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There are two questions with 20 points each (total of 40/40)

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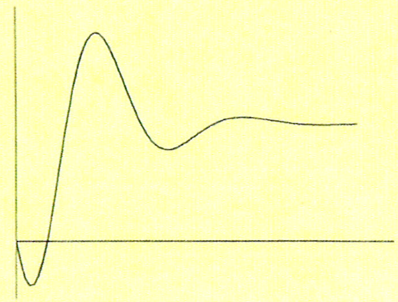
**Question #1 [Points of individual subquestions i)-iv): 2+8+4+6]:**

Consider the lumped scalar linear system with transfer function

$$P(s) = \frac{1 + s - s^2 - s^3}{(1 + s + s^2)^2}$$



The step response $y(t)$ (under zero initial conditions) is shown on the right, this is

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}P(s)\right\}.$$

Determine, explain, or prove the following:

**i) What is the steady-state value as $t \to \infty$?**

**ii) What is the general form of the step response, the period of the oscillations, and the attenuation rate?** (You don't need to compute the constant coefficients in the mathematical expression for the step response. Just indicate the nature of the various terms present.)

**iii) Show that the step response takes negative values for some time, before it rises and oscillates towards the steady state value.** (As you can see, it "undershoots", and you are asked to prove that this is so. Hint: you may use the "initial value theorem.")

**iv) Show that for a stable linear system with transfer function**

$$P(s) = \frac{b_0 + b_1 s + \ldots + b_{n-1}s^{n-1}}{a_0 + a_1 s + \ldots + a_{n-1}s^{n-1} + s^n},$$

**with $b_{n-1} \neq 0$ and $b_0 > 0$, $a_0 > 0$, if $P(s)$ has an odd number of zeros in the right half plane, then the step response turns negative before it eventually settles to a positive steady-state value.**

**Question #2 [Points of individual subquestions i)-v): 4 each]:**
Assume that a physical system is modeled by the transfer function

$$P(s) = \frac{1-s}{1-s/3}.$$

As all models, this is thought only as an approximation of the real system. The system is to operate within a stabilizing feedback loop.
Do the following:

i) Draw the Bode plot (magnitude and phase) and the Nyquist plot for this transfer function.

ii) Determine a constant feedback gain to stabilize the system model $P(s)$.

iii) Verify that this constant feedback control indeed stabilizes the system by using Nyquist's theory.

iv) Show that this constant gain controller however is not robust to small modeling errors at high frequencies. More specifically you need to show that the feedback loop is destabilized when small time-delays are introduced in the loop (such as computational or transmission delays, no matter how small). You need to analyze and explain this using Nyquist's theory.

v) Design a control law $C(s)$ which can stabilize the physical system which is now robust to small high frequency modeling errors. For instance, you may assume that the controller is of the form

$$C(s) = K\frac{1+bs}{1+as},$$

specify suitable values that you can then verify using one or more of the Routh, root locus, or Nyquist criteria. (Hint: you may use root locus and place the controller poles and zeros so that the unstable poles shift to the left half plane without going through the point at infinity.)