1)

a) For a sequence $b[n]$ with z-Transform $B(z)$ and ROC $= R_b$, the multiplication property of the z-Transform states that

$$a^n b[n] \stackrel{\mathcal{Z}}{\longleftrightarrow} B(x/a)$$

where the ROC of $a^n b[n]$ is $|a|R_b$. Here $a = -1$, implying that the ROC of $B(z)$ and $B(-z)$ are the same. At any rate, since the problem states that $b[n]$ is a finite sequence, we know that the ROC of $B(z)$ is the entire z-plane, except possibly $z = 0$ or $z = \infty$.

Taking the inverse z-Transform of each side, it follows that

$$b[n] + (-1)^n b[n] = 2c\delta[n].$$

When $n = 0$, this implies that $2b[0] = 2c$, or $b[0] = c$. When n is not equal to zero but otherwise even, we have $b[n] + b[n] = 0$, implying that $b[n] = 0$. Finally, for n odd, the condition provides no information about $b[n]$, so we conclude that $b[n]$ may be arbitrary for n odd.

b) There are many possibilities, the simplest of which is $b[n] = c\delta[n]$.

2)

a) We can find $h[n]$ using Fourier Transform properties. First we recall (or compute from the definition) that an ideal low-pass filter $\ell[n]$ with cutoff frequency ω_c corresponds to the FT pair:

$$\frac{\sin(\omega_c n)}{\pi n} = \ell[n] \stackrel{\mathcal{F}}{\longleftrightarrow} L(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| < \pi \end{cases}$$

Further, we can express the highpass filter $h[n]$ as a frequency shifted version of this lowpass filter. That is, if we take an ideal lowpass filter with cutoff frequency $\omega_c = \pi/2$ and shift it in frequency (to the right or left) by π we get $h[n]$. Since

$$e^{j\omega_0 n} x[n] \stackrel{\mathcal{F}}{\longleftrightarrow} X(e^{j(\omega-\omega_0)}),$$

we conclude that

$$h[n] = e^{j\pi n} \frac{\sin(\pi n/2)}{\pi n} = (-1)^n \frac{\sin(\pi n/2)}{\pi n}.$$

b) Given the expression for $h[n]$ computed in part (a), we see that

$$y[n] = a(-1)^{(n-n_0)} \frac{\sin(\pi(n-n_0)/2)}{\pi(n-n_0)}$$

c) Note first that

$$h[n] = \begin{cases} \frac{a}{2}, & n = 0 \\ 0, & \text{even } n \neq 0 \\ \frac{a(-1)^{(3n-1)/2}}{\pi n}, & n \text{ odd} \end{cases}$$

and in particular, the only three consecutive nonzero values occur for $n = \{-1, 0, 1\}$. Now, since $y[n]$ is just a time-shifted version of $h[n]$, we can just search for three consecutive nonzero values of $y[n]$. The location of the middle nonzero is n_0.

d) Following the discussion above, we have $a = 2y[n_0]$.

e) Given that $g[n] = x[n] + a\delta[n - n_0]$, we have that $x[n] = g[n] - 2y[n_0]\delta[n - n_0]$.

3)

a) It is easy to see that $\phi_1[m] = x_i[m] * x_i[-m]$, and so $\Phi_1(e^{j\omega}) = X_i(e^{j\omega})X_i(e^{-j\omega})$.

b) Since $x[n]$ is real, so will be $x_i[n]$. Thus, the magnitude spectrum of its Fourier Transform is symmetric in ω, implying that the bandwidth of $\Phi_1(e^{j\omega})$ is the same as the bandwidth of $X_i(e^{j\omega})$.

c) The act of upsampling a discrete sequence by an integer factor L, followed by filtering using an ideal lowpass filter with cutoff frequency π/L, amounts to compressing the spectrum of the sequence in the frequency domain by a factor of L. Since a sequence and its corresponding autocorrelation sequence have the same bandwidth, we can interchange the order of the autocorrelation block with the upsampling/filtering block.

Thus, $H(e^{j\omega})$ corresponds to an ideal lowpass filter with cutoff frequency π/L.