Problem 1

Solutions: WPE/Communications, Apr. 14, 2012

Part A: Using the trigonometric identities, it can be easily shown that the energy of each signaling waveform is equal to 1, and that $s_1(t) \perp s_2(t) \perp s_3(t) \perp s_4(t)$. Hence $\{s_i(t)\}_{i=1}^{4}$ form an orthonormal basis; the given system employs $4-\perp$ modulation with $E_s = 1$ and thus $E_b = \frac{1}{2}$. The optimal receiver structure is a bank of 4 correlators, each matched to one of the $s_i(t)$'s, followed by an optimal detector that takes as input the outputs of these correlators r_1, r_2, r_3, r_4, and outputs $\widehat{i} = \arg\max_i r_i$. The SER cannot be computed in closed form, however it can be estimated using the union bound considering pairs of constellation points:

$$SER \leq (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right),$$

where M is the number of constellation points; i.e.,

$$SER \leq 3Q\left(\sqrt{\frac{1}{N_0}}\right),$$

in this case.

Part B: For 4-PAM, we wish to ensure that neighboring constellation points differ in only one bit (*a-la* Gray mapping) so that the most likely symbol errors yield only one bit error. There is a total of $4! = 24$ permutations of the 4 labels, out of these the following 8 are optimal:

00 01 11 10 and its 3 circular shifts

00 10 11 01 and its 3 circular shifts

For orthogonal modulation, all constellation points are at the same distance from all others, a perfectly symmetric situation; hence bit mapping is irrelevant, or, any bit mapping is equally good. For the last question: the answer is yes, using orthogonal modulation of appropriately high order (= # dimensions). This is possible because $\frac{E_b}{N_o} = 1$ (0 dB) is above the Shannon limit, hence we can attain arbitrarily low BER at the cost of bandwidth (over-)expansion.