This problems tests your knowledge of synthesizing combination and sequential logic, state machines, and switching circuits.

1. **Representations of Boolean Functions** [*1.0 points*]

   For the function defined by the following truth table:

a	b	c	d	$f(a,b,c,d)$
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

   (a) Give the Boolean expression corresponding to a minimal two-level AND-OR circuit (i.e., AND gates in the first level and an OR gate in the second level).

   (b) Give the Boolean expression corresponding to a minimal two-level OR-NAND circuit (i.e., OR gates in the first level and a NAND gate in the second level).

   (c) Give the Boolean expression corresponding to minimal two-level OR-AND circuit (i.e., OR gates in the first level and an AND gate in the second level).

   (d) Give the Boolean expression corresponding to minimal two-level AND-NOR circuit (i.e., AND gates in the first level and a NOR gate in the second level).

   (e) Given the Boolean expression corresponding to a minimal two-level AND-XOR circuit (i.e., AND gates in the first level and an exclusive-OR gate in the second level). Do *not* negate any of the variables in this representation.

Answer

(a)

$$abd + ab'd' + ac + a'b'd + b'c$$

(b)

$$((a' + b' + d')(a' + b + d)(a' + c')(a + b + d')(b + c'))'$$

(c)

$$(a' + b + c + d')(a + b')(a + c + d)(b' + c + d)$$

(d)

$$(ab'c'd + a'b + a'c'd' + bc'd')'$$

(e)

$$a \oplus d \oplus c \oplus ab \oplus ac \oplus bd \oplus bc \oplus dc \oplus abd \oplus bdc \oplus abdc$$

2. **State Tables and Graphs** [1.0 points]

A sequential circuit has two inputs (X_1, X_2) and one output (Z). The output remains a constant value unless one of the following input sequences occurs:

- The input sequence X_1, X_2 = 01, 11 causes the output to become 0.
- The input sequence X_1, X_2 = 10, 11 causes the output to become 1.
- The input sequence X_1, X_2 = 10, 01 causes the output to change value.

Assume that this is a Moore machine. Provide a state transition table and state graph for this circuit.

Answer

state	next state				Z
	X_1, X_2				
	00	01	10	11	
S_0	S_0	S_1	S_2	S_0	0
S_1	S_0	S_1	S_0	S_0	0
S_2	S_0	S_3	S_2	S_3	0
S_3	S_3	S_4	S_5	S_3	1
S_4	S_3	S_4	S_3	S_0	1
S_5	S_3	S_0	S_5	S_3	1

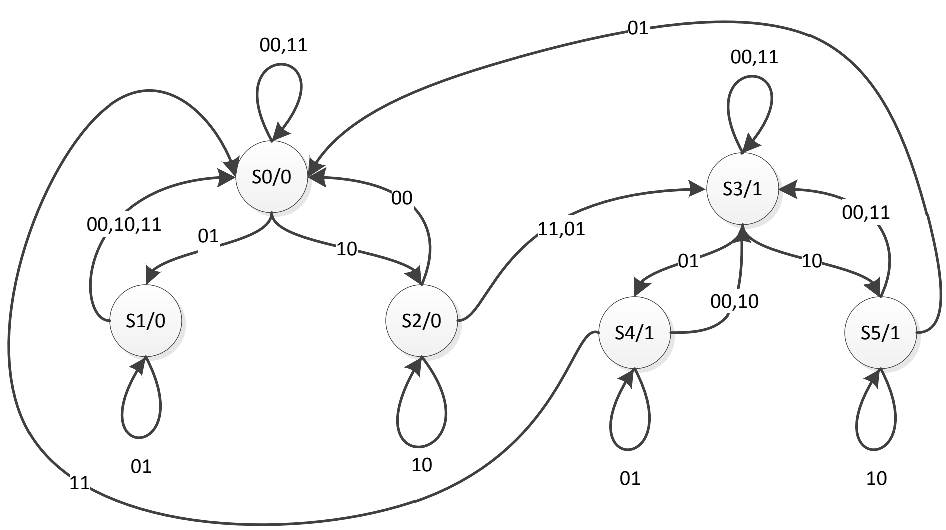

Figure 1: State Graph

3. **Flip-Flop Input Equations** [1.0 points]

Consider the following state table for a sequential circuit with input X and output Z.

state	next state		Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
S_0	S_1	S_2	0	0
S_1	S_3	S_2	0	0
S_2	S_1	S_4	0	0
S_3	S_5	S_2	0	0
S_4	S_1	S_6	0	0
S_5	S_5	S_2	1	0
S_6	S_1	S_6	0	1

Derive flip-flop input equations for an implementation of the circuit with:

(a) D flip-flops

(b) J-K flip-flops

Answer

(a) Let's use the following state assignment:

state	next state	
	$X = 0$	$X = 1$
abc	$a^+b^+c^+$	$a^+b^+c^+$
a' b' c'	0 0 1	0 1 0
a' b' c	0 1 1	0 1 0
a' b c'	0 0 1	1 0 0
a' b c	1 0 1	0 1 0
a b' c'	0 0 1	1 1 0
a b' c	1 0 1	0 1 0
a b c'	0 0 1	1 1 0
a b c	x x x	x x x

The input equations for D flip-flops D_a, D_b and D_c are:

$$\begin{aligned} D_a &= (xc' + x'c)(a + b) \\ D_b &= a'b'c + x(a + b' + c) \\ D_c &= x' \end{aligned}$$

(b) Using the same state assignment, the input equations for J-K flip-flops are:

$$
\begin{aligned}
J_a &= bc'x + bcx' \\
K_a &= ac'x' + acx \\
J_b &= a'b'x + a'cx' + ab'x \\
K_b &= bc'x' + a'bc' + a'bx' \\
J_c &= c'x' \\
K_c &= cx
\end{aligned}
$$

4. **Switching Circuit** [*1.0 points*]

For the switching circuit in Figure 2, write the Boolean function implemented between S and D. Each switch is closed if the corresponding variable is 1 and open if it 0. The function evaluates to 1 if there is a closed path from S to D and 0 otherwise.

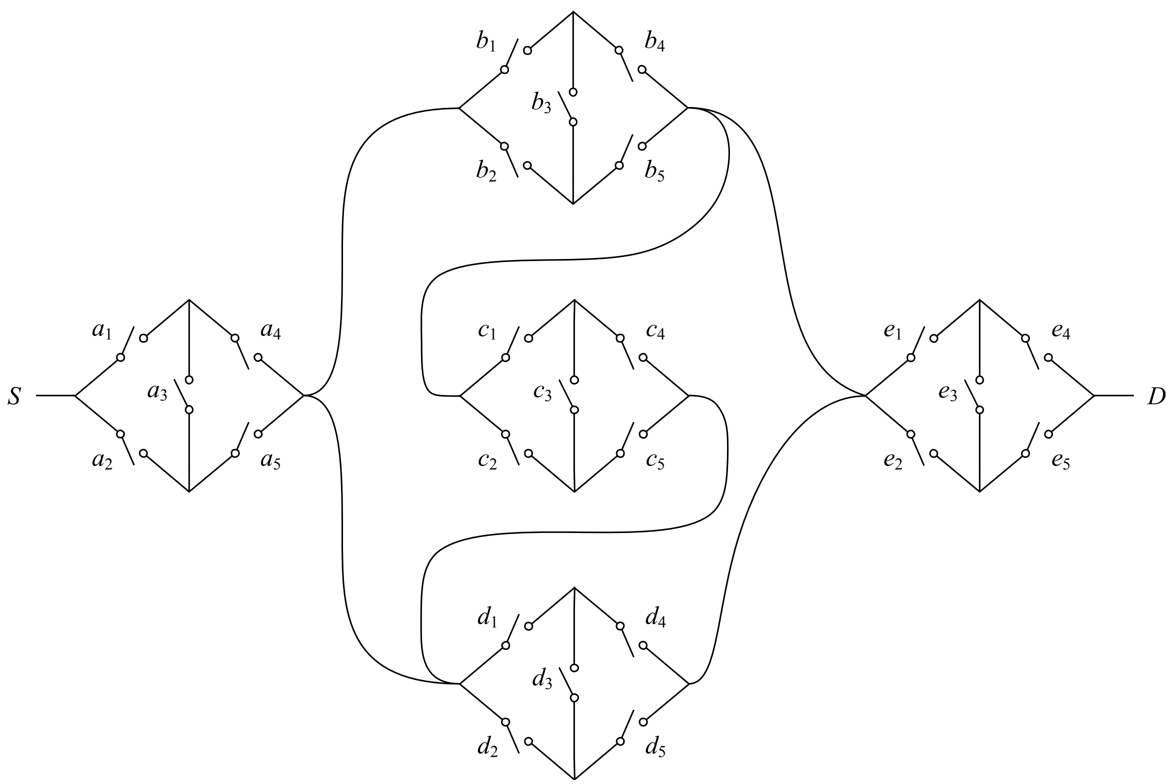

Figure 2: A switching circuit.

Answer

$$
\begin{aligned}
a &= a_1a_4 + a_2a_5 + a_1a_3a_5 + a_2a_3a_4 \\
b &= b_1b_4 + b_2b_5 + b_1b_3b_5 + b_2b_3b_4 \\
c &= c_1c_4 + c_2c_5 + c_1c_3c_5 + c_2c_3c_4 \\
d &= d_1d_4 + d_2d_5 + d_1d_3d_5 + d_2d_3d_4 \\
e &= e_1e_4 + e_2e_5 + e_1e_3e_5 + e_2e_3e_4 \\
f_{S-D} &= ae(b + c + d)
\end{aligned}
$$