

- a) Consider the resonator shown in fig. 1 consisting of two flat mirrors separated by a distance d and one lens of focal length f in between the two mirrors and in contact with the right mirror. Assume f is chosen to make the system stable. Where is the waist of the Gaussian beam corresponding to the fundamental mode of this system? (0.5 points)

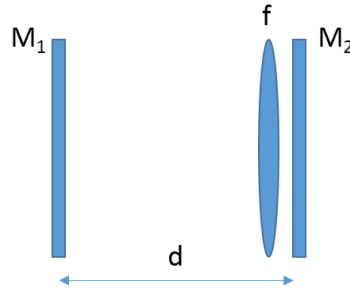


Fig. 1

- b) Assuming that a mode waist of size ω_0 is desired with a corresponding Rayleigh range of $z_0 = \pi\omega_0^2/\lambda$ (where λ is the wavelength of light), calculate the separation d necessary between the two mirrors to produce this mode size. You may leave the result in terms of the Rayleigh range if you wish. (1 point)
- c) What is the maximum size d_{\max} that ensures this system is optically stable? (0.5 points)
- d) Describe the effect on stability by moving the lens to the center of the cavity. (0.5 points)
- e) An external source is to be coupled into the resonator to excite only the fundamental Gaussian mode. The source is placed at the front focal plane of an external coupling lens, and the back focal plane of this lens is incident on the left mirror of the cavity (see fig. 2). **Using the ABCD matrix method**, calculate the focal length of the external lens f_2 that will couple light from a Gaussian source of size ω_0' into the cavity to excite only the fundamental mode of the cavity. (1.5 points)

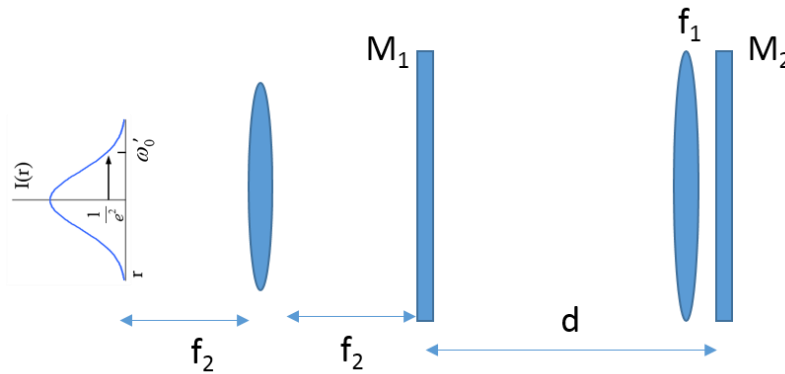
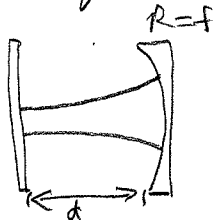


Fig. 2

Optics Problem # 7

- a) This resonator is equivalent to a curved mirror with $\text{roc} = f$



(because a double passed lens has a focal length of $f/2$, but a mirror with $\text{roc} R$ has a focal length of $R/2$)

In this configuration, the beam waist is at the flat mirror on the left.

- b) We want to establish a mode waist size of w_0 , corresponding to a Rayleigh range of $z_0 = \pi w_0^2 / \lambda$. We know that the roc of the Gaussian beam at the curved mirror has to match the roc of the mirror (for a stable cavity). Thus, from the Gaussian beam propagation equations:

$$R = z \left[1 + \frac{z_0^2}{z^2} \right] = d \left[1 + \frac{z_0^2}{d^2} \right]$$

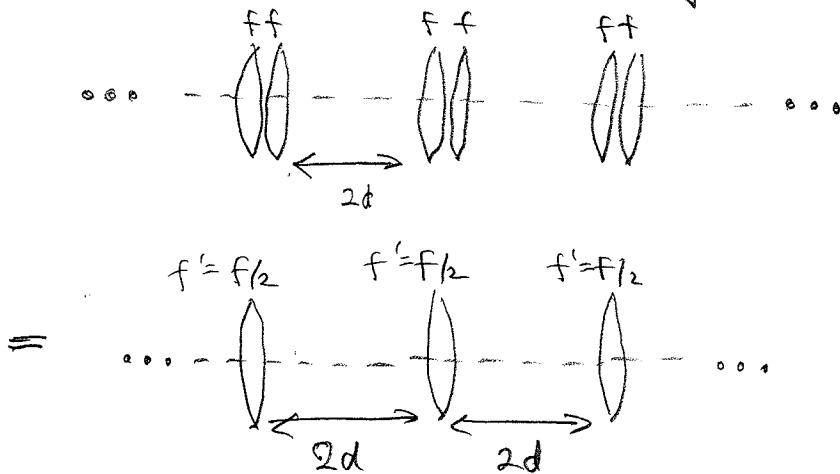
for a propagation distance of d .

$$\therefore R = d + \frac{z_0^2}{d} \quad \text{or} \quad z_0^2 - Rd + d^2 = 0$$

$$\Rightarrow z_0 = \frac{Rd \pm \sqrt{(Rd)^2 - 4d^2}}{2} = \frac{R}{2} \pm \sqrt{\frac{R^2}{4} - d^2}$$

$$\text{where } w_0 = \frac{\pi w_0^2}{\lambda}$$

c) The resonator has the following lens train analog:



For a single-lens system, the stability is given by

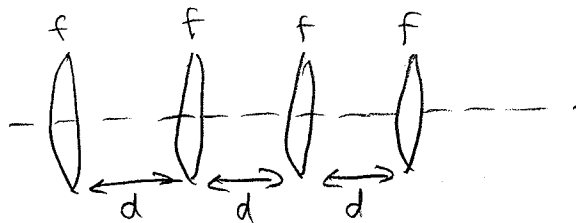
$$0 \leq d' \leq 4f'$$

$$\text{where } d' = 2d$$

$$f' = f/2$$

$$\therefore 0 \leq 2d \leq 2f, \text{ or } d \leq f$$

d) If the lens is move to the center of the resonator, then the lens train analog is given by:



$$\text{and we have } 0 \leq d \leq 4f$$

