Problem 1:

Right hand circularly polarized light: $\begin{bmatrix} 1 \\ -i \end{bmatrix}$

Half wave plate: $\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$

Rotated half-wave plate: $\underbrace{\begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}}_{R(+\psi)} \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} \underbrace{\begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}}_{R(-\psi)}$

Conversion of R-H circularly polarized light through rotated half-wave plate:

$$\begin{bmatrix} V_x' \\ V_y' \end{bmatrix} = R(\psi) \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} R(-\psi) \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

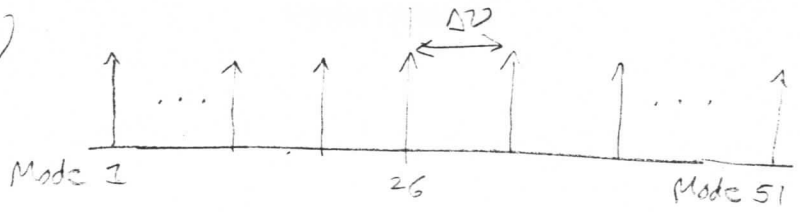
$$= \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} -i\, e^{-i\psi} \\ e^{-i\psi} \end{bmatrix}$$

$$= e^{-i\psi} \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$= e^{-i\psi} \begin{bmatrix} -i e^{-i\psi} \\ e^{-i\psi} \end{bmatrix} = i e^{-i2\psi} \begin{bmatrix} 1 \\ +i \end{bmatrix}$$

= left-hand polarized light, independent of ψ.

Part 2. a)



$$\Delta\nu = \frac{c}{2nl} = \frac{3\times10^8\ m}{2m\ \ s} = 150\ MHz$$

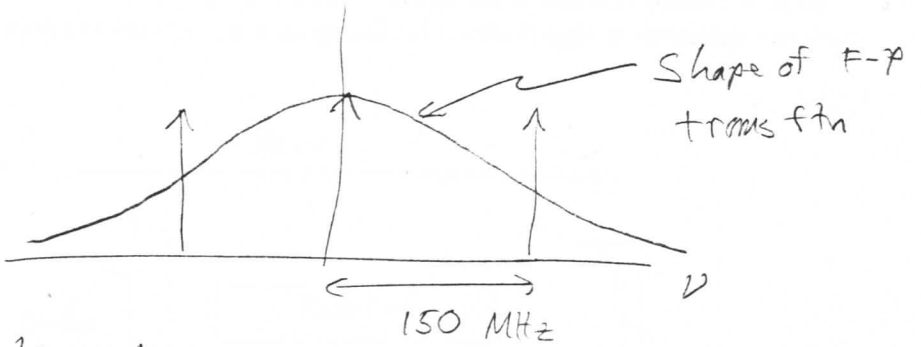
FSR of FP must be at least (150)25 MHz

$= 3.75$ GHz

and $\frac{c}{2nl} > 3.75\ GHz \Rightarrow l < 2.7\ cm$

$(n = 1.5)$

b) If etalon is centered on the desired (ongitudinal mode, then we desire a peak transmittance of the shape



such that the FWHM of the etalon is < 300 MHz

Since the Finesse is defined as $\mathcal{F} = \frac{FSR}{FWHM}$, we have

$$\mathcal{F} > \frac{3.75\times10^9}{3\times10^8} = 12.5$$

The reflectivity is then given by $F = \frac{\pi\sqrt{R}}{1-R}$

or $R = (2F^2+\pi^2) \pm \sqrt{(2F^2+\pi^2)^2 - 4F^2} = 0.78$