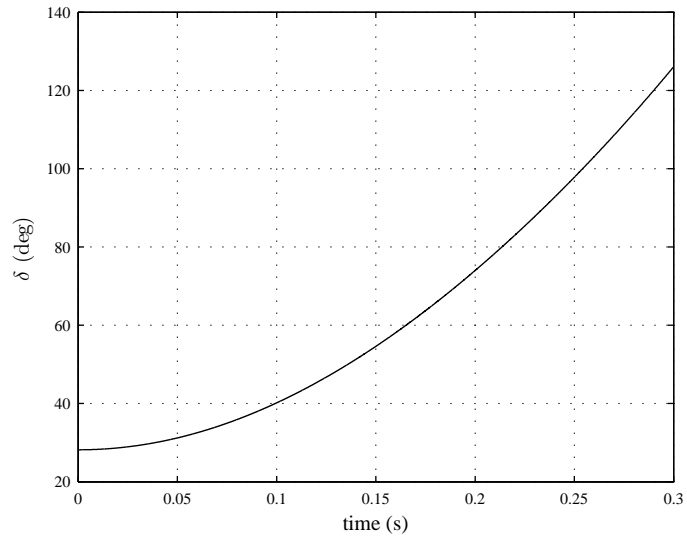


Figure 5: Evolution of  $\delta$  during fault

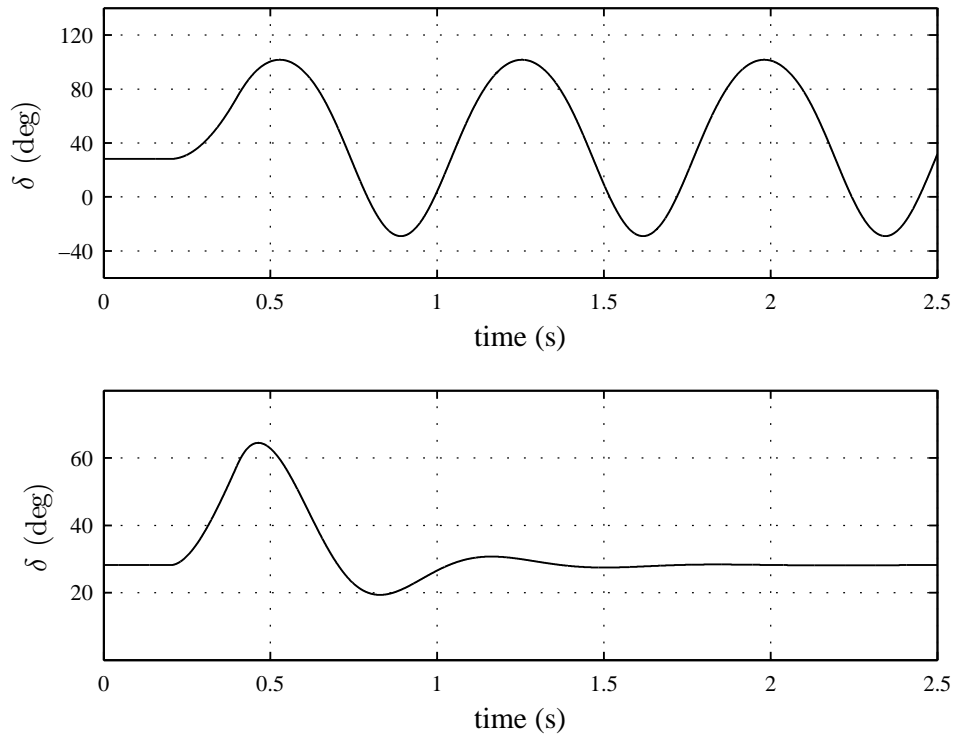


Figure 6: Plot of  $\delta$  without and with damping in MATLAB

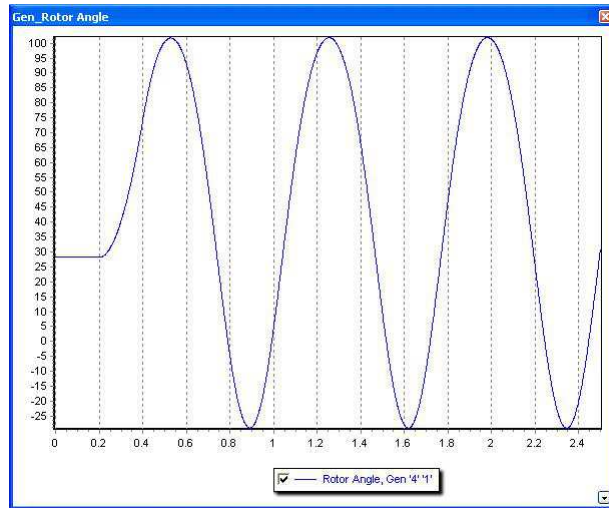


Figure 7: Plot of  $\delta$  without damping in PowerWorld

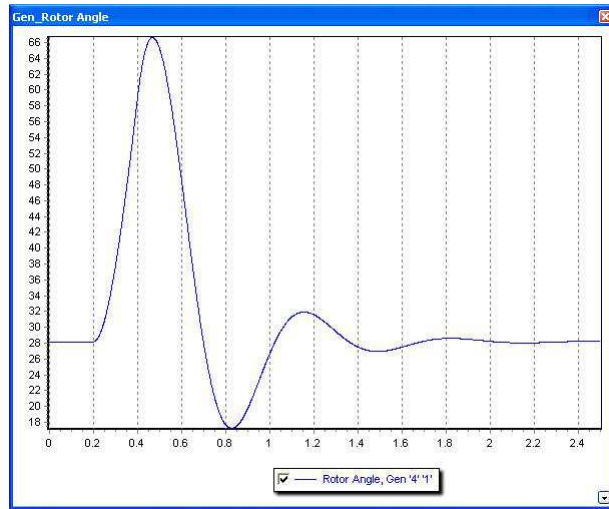


Figure 8: Plot of  $\delta$  with damping in PowerWorld

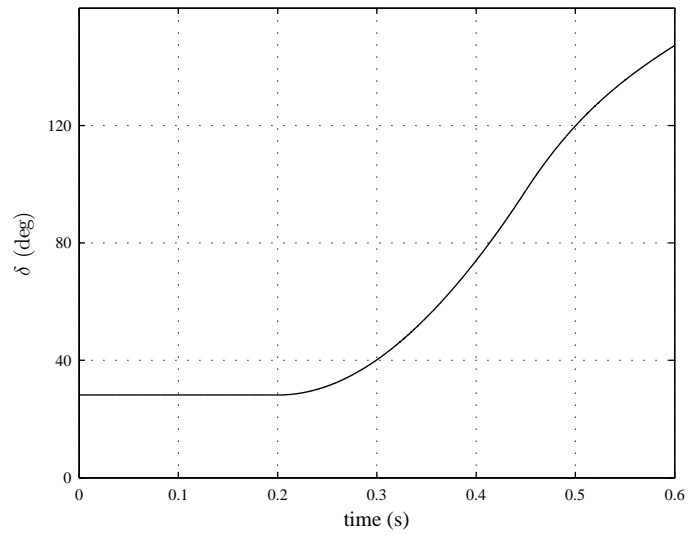


Figure 9: unstable case: Plot of  $\delta$  in MATLAB

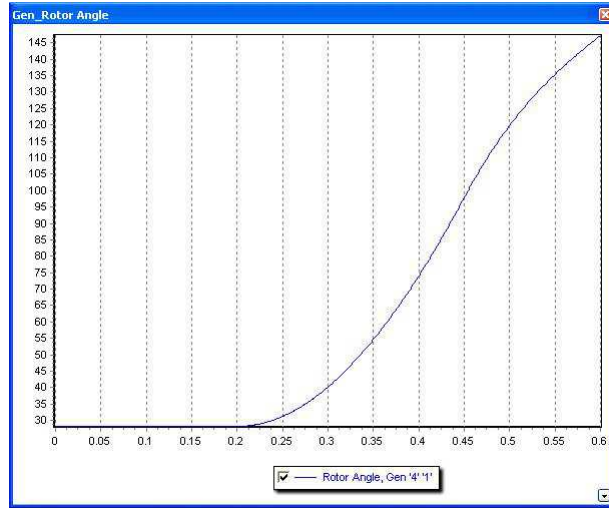


Figure 10: unstable case: Plot of  $\delta$  in PowerWorld

is the only way out. This problem has been completely solved using MATLAB and results match with those obtained from a simulation of the identical system in PowerWorld.