(a) Closed Book and Closed Notes Exam (Calculators are allowed)
(b) Perfect Score: 40
(c) Nominal Duration: 60 minutes

Student Number:________________________

Problem 1 (6 points) Consider the following two passband signals:

$$u_p(t) = \sqrt{2}\,\mathrm{sinc}(2t)\cos 100\pi t$$

and

$$v_p(t) = \sqrt{2}\,\mathrm{sinc}(t)\sin\left(101\pi t + \frac{\pi}{4}\right).$$

(a) (2 points) Find the complex envelopes $u(t)$ and $v(t)$ for $u_p(t)$ and $v_p(t)$, respectively, with respect to the carrier frequency $f_c = 50$. [Recall the definition $\mathrm{sinc}(x) := (\sin(\pi x))/(\pi x)$.]

(b) (2 points) Find the inner product $< u_p(t), v_p(t) >$.

(c) (2 points) Find the convolution $y_p(t) = (u_p \star v_p)(t)$.

Problem 2 (9 points) Consider the pulse $s(t) = \mathrm{sinc}(at)\,\mathrm{sinc}(bt)$, where $a \geq b > 0$).

(a) (2 points) Sketch the frequency-domain response $S(f)$ of the pulse.

(b) (4 points) Suppose that the pulse is to be used over a passband channel spanning the frequencies 2.4–2.42 GHz. Assuming that we use 64-QAM signaling at 60 Mbit/s, choose a and b so that the pulse is Nyquist and exactly fills the channel bandwidth.

(c) (3 points) If $s(t)$ is used as a transmit-filter to form a linearly modulated waveform $x(t)$, argue that $|x(t)|$ is always finite.

Problem 3 (10 points) Consider a BPSK system in which the receiver's estimate of the carrier phase is off by θ. Supposing $0 < \theta < \pi/2$, find the bit error rate as a function of θ and E_b/N_0.

Problem 4 (15 points) The receiver in a binary communication system employs a decision statistic x which behaves as follows: $x = n$ if 0 is sent, and $x = 4 + n$ if 1 is sent, where n is modeled as Laplacian with density $p(n) = \frac{1}{2}\exp(-|n|)$.

(a) (5 points) Find the log likelihood ratio $\ell(x) := \log[p(x|1)/p(x|0)]$, where $p(x|i)$ denotes the conditional density of x given that i is sent ($i = 0, 1$).

(b) (5 points) Find $P_{e|1}$, the conditional error probability given that 1 is sent, for the decision rule

$$\delta(x) = \begin{cases} 0, & x < 1 \\ 1, & x \geq 1. \end{cases}$$

(c) (5 points) Is $\delta(x)$ in (b) the minimum probability of error (MPE) rule for any choice of prior probabilities? If so, specify the prior probability $\pi_0 = P[0 \text{ sent}]$ for which it is the MPE rule. If not, say why not.