(1) (12 points) Suppose Z_i, $i = 1, \ldots, n$ are i.i.d. random variables with the following distribution:

$$Z_i = \begin{cases} -1 & \text{with prob. } p \\ 0 & \text{with prob. } 1-2p \\ 1 & \text{with prob. } p. \end{cases}$$

Let,

$$Z = \sum_{i=1}^{n} Z_i.$$

a. Find $\mathbb{E}Z$ and $\mathbb{E}Z^2$. 4

b. Find the characteristic function of Z. 5

Now consider a discrete time signal X_i is transmitted over an additive-noise channel. At the output of the channel we obtain $i = 1, \ldots, n$:

$$Y_i = X_i + Z_i.$$

If $[X_1, X_2, \ldots, X_n]$ is a Gaussian vector with covariance matrix $= 10I$, where I is an $n \times n$ identity matrix, then,

c. Find the Signal to Noise ratio for this channel when $p = 0.2$. 3

(2) (8 points) Suppose, you need to sample and transmit the following signal using an 8-bit PCM (pulse code modulation):

$$X(t) = 32\cos(8\pi t).$$

a. What is the bits/sec. transfer rate (assume Nyquist rate sampling)? 3

b. What is the mean square quantization error in the PCM? You can assume that the quantization noise is uniformly distributed. 5

(3) (20 points) Consider the following Octal Phase-Shift-Keying (PSK) scheme. Each of the 8 signal waveforms are represented as (below $f_c = 1$ Hz):

$$s_n(t) = \left|t - \frac{1}{2}\right| \cos\left(2\pi f_c t + \frac{\pi}{4}(n-1)\right), \quad n = 1, 2, \ldots, 8, \quad 0 \le t \le 1.$$

a. Compute the energy of the signal waveform $s_n(t)$. 5

b. Write an basis for this set of signals. There should be only two signals in this basis. 5

c. Represent $s_1(t)$ and $s_2(t)$ as a linear combination of above two basis signals. 5

d. How many bits of information can be sent in the interval $0 \le t \le 1$? Suppose, signals with adjacent phases can be confuseed at the receiver. How many bits can still be sent so that information retrieval with certainty is possible? 5