There are four Parts, assigned 1 point each, for a total of 4 points.

Part I (1 point):
A feedback system is shown in Figure 1 where $P(s)$ is a system model and $e^{-\tau s}$ represents transmission delay in the feedback path. The system transfer function can be fairly accurately

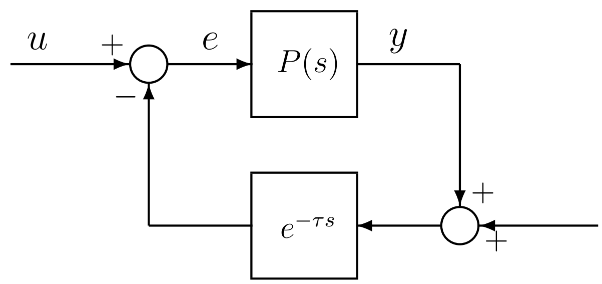

Figure 1: Feedback system.

modeled over the range of frequencies that are relevant to stability analysis by

$$P(s) = \frac{2}{\sqrt{s}}$$

for $s = j\omega$ with ω measured in radians/sec. It is also given that when the transmission delay τ is sufficiently small or zero, the feedback system is bounded-input/bounded-output stable.
Determine the maximal amount of time delay that the feedback system can tolerate before it becomes unstable.

Space for your work:

Part II (1 point):
Determine the maximal interval $[0, K_{\mathrm{maximal}})$ for the gain K for which the delay-differential equation

$$\dot{y}(t) = u(t) - Ky(t) - 2Ky(t-1) - Ky(t-2)$$

is stable. Here, $u(t)$ is thought of as the input to the system and $y(t)$ as the output.

Space for your work:

Part III (1 point):
Consider the feedback system in Figure 2. Do the following:

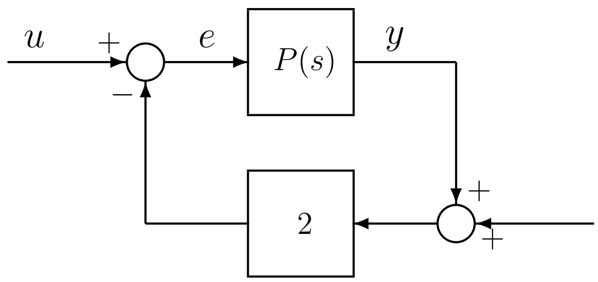

Figure 2: Feedback system.

i) Determine whether, for $P(s) = \frac{s}{1-s}$, the feedback loop is stable or not.
ii) Determine whether, for $P(s) = \frac{s}{(1-s)(1+\epsilon s)}$ and sufficiently small ϵ, the feedback loop is stable or not.
iii) Determine whether, for $P(s) = \frac{se^{-\tau s}}{1-s}$ and sufficiently small τ, the feedback loop is stable or not.
iv) What conlcusions do you reach regarding the theoretical answer to part i)?

Space for your work:

Part IV (1 point):
Consider the Nyquist plot computed for a transfer function $G(s)$ and shown in Figure 3. Determine whether $G(s)$ is stable or not, and explain your reasoning.

[Warning: the above question is not about the stability of the feedback system with $G(s)$ in the forward path and, perhaps, negative unity feedback. The question is about whether $G(s)$ itself is stable or not.]

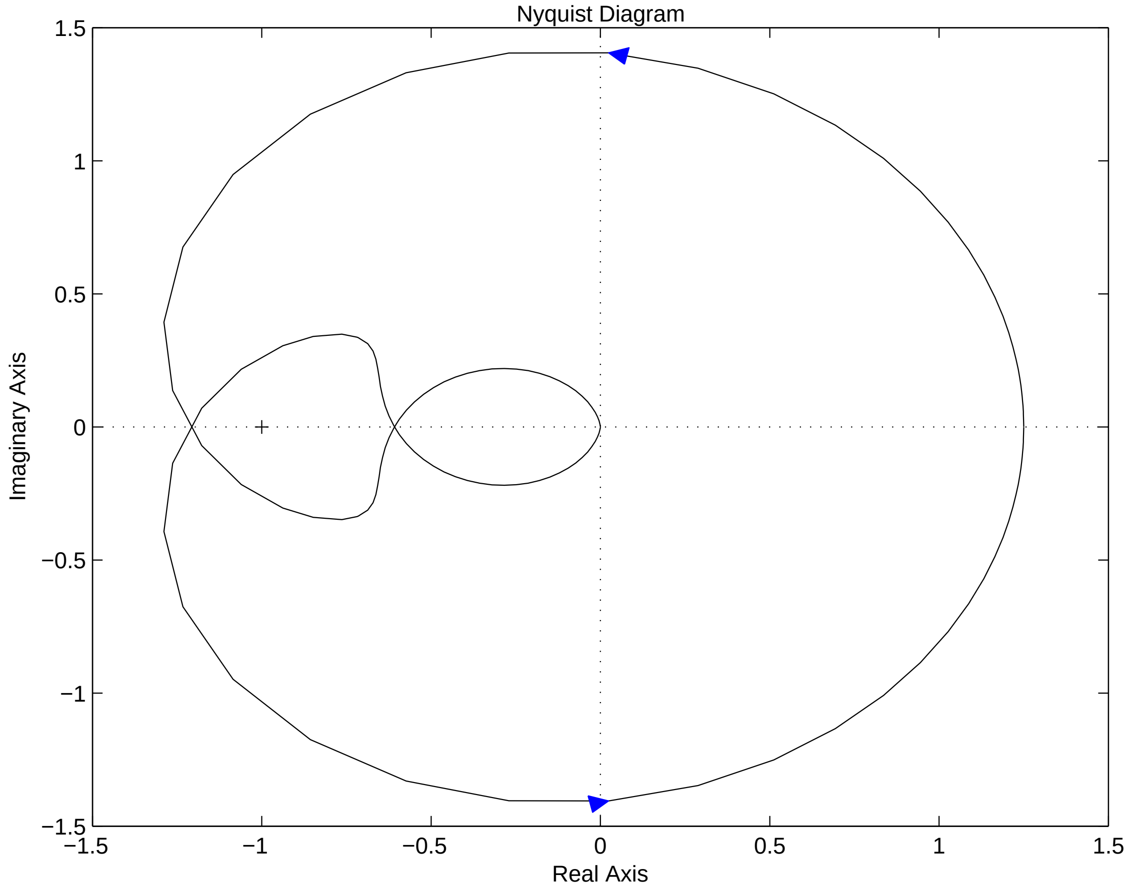

Figure 3: Nyquist plots

Space for your work: