

**Solution to Question #1:**

i) The transfer functions are:

$$T_A(s) = \frac{20}{(s+1)(s-10)}, \text{ and } T_B(s) = \frac{50(s-1)}{(s-10)^2}.$$

The precise values are not important. What is important is that: system A has a DC gain of about  $-2$ , whereas system B has a DC gain of about  $-1/2$ , then also, the approximate location and nature of poles/zeros.

ii) Nyquist drawn in Figures 3, 4.

iii) Negative unity-gain feedback stabilizes the second but not the first. The reason being that the Nyquist-diagram of the first has an encirclement count of  $N = 1$  whereas for the second,  $N = -2$ . Clearly, the second system must have two unstable open-loop poles and the closed-loop system with system B in the forward path is stable.

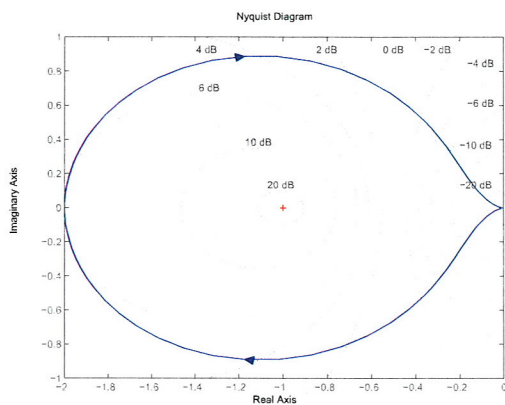


Fig. 3. Nyquist plot for system 1.

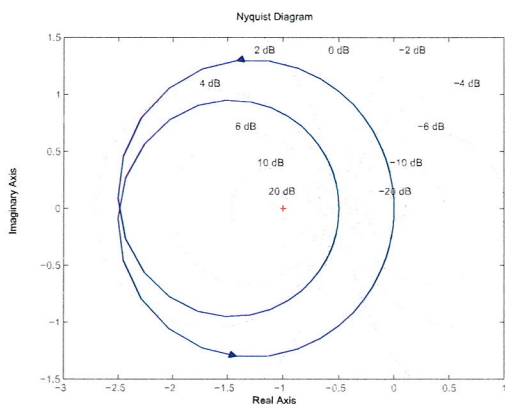


Fig. 4. Nyquist plot for system 2.

**Solution to Question #2:**

i) Using the final-value theorem, the steady-state value of the output is

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \frac{1}{s} \frac{k/(s+1)}{1 + k/(s+1)} = \frac{k}{k+1}.$$

Since

$$\begin{aligned} \frac{k}{k+1} &= 2 - \sqrt{2} \\ &= \sqrt{2}(\sqrt{2} - 1) \\ &= \frac{\sqrt{2}}{\sqrt{2} + 1} \end{aligned}$$

it is clear that  $k = \sqrt{2}$ .

ii) Consider the Nyquist plot for  $\sqrt{2}/(s+1)$ . This is a circle with center at  $s = \sqrt{2}/2$  and diameter  $\sqrt{2}$ . Thus, the point on the Nyquist diagram which is at a distance 1 from the origin is the point with coordinates  $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ , whose angular distance from  $(-1, 0)$  is 135 degrees, and therefore the phase margin is  $135^\circ$ .

**Solution to Question #3:**

Simply choose a state-feedback gain  $K = [0 \quad -1]$  so that  $A - BK = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ .