Ph.D. Qualifying Exam on Communications, November 2010

(a) Closed Book and Closed Notes Exam (Calculators are allowed)

(b) Perfect Score: 40

(c) Nominal Duration: 60 minutes

Student Number:_____

Problem 1 (5 points) A discrete source generates independent, identically distributed symbols $X_i, i = 1, 2, 3, \ldots$, according to the following distribution: $P(X_i = 1) = 1/2$, $P(X_i = 2) = 1/4$, $P(X_i = 3) = 1/8$, and $P(X_i = 4) = 1/8$. What is the minimum average number of bits per symbol needed to encode the source symbols without loss of information (meaning that one can perfectly recover the source symbols from the encoded ones)?

Problem 2 (5 points) It is desired to transmit an independent, identically distributed sequence of zeros and ones occurring equi-probably using baseband binary (antipodal) pulse amplitude modulation (PAM) that relies on the pulse shown below.
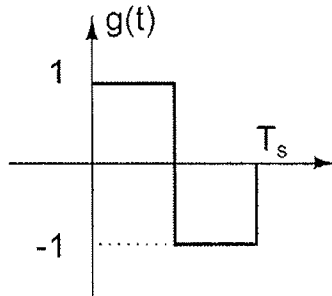

Find the power spectral density of the transmitted waveform in terms of the pulse parameters.

Problem 3 (15 points) A 5-ary communication system uses the following pulses to represent the information-bearing symbols.
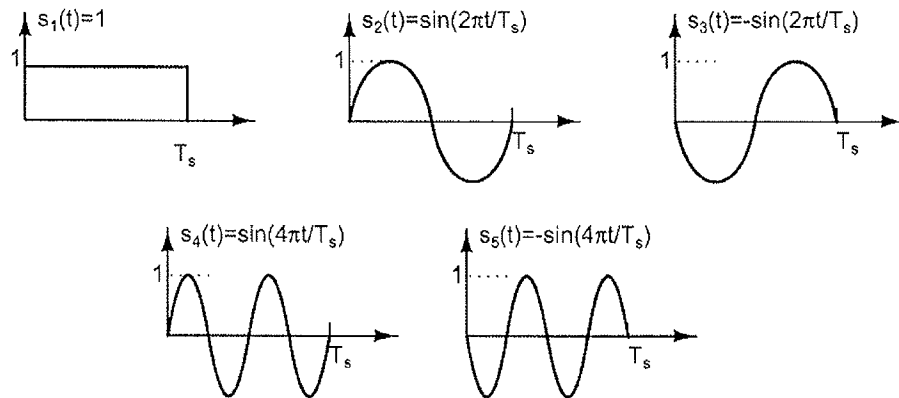
(a) (3 points) Find a set of basis functions and use them to express linearly the given pulses;

(b) (2 points) What is the dimensionality of the space spanned by $\{s_m(t), m = 1, \ldots, 5\}$?

(c) (3 points) What is the energy of $s_3(t)$?

(d) (2 points) What is the distance between $s_2(t)$ and $s_4(t)$?

(e) (5 points) With fixed energy per bit, show that for *any* M-ary orthogonal modulation it holds that: (e1) the distance between any two transmitted waveforms per symbol duration increases to infinity as $M \to \infty$; and (e2) the required bandwidth also increases to infinity as $M \to \infty$.

Problem 4 (15 points) Consider the digital communication system depicted below, and suppose that: i) the messages m_0 and m_1 are equi-probable; ii) the transmitted signals are antipodal, that is $s_1 = -s_0 = \sqrt{E_b}$; iii) the noise terms n_1 and n_2 are zero-mean, Gaussian, with variance σ^2; and iv) the transmitted signals as well as the noise terms are all statistically independent.
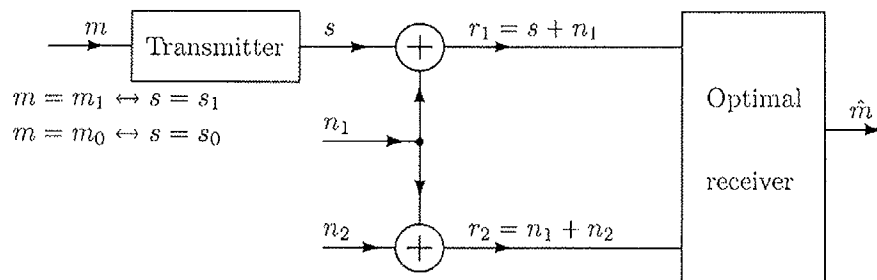
(a) (5 points) Find the conditional probability density function $f(r_1, r_2|s)$.

(b) (5 points) Develop the optimal (in the sense of minimum probability of error) decision rule, and simplify it as much as possible.

(c) (5 points) Express the probability of error in terms of the Gaussian tail function $Q(\cdot)$.

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