Problem 9 (a) A buck converter is to be designed to deliver power from a DC input with voltage 12 V to an output of 5 V. The switching frequency is chosen to be $f = 25$ kHz. The specifications call for a 20 mV peak-to-peak output-voltage ripple, and a 0.8 A peak-to-peak inductor-current ripple. Assume all switching and filter components are ideal.

(i) What is the duty cycle that the converter should operate at?

(ii) What value of filter inductance would meet the specifications?

(iii) What value of filter capacitance would meet the specifications?

(iv) Assuming a load resistance, $R = 500\,\Omega$, what is the critical filter inductance for the converter? (Recall, the converter operates at the boundary of continuous- and discontinuous-current conduction modes when the inductance is chosen to be the *critical filter inductance*.)

Solution 9 (a)

(i) The input voltage, $V_{\text{in}} = 12$ V; and output voltage, $V_{\text{out}} = 5$ V. The duty cycle, D, is given by

$$D = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{5}{12} \approx 41.67\%. \tag{1}$$

(ii) With the active switch turned on, we can write

$$V_{\text{in}} - V_{\text{out}} = L\frac{di}{dt}, \tag{2}$$

where i denotes the instantaneous inductor current. With a straight-line approximation for the inductor current, we get

$$V_{\text{in}} - V_{\text{out}} \approx L\frac{\Delta i}{DT}, \tag{3}$$

where $T = f^{-1}$ is the switching period and Δi is the current ripple. Substituting the specifications of the converter and the duty cycle from (1),

$$L = \frac{(V_{\text{in}} - V_{\text{out}})D}{\Delta i \cdot f} = \frac{(12-5)5}{12 \times 0.8 \times 25 \times 10^3} \approx 146\,\mu\text{H}. \tag{4}$$

(iii) The instantaneous capacitor current is given by

$$i = C\frac{dv}{dt} \tag{5}$$

Since the average capacitor current is zero, assuming the inductor ripple current is completely absorbed by the capacitor, we can write

$$\int_{i_{\text{ripple}}>0} i_{\text{ripple}} dt = \frac{1}{2} \times \frac{T}{2} \times \frac{\Delta i}{2} = C\Delta v \tag{6}$$

Substituting the values of the allowed current ripple Δi and the allowed voltage ripple Δv, we get

$$C = \frac{T \times \Delta i}{8 \times \Delta v} = \frac{0.8}{8 \times 25 \times 10^3 \times 20 \times 10^{-3}} = 200\,\mu\text{F}. \tag{7}$$

(iv) Denote the *critical inductance* of the dc-dc buck converter by L_{crit}. Recall that the critical inductance is the minimum inductance required to avoid discontinuous conduction mode (DCM) under all operating conditions. That is, if the chosen inductor for the dc-dc buck converter, $L > L_{\text{crit}}$, then DCM is avoided. On the other hand, if the dc-dc buck converter inductor $L < L_{\text{crit}}$, then the converter always operates in DCM. For $L = L_{\text{crit}}$, $\Delta i = 2I_{\text{out}}$, where $I_{\text{out}} = V_{\text{out}}/R$ is the average output current. With this operating mode in mind, we get

$$L_{\text{crit}} = \frac{(V_{\text{in}} - V_{\text{out}})D}{\Delta i \cdot f} = \frac{(V_{\text{in}} - V_{\text{out}})D}{2I_{\text{out}} \cdot f} = \frac{(12-5)5}{12 \times 2(5/500) \times 25 \times 10^3} \approx 5.83\,\text{mH}. \quad (8)$$