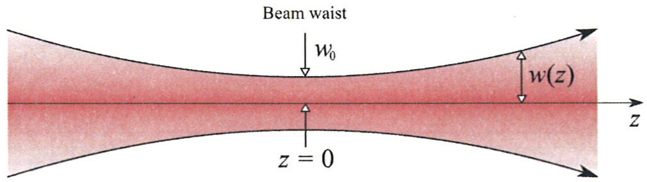
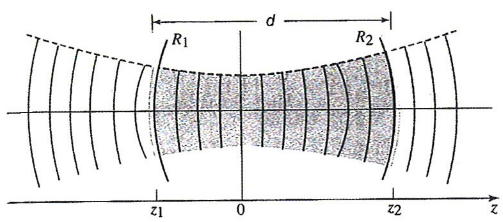
Confinement Condition for a Gaussian Beam

Consider a Gaussian beam whose width (beam radius) $W(z)$ and wavefront radius of curvature $R(z)$ are given by

$$W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \qquad R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$



A Gaussian beam reflected from a spherical mirror will retrace the incident beam if the radius of curvature of its wavefront is the same as that of the mirror radius. Let's fit a Gaussian beam to two mirrors separated by a distance d, as shown below. Their radii of curvature are R_1 and R_2. Both mirrors are taken to be concave (i.e. $\mathbf{R_1 < 0}$ and $\mathbf{R_2 < 0}$) The center of the beam is assumed to be at the origin $z = 0$; mirrors R_1 and R_2 are located at positions z_1 , and $z_2 = z_1 + d$



Prove the following relations:

*Pay careful attention to the sign of $R(z)$: A concave mirror has a negative radius (i.e. $\mathbf{R_1 < 0}$ and $\mathbf{R_2 < 0}$). But the beam radius of curvature is defined to be positive for $z > 0$ (right mirror) and negative for $z < 0$, meaning $\mathbf{R_1} = R(z_1)$, $\mathbf{-R_2} = R(z_2)$

(a) (1 point) $$z_1 = \frac{-d(R_2+d)}{R_2+R_1+2d}$$

(b) (1 point) $$z_0^2 = \frac{-d(R_1+d)(R_2+d)(R_1+R_2+d)}{(R_2+R_1+2d)^2}$$

In order that the above solution represents a Gaussian beam, z_0 must be real. Using that condition, derive the following condition for the confinement of a Gaussian beam:

(c) (2 points) $$0 \le \left(1+\frac{d}{R_1}\right)\left(1+\frac{d}{R_2}\right) \le 1$$