
`There are four Parts; assigned 1 point each, for a total of 4 points.`

Part I (1 point):
Heat conduction across a slab of metal obeys a partial differential equation where the heat flow per unit time depends on the temperature gradient and the thermal conductivity. A system which consists of such a slab of metal, a heating element on one end, and a temperature sensor at some fixed point is considered. This is a distributed-parameter system with a transfer function which is not a rational function of s. The transfer function between the rate of heat $u(t)$ provided at one end as the input and the temperature $y(t)$ being recorded at the sensor location (all in appropriate units) is given and it is

$$G(s) = e^{-\sqrt{s}}.$$

Suppose that $u(t)$ varies periodically and is

$$u(t) = \sin(\frac{2\pi}{T}t)$$

(where t denotes time and T the period of oscillation) and suppose that the oscillation recorded at the sensor location trails that of the input by a quarter of the period, i.e., that

$$y(t) = A\sin(\frac{2\pi}{T}(t - \frac{T}{4})).$$

Determine the amplitude A of the temperature oscillations as well as the period T.

Part II (1 point):
Consider a dynamical system modeled as a lumped scalar linear system

$$\frac{d^3x(t)}{dt^3} = u(t).$$

Here $u(t)$ represents an input to the system, while $x(t)$ represents its position at time $t \in [0, \infty)$. Determine a second-order stabilizing controller which measures and processes $x(t)$ as its input, and determines the value for the input $u(t)$ to the given system. Explain why your design works.

Part III (1 point):

A physical system is modeled by a second order differential equation

$$\ddot{y}(t) = -\dot{y}(t) + u(t - \tau)$$

where $y(t)$ is the output, $u(t)$ is the input, and τ represents a time-delay measured in seconds. Thus, the transfer function from

$$u(t - \tau) \text{ to } y(t)$$

is

$$G(s) = \frac{1}{s(s+1)}.$$

frequency	magnitude	phase in degrees
1.1716	0.5541	-139.5176
1.4921	0.3731	-146.1702
1.8126	0.2665	-151.1152
2.1332	0.1990	-154.8834
2.4537	0.1538	-157.8266
2.7742	0.1222	-160.1776
3.0947	0.0994	-162.0928
3.4153	0.0823	-163.6798
3.7358	0.0692	-165.0143
4.0563	0.0590	-166.1511
4.3768	0.0509	-167.1302
4.6974	0.0443	-167.9820
5.0179	0.0389	-168.7294

You are given the magnitude and the phase of $G(j\omega)$ for a range of frequencies between .1716 and 5.0179 [rad/sec] in the table above (and to the right). The system is controlled using negative feedback; that is, the control input is proportional to the difference of an external reference signal $r(t)$ and the output $y(t)$, namely

$$u(t) = K\left(r(t) - y(t)\right),$$

with K a gain factor which is now taken to be $K = 10$. Determine the range of values for the time delay for which the closed loop system is stable.

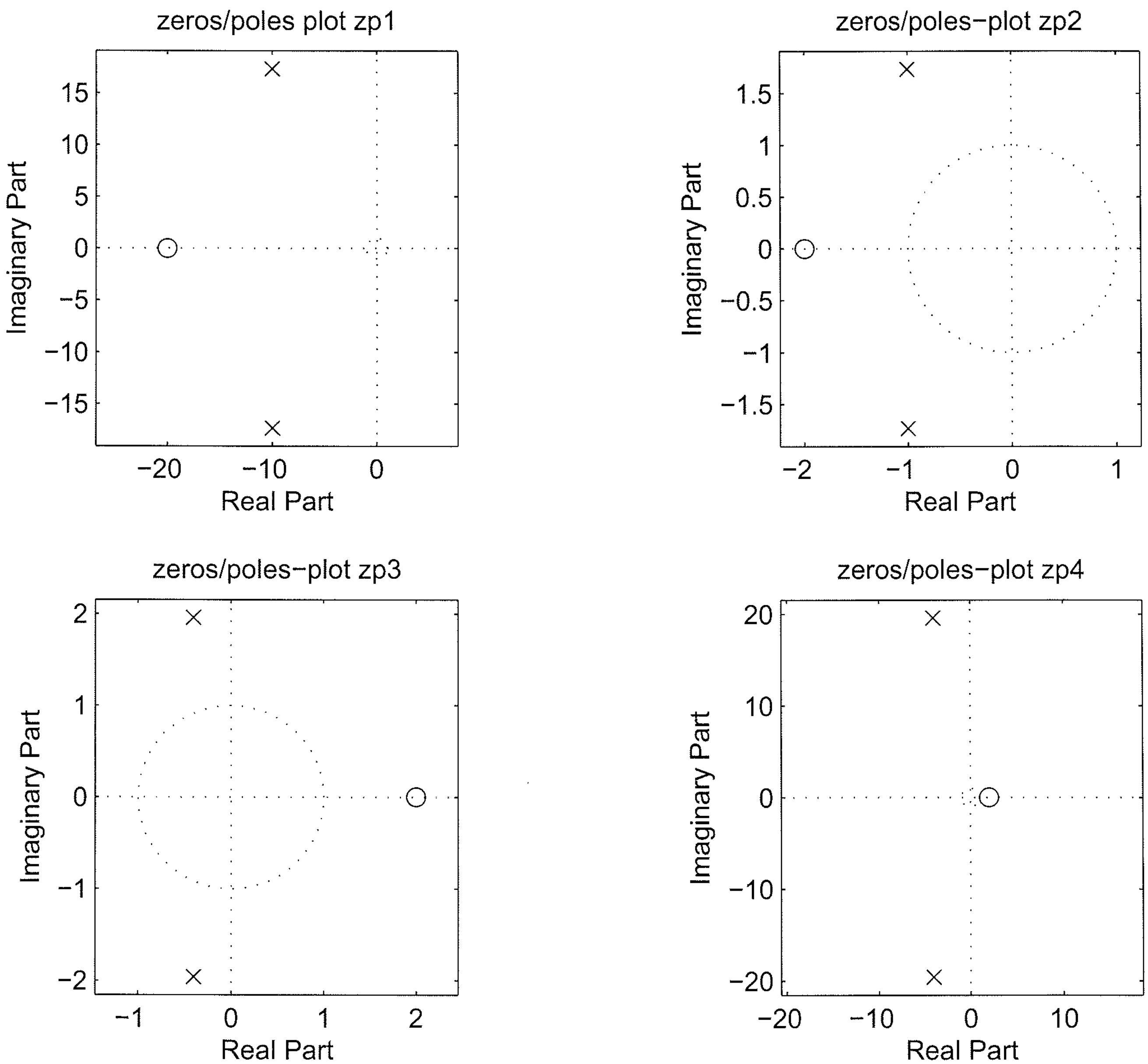
Part IV (1 point):
Consider four systems with the following transfer functions:

$$
\begin{aligned}
G_A(s) &= \frac{20s + 400}{s^2 + 20s + 400}, \\
G_B(s) &= \frac{200s - 400}{s^2 + 8s + 400}, \\
G_C(s) &= \frac{2s + 4}{s^2 + 2s + 4}, \\
G_D(s) &= \frac{-2s + 4}{s^2 + 0.8s + 4}.
\end{aligned}
$$

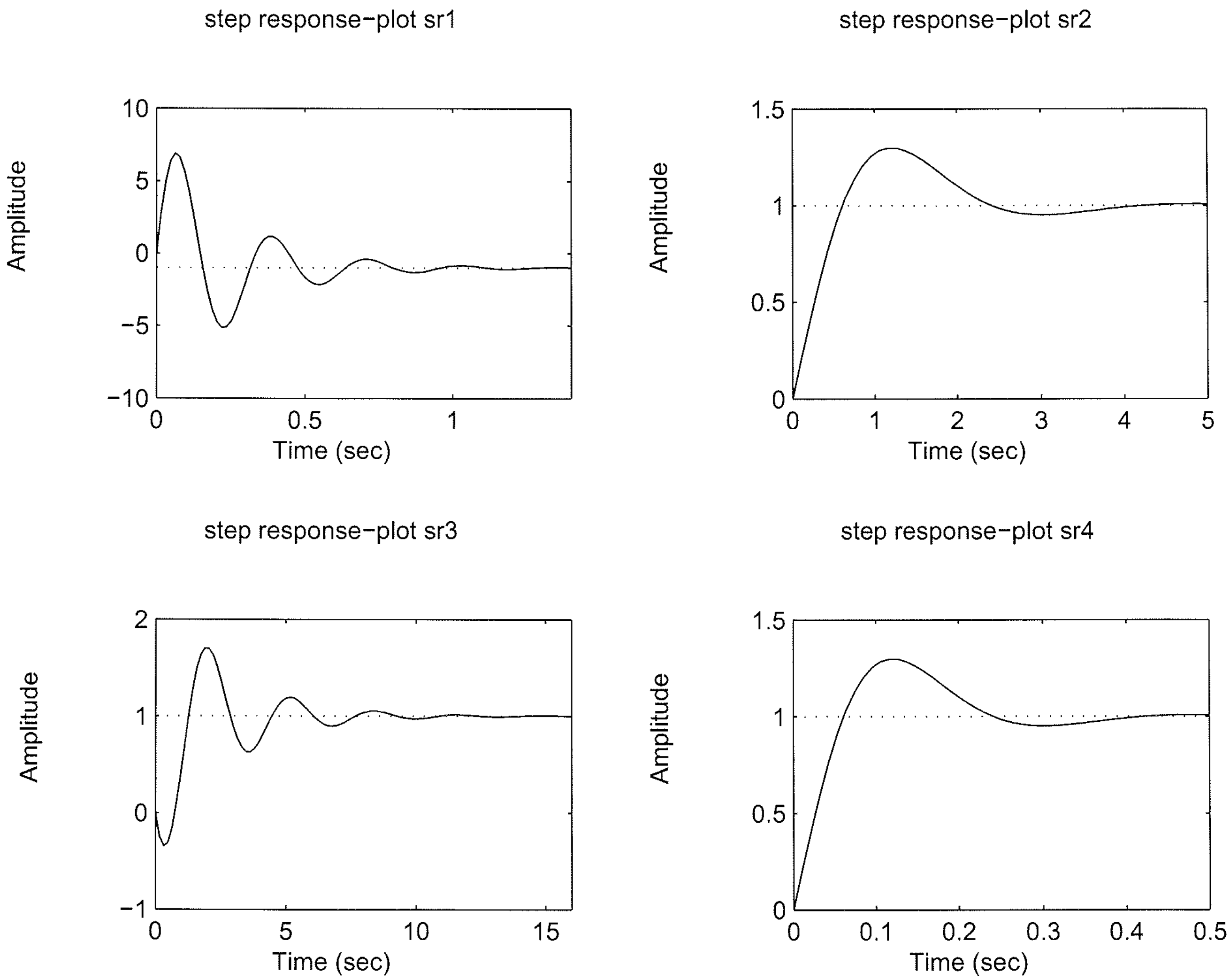
You are required to match these with corresponding pole/zero plots, step responses, Bode plots, and Nyquist plots shown in the next four pages. (E.g., $G_A(s)$ corresponds to the first pole-zero plot. Then we should mark zp1 in the corresponding position as shown, etc.)

Transfer fn	Pole-zero plot	Step response	Bode plot	Nyquist plot
A	zp1			
B				
C				
D				

Note that the Nyquist plots $nq2$ and $nq4$ are identical. In this case mark the possibilities in the above table, and explain why these two plots are identical.

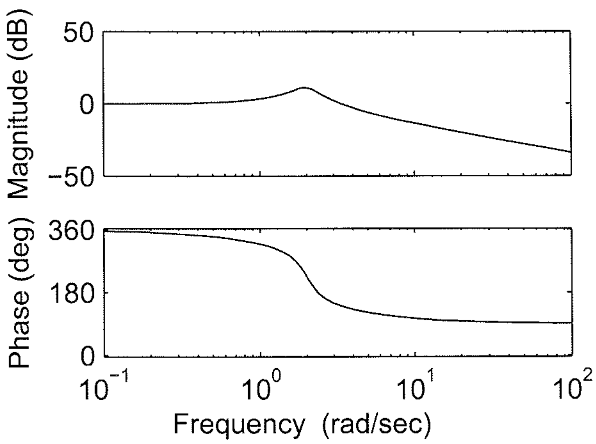


Pole-zero plots

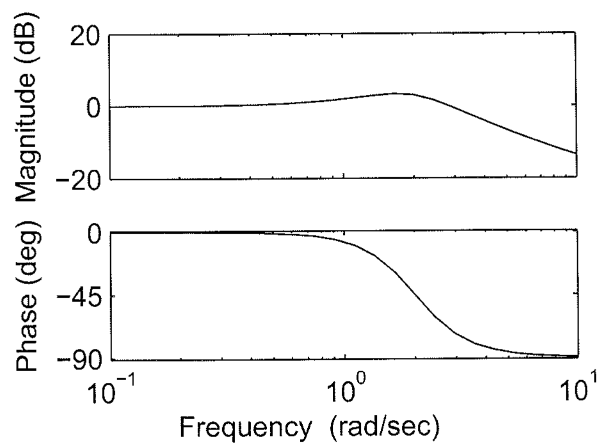


Step responses

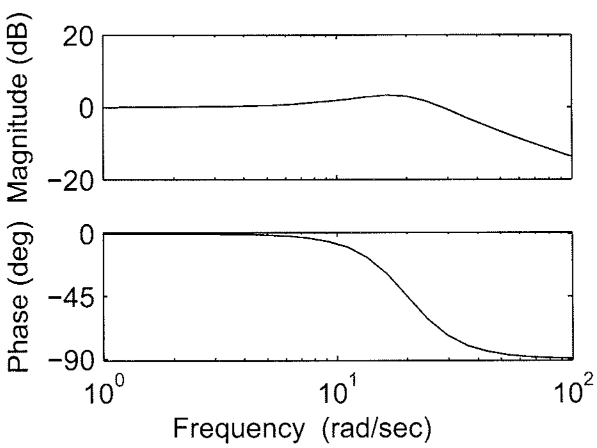
Bode plot bd1



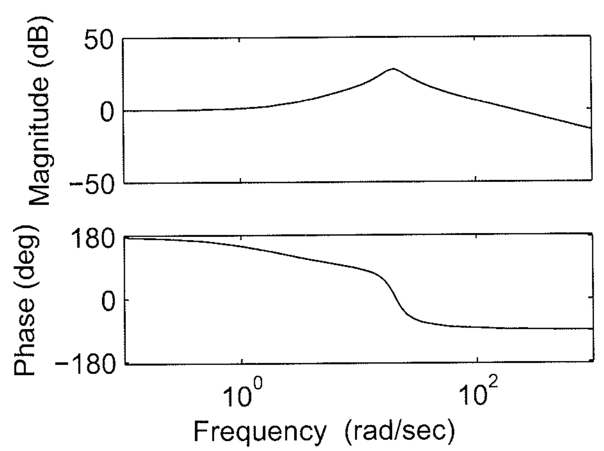
Bode plot bd2



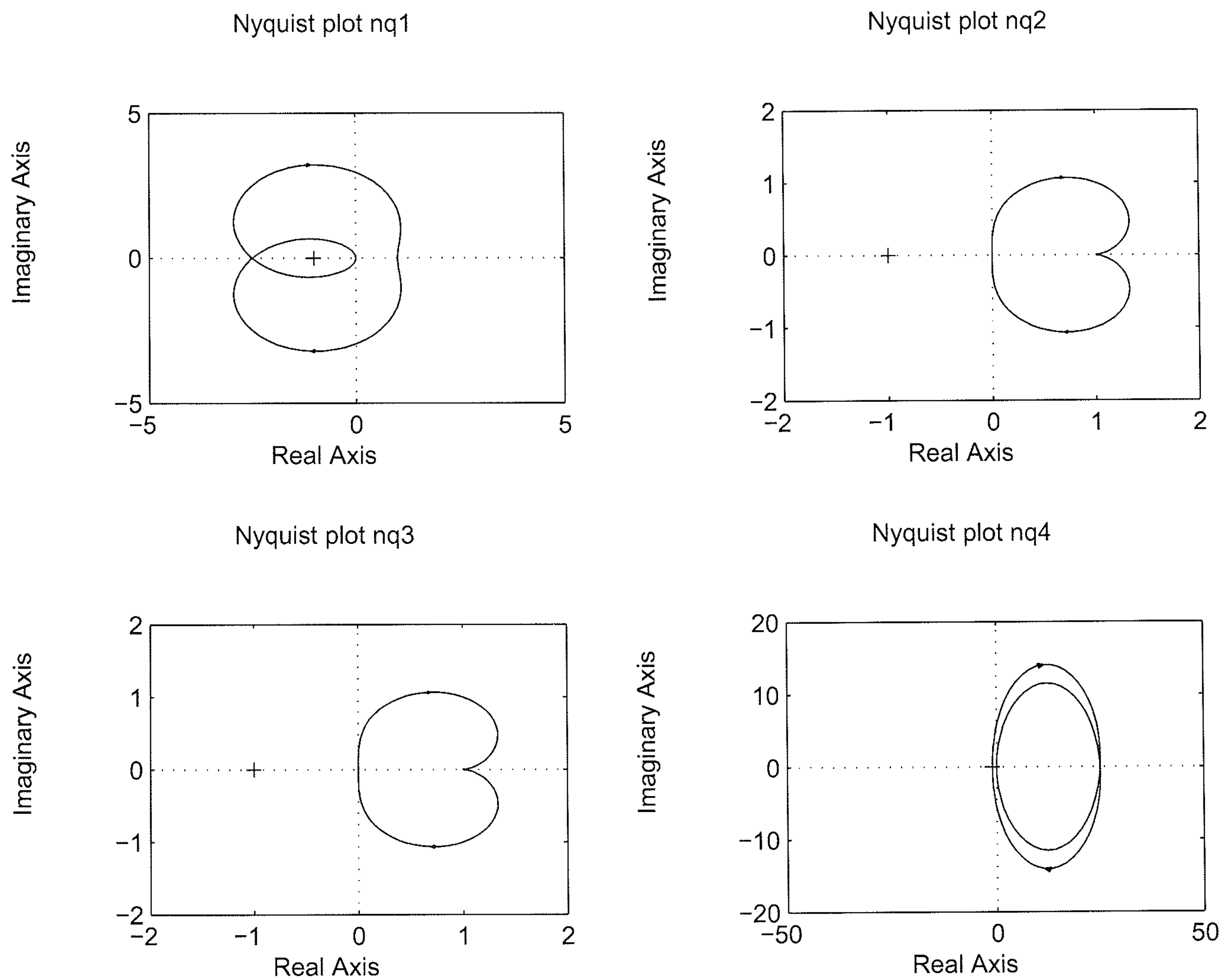
Bode plot bd3



Bode plot bd4



Bode plots



Nyquist plots