1) **[7 points total]** A finite sequence $b[n]$ is such that its z-Transform $B(z) = \sum_{n=-\infty}^{\infty} b[n]z^{-n}$ satisfies

$$B(z) + B(-z) = 2c,$$

where $c \neq 0$ is a real constant.

a) **[5 points]** What constraints does this place on the signal $b[n]$? In particular, what can be said about $b[n]$ for the following cases?

- $n = 0$,
- n odd,
- n even ($n \neq 0$)

b) **[2 points]** Give one example of such a sequence $b[n]$.

2) **[20 points total]** Consider a real valued sequence $x[n]$ whose Fourier transform $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ is known to satisfy

$$X(e^{j\omega}) = 0, \quad \text{for } \frac{\pi}{4} \leq |\omega| \leq \pi.$$

One value of the sequence $x[n]$ may be corrupted; we would like to devise a strategy to recover it.

Let $g[n]$ denote the corrupted signal, which is real-valued, and let n_0 denote the (unknown) location of the corrupted value. Thus, we can write

$$g[n] = x[n], \quad n \neq n_0,$$

and

$$g[n] = x[n] + a\delta[n - n_0],$$

for some real-valued a. Our recovery approach begins by filtering $g[n]$ with an ideal high pass filter $h[n]$ with cutoff frequency $\omega_c = \pi/2$.

a) **[5 points]** Find an expression for the entries of the filter $h[n]$. (*You might choose to use the Fourier inversion formula:* $h[n] = \frac{1}{2\pi}\int_{2\pi} H(e^{j\omega})e^{j\omega n}d\omega$.)

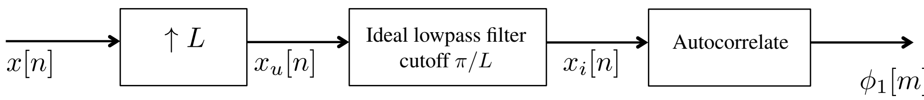
b) **[2 points]** Let $y[n] = g[n] * h[n] = (g[n] = x[n] + a\delta[n - n_0]) * h[n]$. Use your expression from part (a) to find a simple expression for $y[n]$.

c) **[5 points]** Explain how you could identify n_0 by examining the values of the filtered sequence $y[n]$. (*Hint: It may help to sketch* $y[n]$.)

d) **[3 points]** Find a simple expression for the unknown amplitude a in terms of the original sequence $g[n]$. (Your answer will be a function of n_0, which you would be able to identify following the procedure you describe in part (c)).

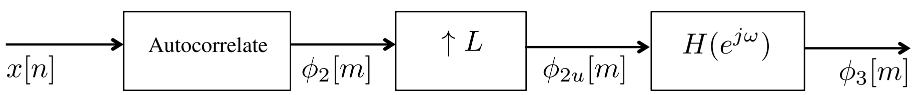
e) **[5 points]** Find a (simple) expression for the original sequence $x[n]$ in terms of the quantities $g[n], y[n]$, and the (now) known n_0.

3) **[13 points total]** Suppose we wish to compute the autocorrelation function of an upsampled real signal using the system below



$$\phi_1[m] = \sum_{n=-\infty}^{\infty} x_i[n+m]x_i[n]$$

It was suggested that this can be accomplished by a different system, for an appropriate $H(e^{j\omega})$



a) **[5 points]** Find an expression relating the Fourier transform of $\phi_i[m]$, denoted $\Phi_1(e^{j\omega})$, to the Fourier transform of $x_i[n]$, denoted $X_i(e^{j\omega})$.

b) **[3 points]** How does the bandwidth of $\Phi_1(e^{j\omega})$ compare with the bandwidth of $X_i(e^{j\omega})$?

c) **[5 points]** What should $H(e^{j\omega})$ be to ensure that the outputs of the two systems are the same (i.e., to ensure that $\phi_3[m] = \phi_1[m]$ for all m)?