Solution

a) Majority carrier concentration: $n_{n0} = 2\times10^{17}\ \text{cm}^{-3}$
So minority carrier concentration:

$$p_{n0} = n_i^2 / n_{n0} = \frac{(10^{10})^2}{2\times10^{17}} = 5\times10^2\ \text{cm}^{-3}$$

At x=0, differential equation for the minority carrier concentration is:

$$\frac{d\,p_n}{dt} = G - \frac{p_n - p_{n0}}{\tau_p}$$

At steady-state: $d\,p_n / dt = 0$, so

$$p_n(0) \simeq p_n(x=0) - p_{n0} = G\tau_p = 10^{20}\times50\times10^{-6} = 5\times10^{15}\ \ \text{cm}^{-3}$$

b) From current continuity, the differential equation for $p_n(x)$ is:

$$\frac{\partial p_n}{\partial t} = -\frac{p_n - p_{n0}}{\tau_p} + D_p\frac{\partial^2 p_n}{\partial x^2}$$

At steady-state: $\partial p_n / \partial t = 0$, thus

$$\frac{\partial^2 p_n}{\partial x^2} = \frac{p_n - p_{n0}}{D_p\tau_p} = \frac{p_n - p_{n0}}{L_p^2}$$

where $L_p = \sqrt{D_p\tau_p}$ is the carrier diffusion length.

From Einstein relation: $D_p = (kT/q)\mu_p = 1.38\times10^{-23}\times300/1.6\times10^{-19}\times250 = 6.5\ [\text{cm}^2\text{s}^{-1}]$ is the diffusion constant for holes.

Therefore, $L_p = \sqrt{D_p\tau_p} = \sqrt{6.5\times50\times10^{-6}} = 1.8\times10^{-2}$ cm=180 μm.

c) Boundary conditions are: at x=0, $p_n = p_n(0)$; at x=W, $p_n(x=W) = p_{n0}$ because all of the excess carriers are extracted.
The general solution of the differential equation is: $p_n(x) - p_{n0} = Ae^{-x/L_p} + Be^{x/L_p}$
At x=0, $p_n(0) - p_{n0} = A + B$
At x=W, $0 = Ae^{-W/L_p} + Be^{W/L_p}$
So:

$$A = -\left[p_n(0) - p_{n0}\right]\frac{e^{W/L_p}}{e^{-W/L_p} - e^{W/L_p}} = \left[p_n(0) - p_{n0}\right]\frac{e^{W/L_p}}{2\sinh(W/L_p)}$$

$$B = \left[p_n(0) - p_{n0}\right]\frac{e^{-W/L_p}}{e^{-W/L_p} - e^{W/L_p}} = -\left[p_n(0) - p_{n0}\right]\frac{e^{-W/L_p}}{2\sinh(W/L_p)}$$

Finally,

$$p_n(x) - p_{n0} = \left[p_n(0) - p_{n0}\right]\frac{e^{(W-x)/L_p}}{2\sinh(W/L_p)} - \left[p_n(0) - p_{n0}\right]\frac{e^{-(W-x)/L_p}}{2\sinh(W/L_p)}$$

$$= \left[p_n(0) - p_{n0}\right]\frac{\sinh\left[(W-x)/L_p\right]}{\sinh(W/L_p)}$$

d) Current density by diffusion at x=W is:

$$J_p = -qD_p \left.\frac{dp_n}{dx}\right|_{x=W} = \frac{qD_p\left[p_n(0) - p_{n0}\right]}{\sinh(W/L_p)}\frac{d}{dx}\sinh\left[(W-x)/L_p\right]\Big|_{x=W}$$

$$= \frac{qD_p\left[p_n(0) - p_{n0}\right]}{\sinh(W/L_p)}\frac{1}{L_p}\cosh\left[(W-x)/L_p\right]\Big|_{x=W}$$

$$= \frac{qD_p\left[p_n(0) - p_{n0}\right]}{L_p\sinh(W/L_p)}$$

$$= \frac{1.6\times10^{-19}\times6.5\times5\times10^{15}}{1.8\times10^{-2}\times\sinh(500/180)} = 0.036\ \text{A/cm}^2$$

So total current is:

$$I(W) = J_pLH = 0.36\times0.05\times0.02 = 3.6\times10^{-5}\ \text{A=72}\ \mu\text{A}$$