(1) (20 points) Suppose, you want to transmit over a channel characterized by an additive noise Z, such that the probability density function of Z is given by

$$f_Z(z) = \begin{cases} (2 - |z|)/4, & -2 \leq z \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

You may transmit either a $+1$ or a -1 over this channel, and these two options are equally likely. This means, if you transmit $X \in \{+1, -1\}$, then you receive $Y = X + Z$.

a. What is the optimal strategy to recover an estimate of X from Y? 3

b. What is the probability of error in the above estimate? 3

c. Suppose you send the following three: X_1, X_2 and X_1X_2, where $X_1, X_2 \in \{+1, -1\}$. What is the probability that the errors remain unnoticed (Hint: Any one error will be noticed)? 4

Consider the following protocol to recover X:

$$\hat{X} = \begin{cases} -1 \text{ if } Y \leq -0.5 \\ +1 \text{ if } Y \geq 0.5 \\ \text{Retransmit otherwise} \end{cases}$$

d. What is the average number of transmission that has to be performed then to transmit a vector of $+1$ and -1 of length 100? 5

e. What is the probability for any symbol (+1 or -1) to be wrongly estimated? 5

(2) (20 points) Suppose the signal $f(t)$ is going to be transmitted with double-sideband (DSB-SC) amplitude modulation (AM). That is, the signal $f(t)\cos(2.4\pi \times 10^6 t)$ is transmitted.

a. What can be the maximum bandwidth of $f(t)$ for distortionless reception? 3

b. If you are allowed to transmit only within the band of 1 MHz to 1.4MHz, what is the maximum bandwidth of $f(t)$ that you can support? 2

At the receiver end, you receive a phase-shifted version because of asynchronous communication:

$$X(t) = f(t)\cos(2.4\pi \times 10^6 t + \Theta),$$

where Θ is a random phase sampled from the uniform distribution in $[0, 2\pi]$.

c. Find expected value and autocorrelation function of $X(t)$. 3+4

d. Is $X(t)$ a stationary process? 1

Suppose, $f(t)$ is band-limited according to part **b.** above. Let us sample $f(t)$ at rate 25% above the Nyquist rate and use PCM (pulse-coded modulation) to transmit this signal.

e. What is the sampling rate? 1

f. Let $|f(t)| \leq 100$ and each sample drawn above is quantized into levels of size 0.25. Determine the number of binary pulses required to encode each sample? 3

g. Determine the bits per second transmission rate and the minimum bandwidth required to transmit the signal. 3