

- Problem 1.

(a) Find α_0, α_1

3 conditions for PDS

$$\begin{cases} \int_{-4}^4 p(s) ds = 1 \\ -4\alpha_1 + \alpha_0 = 0 \\ \alpha_0 \geq 0 \end{cases}$$

$$\Rightarrow \begin{cases} 2 \times \frac{\alpha_0 + (-4\alpha_1 + \alpha_0)}{2} \times 4 = 1 \\ \alpha_0 = 4\alpha_1 \\ \alpha_0 \geq 0 \end{cases}$$

$$\Rightarrow \begin{cases} \alpha_0 - 2\alpha_1 = \frac{1}{8} \\ \alpha_0 = 4\alpha_1 \\ \alpha_0 \geq 0 \end{cases}$$

$$\Rightarrow \begin{cases} \alpha_0 = \frac{1}{4} \\ \alpha_1 = \frac{1}{16} \end{cases} \quad \#$$

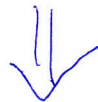
(b) 4 Quantization level $\Rightarrow \Delta = \frac{4 - (-4)}{4} = 2 \quad \#$

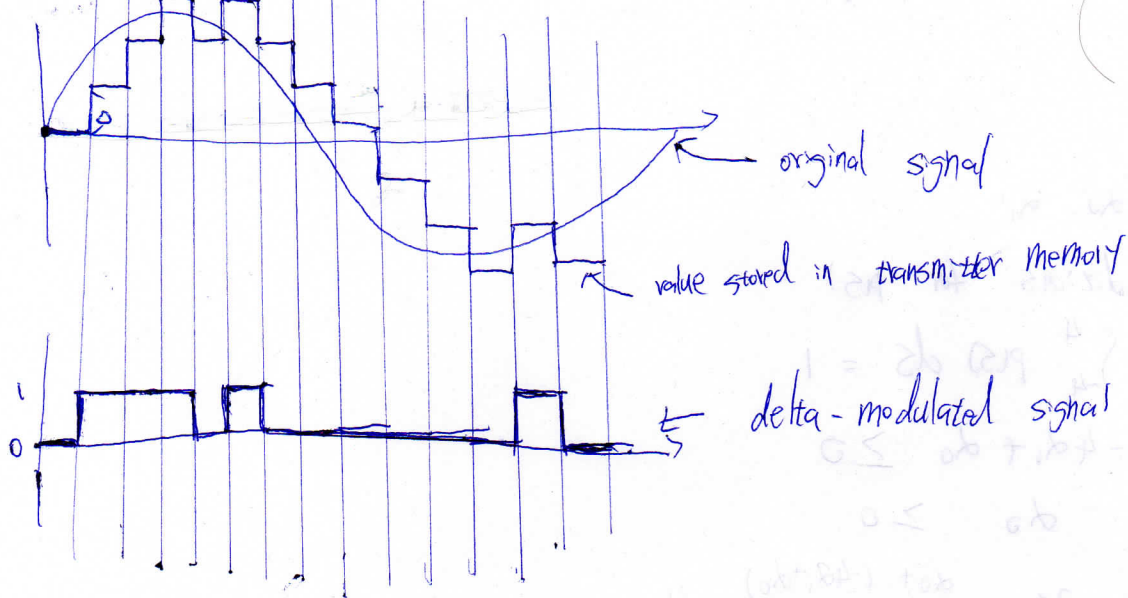
Quantized signal level = $\{-3, -1, 1, 3\} \quad \#$

(c) In delta modulation, the modulated signal is

- 1, if the sample value is larger than the transmitter memory
- 0, if the sample value is smaller than the transmitter memory.

In other words, delta modulation use 1 bit to keep track of the original signal.





d) To avoid slope overloading, we need the signal value change in adjacent samples to be less than Δ .

$$\Delta \geq |s(n+1)T - s(n)T|, \quad \forall n \in \mathbb{N}$$

$$= A |\sin(\omega(n+1)T) - \sin(\omega n T)|$$

$$= 2A \left| \sin\left(\omega n T + \frac{\omega T}{2}\right) \right| \times \left| \cos\left(\frac{\omega T}{2}\right) \right|$$

$$\Rightarrow A \leq \frac{\Delta}{2 \left| \cos\left(\frac{\omega T}{2}\right) \right|} \quad \#$$

(a). The highest bit rate without ISI is the Nyquist sampling rate,
which is $2 \times B = 2 \times 1200 = \underline{2400 \text{ (Hz)}} \#$

(b). Using BPSK, the error rate is

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

If we want $P_e = 10^{-7}$, we need $\frac{E_b}{N_0}$ at least equal to

$$\begin{aligned} \frac{E_b}{N_0} &\geq \left[\text{erfc}^{-1}\left(2 \times 10^{-7}\right) \right]^2 \\ &= (3.6765)^2 = \underline{13.5166} \# \end{aligned}$$

(c) Since the channel has 1 dB/km attenuation, the received signal power at each repeater is

$$E_b = \frac{\gamma^{50} \times P}{R}, \text{ where } \gamma = 10^{-0.1}$$

P is the transmitted power at each repeater
 $R = 2400 \text{ (bits/sec)}$,

To meet the required $\frac{E_b}{N_0}$ in (b), we need

$$\begin{aligned} \frac{\frac{\gamma^{50} \times P}{R}}{N_0} &\geq 13.5166 \\ \Rightarrow P &\geq \frac{R \times N_0 \times 13.5166}{\gamma^{50}} = \underline{1.33 \times 10^{-11}} \# \end{aligned}$$