

Solutions
Spring 2014

Problem 8
Transmission Lines and Fields

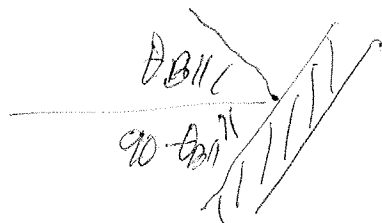
A (a) $E = 10 \text{ V/m}$ Power density = $\frac{|E|^2}{2\eta_0} \text{ W/m}^2$
 $= \frac{100}{2 \times 377} = 0.1326 \text{ W/m}^2$

(b) A plane wave may be resolved into two contra-rotating circular polarized waves each carry half the power density.

Hence - Power = $\frac{1}{2} \times 0.1326 \times 1 = 66.31 \mu\text{W}$
 power received by antenna

c) At the Brewster angle

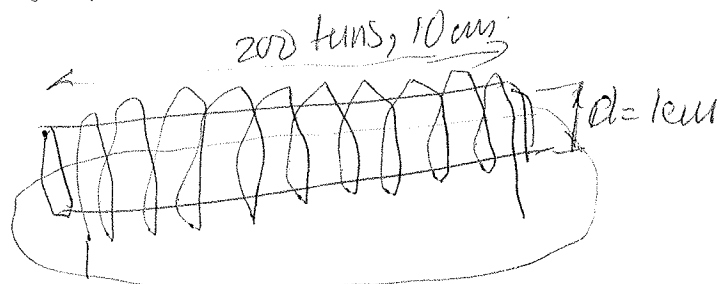
$$\sin \theta_{B11} = \frac{1}{\sqrt{1 + 1/4}}$$



$$\theta_{B11} = \sin^{-1} \frac{1}{\sqrt{1.25}} = 0.89443 \text{ rad} = 0.89444$$

$$= 63.4^\circ$$

B



$$\int H \cdot dl = NI$$

Or $H \cdot 0.1 = 200 I$, $H = 2000 I$

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 100 \times 2000 I = 0.25 I$$

$$\Phi = A \cdot B = \pi \times 10^{-4} \times 0.25 I$$

$$\Phi_{100} = 1.973 \times 10^{-5} I$$

$$\Lambda_{20} = N\Phi_{200} = 1.973 \times 10^{-5} \times 200 I = 3.9478 \times 10^{-3} I$$

$$\text{Hence } L_1 = 3.95 \times 10^{-3} \text{ H}$$

50 turn coil . . . $\int H dl = NI$

$$H_{50} = \frac{50 I}{2.5 \times 10^{-2}} = 200 I$$

$$B_{50} = \mu_r \mu_0 H_{50} = 0.02513 I$$

$$\Phi_{50} = B_{50} \times A = 0.02513 I \times \pi \times 0.25 \times 10^{-4}$$

$$= 1.973 \times 10^{-6} I$$

$$\Lambda_{50} = 1.973 \times 10^{-6} \times 50 I = 0.987 \times 10^{-6} I$$

$$L_{50} = \frac{\Lambda_{50}}{I} = 0.987 \times 10^{-6} \text{ H}$$

Mutual Inductance,

$$\Phi_{200} = 1.973 \times 10^{-5} I_1 \text{ T}$$

$$\Lambda_{12} = \Phi_{200} \times 50 = 9.87 \times 10^{-4} I_1$$

$$L_{12} = \frac{\Lambda_{12}}{I_1} = 9.87 \times 10^{-4} \text{ H}$$