This problems tests your knowledge of synthesizing combination and sequential logic, state machines, and switching circuits.

1. **Representations of Boolean Functions** [*1.0 points*]

   For the function defined by the following truth table:

a	b	c	d	$f(a,b,c,d)$
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

   (a) Give the Boolean expression corresponding to a minimal two-level AND-OR circuit (i.e., AND gates in the first level and an OR gate in the second level).

   (b) Give the Boolean expression corresponding to a minimal two-level OR-NAND circuit (i.e., OR gates in the first level and a NAND gate in the second level).

   (c) Give the Boolean expression corresponding to minimal two-level OR-AND circuit (i.e., OR gates in the first level and an AND gate in the second level).

   (d) Give the Boolean expression corresponding to minimal two-level AND-NOR circuit (i.e., AND gates in the first level and a NOR gate in the second level).

   (e) Given the Boolean expression corresponding to a minimal two-level AND-XOR circuit (i.e., AND gates in the first level and an exclusive-OR gate in the second level). Do *not* negate any of the variables in this representation.

2. **State Tables and Graphs** [1.0 points]

   A sequential circuit has two inputs (X_1, X_2) and one output (Z). The output remains a constant value unless one of the following input sequences occurs:

   - The input sequence $X_1, X_2 = 01, 11$ causes the output to become 0.
   - The input sequence $X_1, X_2 = 10, 11$ causes the output to become 1.
   - The input sequence $X_1, X_2 = 10, 01$ causes the output to change value.

   Assume that this is a Moore machine. Provide a state transition table and state graph for this circuit.

3. **Flip-Flop Input Equations** [1.0 points]

Consider the following state table for a sequential circuit with input X and output Z.

state	next state		Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
S_0	S_1	S_2	0	0
S_1	S_3	S_2	0	0
S_2	S_1	S_4	0	0
S_3	S_5	S_2	0	0
S_4	S_1	S_6	0	0
S_5	S_5	S_2	1	0
S_6	S_1	S_6	0	1

Derive flip-flop input equations for an implementation of the circuit with:

(a) D flip-flops

(b) J-K flip-flops

4. **Switching Circuit** [*1.0 points*]

For the switching circuit in Figure 1, write the Boolean function implemented between S and D. Each switch is closed if the corresponding variable is 1 and open if it 0. The function evaluates to 1 if there is a closed path from S to D and 0 otherwise.

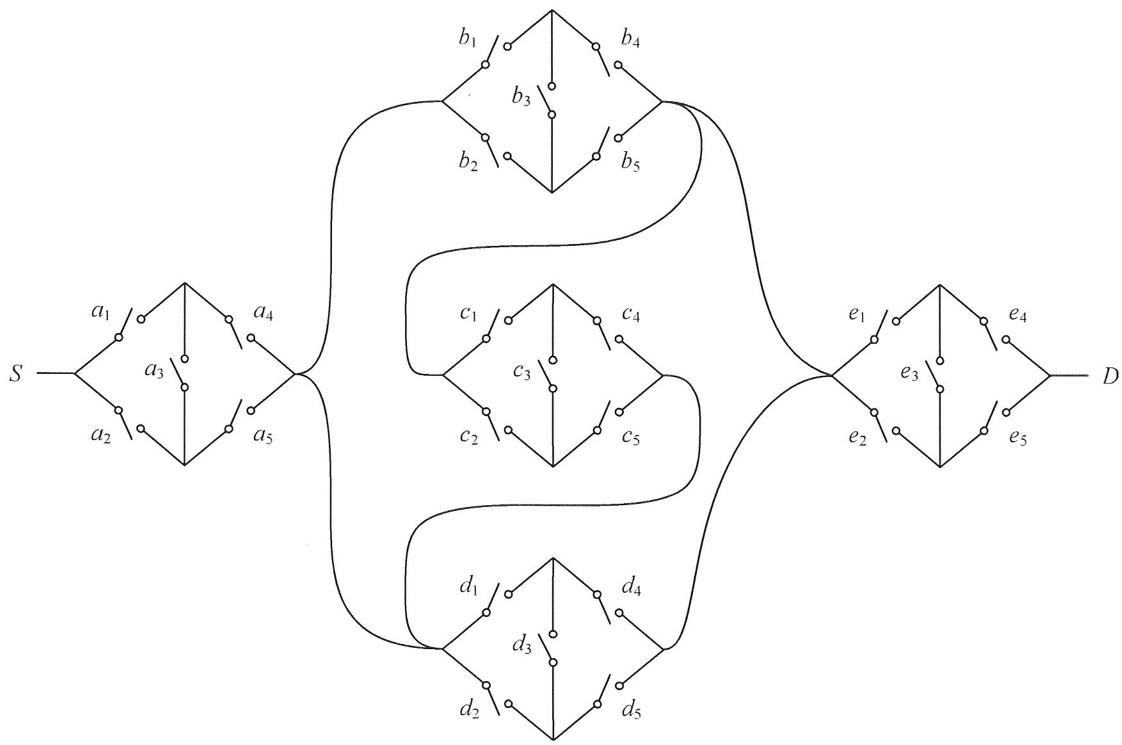

Figure 1: A switching circuit.