Problem 1. (a) For a frequency reference $f_c = 50$, the complex envelope of $u_p(t)$ is given by $u(t) = \text{sinc}(2t)$. For $v_p(t)$, we write

$$\begin{aligned} v_p(t) &= \sqrt{2}\,\text{sinc}(t)\sin\left(101\pi t + \frac{\pi}{4}\right) \\ &= \sqrt{2}\,\text{Re}\left(\text{sinc}(t)\left(-je^{j(101\pi t+\pi/4)}\right)\right) \\ &= \sqrt{2}\,\text{Re}\left(\text{sinc}(t)e^{j(\pi t-\pi/4)}e^{j100\pi t}\right) \end{aligned}$$

(where we have used $-j = e^{-j\pi/2}$), from which we can read off $v(t) = \text{sinc}(t)e^{j(\pi t-\pi/4)}$.

(b) The inner product

$$< u_p, v_p >= \text{Re}(< u, v >) = \text{Re}\left(\int U(f)V^*(f)\text{d}f\right).$$

But $U(f) = \frac{1}{2}I_{[-1,1]}(f)$ and $V(f) = e^{-j\pi/4}I_{[0,1]}(f)$, so that

$$\int U(f)V^*(f)\text{d}f = \frac{1}{2}e^{j\pi/4}.$$

We therefore obtain that $< u_p, v_p >= \frac{1}{2\sqrt{2}}$.

(c) We have $Y(f) = \frac{1}{\sqrt{2}}U(f)V(f) = \frac{1}{2\sqrt{2}}V(f)$ (since $U(f)$ takes the constant value $\frac{1}{2}$ over the support of $V(f)$). This implies that $y_p(t) = \frac{1}{2\sqrt{2}}v_p(t) = \frac{1}{2}\text{sinc}(t)\sin(101\pi t + \pi/4)$.

Problem 2. (a) Letting $s_1(t) = \text{sinc}(at)$, $s_2(t) = \text{sinc}(bt)$, we have $s(t) = s_1(t)s_2(t) \leftrightarrow S(f) = (S_1 \star S_2)(f)$. The spectra S_1, S_2, and S are sketched in Figure 1.

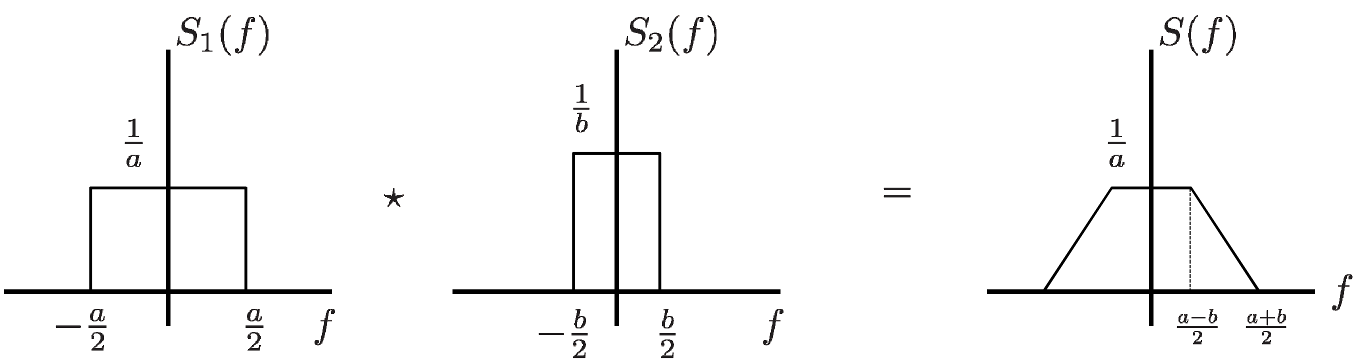

Figure 1: The spectrum of $S(f)$.

(b) The passband system has (two-sided) bandwidth of 20 MHz, and symbol rate $1/T = \frac{60 \text{ Mbps}}{\log_2 64 \text{ bits/symbol}} = 10$ Msymbols/sec. For full occupancy, we have $\frac{a+b}{2} = 10$ MHz, and $\frac{a+b}{2} + \frac{a-b}{2} = 10$ Mhz, which yields $a = b = 10MHz$.

(c) Since $\text{sinc}(at)$, $\text{sinc}(bt)$ both decay as $1/t$, the pulse $s(t)$ exhibits $1/t^2$ decay. Since $\int \frac{1}{t^2}\mathrm{d}t$ is finite, the superposition of waveforms of the form $s(t-nT)$ adds up to a finite value at any point of time.

Problem 3. From Fig. 2, it can be seen that error occurs if the I-channel noise $N \sim N(0,\sigma^2)$ causes a boundary crossing:

$$P_e = \Pr[N > d/2\cos\theta] = Q\left(\frac{d\cos\theta}{2\sigma}\right)$$

Since $E_b = d^2$, we obtain $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\cos\theta\right)$.

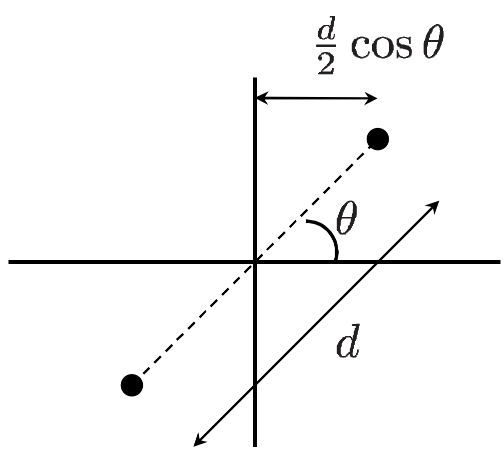

Figure 2: BPSK with phase mismatch.

Problem 4. (a) We have $p(x|0) = \frac{1}{2}e^{-|x|}$ and $p(x|1) = \frac{1}{2}e^{-|x-4|}$, so that the log likelihood ratio is given by $\ell(x) = \log[p(x|1)/p(x|0)] = |x| - |x-4|$.

(b) The conditional error probability given 1 is

$$P_{e|1} = P[x < 1|H_1] = \int_{-\infty}^{1} p(x|1)\mathrm{d}x = \int_{-\infty}^{1} \frac{1}{2}e^{-|x-4|}\mathrm{d}x$$
$$= \frac{1}{2}\int_{-\infty}^{1} e^{x-4}\mathrm{d}x = e^{-3}/2 = 0.025.$$

(c) The region $x < 1$ can be written as $\ell(x) < -2$. The MPE rule compares $\ell(x)$ to $\log\frac{\pi_0}{\pi_1}$. Thus, the rule in (b) is MPE rule if $\log\frac{\pi_0}{\pi_1} = -2$, which yields $\pi_0 = \frac{1}{e^2+1} \approx 0.12$.