Closed book, closed notes, calculators OK. The Communications problem consists of two parts, each comprising multiple questions, points as marked (point sum = 40 = perfect score); nominal duration = 1 hour.

Part A: Modulation, demodulation, performance [sum of points = 20]
A digital modulator produces the following four symbol waveforms at its output

$$s_1(t) = \sqrt{\frac{2}{T}} \cos\left(\frac{2\pi t}{T}\right) w_T(t)$$

$$s_2(t) = \sqrt{\frac{2}{T}} \cos\left(\frac{4\pi t}{T}\right) w_T(t)$$

$$s_3(t) = \sqrt{\frac{2}{T}} \sin\left(\frac{2\pi t}{T}\right) w_T(t)$$

$$s_4(t) = \sqrt{\frac{2}{T}} \sin\left(\frac{4\pi t}{T}\right) w_T(t)$$

where $w_T(t) := u(t) - u(t-T)$ is a rectangular window of length equal to the symbol period T ($u(t)$ is the usual unit step function, equal to one if $t \geq 0$, zero otherwise).

- **[3 points]** Compute the energy of each symbol.
- **[5 points]** Construct an orthonormal basis for these four signaling waveforms.
- **[6 points]** Specify and sketch the optimal demodulator, assuming equiprobable symbols and transmission over an additive white Gaussian noise channel with no intersymbol interference.
- **[6 points]** Assume that the noise at the input of the decision device has variance $\sigma^2 = \frac{N_0}{2}$ per signaling dimension. Use the union bound to derive a closed-form estimate of the symbol error rate.

Trigonometric identities:

$$\sin(A)\sin(B) = (1/2)\cos(A-B) - (1/2)\cos(A+B)$$

$$\cos(A)\cos(B) = (1/2)\cos(A-B) + (1/2)\cos(A+B)$$

$$\sin(A)\cos(B) = (1/2)\sin(A-B) + (1/2)\sin(A+B)$$

Part B: Bit mapping, link budget [sum of points = 20]

- **[7 points]** Assuming equiprobable symbols and zero-mean additive Gaussian noise, find the best (from a BER-minimizing viewpoint) mapping of pairs of bits to constellation points for uniform 4-level pulse amplitude modulation (4-PAM). Is the solution unique? If yes, explain why; else list all solutions.
- **[7 points]** Repeat, this time for 4-orthogonal ($4-\perp$) modulation.
- **[6 points]** Is there a digital transmission system that can attain BER equal to 10^{-6} at $\frac{E_b}{N_0} = 1$ over an AWGN channel without error control coding/decoding? Justify your answer.