Problem 1 [40 pts] Assume that an uncompensated operational amplifier (Op-Amp) has the following transfer function from the differential voltage $V_p - V_n$ to the output voltage V_o.

$$a(s) = \frac{10^5}{(1 + 10^{-4}s)(1 + 10^{-6}s)}$$

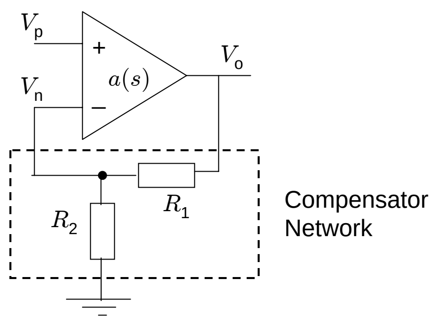

Figure 1: The Op-Amp with the static compensator

1. (5pts) Carefully sketch the bode plot corresponding to $a(s)$. Make sure that your axis are correctly labeled and the asymptotes are evident.
2. (10pts) If the Op-Amp is compensated by the resistive network shown in Figure 1, Provide values for the two compensator resistors so that the closed loop transfer function from V_p to V_o has a DC gain equal to 10.

   To answer this question, assume that V_o is not affected by the load provided by the compensator network, and that the current flowing into the (-) terminal is zero, thus that V_n is determined by the compensator transfer function. *The resistive network realizes a proportional controller; assume this gain to be K.*
3. (10 pts) Describe what is the unit step response you expect from the compensated amplifier, and why. *Hint: Find the gain cross-over frequency and phase margin.*
4. (5pts) Provide the relationship between the pole and zero of a first order lead compensator. To improve the response, the control engineers decide to use a lead compensator. Such a compensator can be built by putting a capacitor in parallel to one of the resistors. Modify the compensator circuit using a capacitor that can realize a lead compensator behavior.
5. (10 pts) Keeping the values of the resistors you have found in Part 2, find a value of the capacitor, for the lead compensator in Part 4, that improves the closed loop step response without reducing the closed loop bandwidth. Explain. *Hint: Increase the phase margin.*

Solution

1. The Bode plot is shown in Figure 2.
2. The compensator is static gain of value

$$K = \frac{R_2}{R_1 + R_2}.$$

   Using the assumption that that the output impedance is zero and the input impedance is infinity, we have that

$$V_o(s) = a(s)(V_p(s) - V_n(s)) = a(s)(V_p(s) - KV_o(s))$$

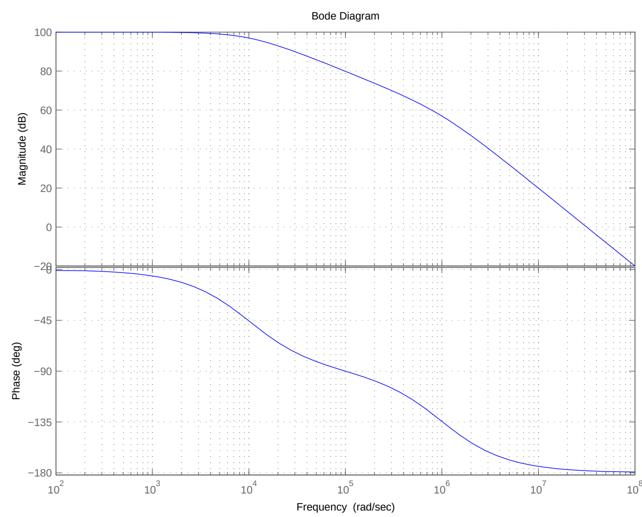

Figure 2: Amplifier Bode plot

Thus, the closed loop transfer function is given by the following expression:

$$\frac{V_o(s)}{V_p(s)} = \frac{a(s)}{1 + a(s)K}.$$

DC gain $= \frac{a(0)}{1+a(0)K} \approxeq \frac{1}{K}$ for $a(0) >> 1$.

It follows that $K = 0.1$ satisfies the DC gain specification. Such K can be achieved by selecting

$$R_1 = 90k\Omega, \qquad R_2 = 10k\Omega$$

3. Since the closed loop system is stable for all $K > 0$, the DC gain corresponds to the steady state step response.

   However, the open loop transfer function $L(s) = a(s)K$, which, with $K = 0.1$, has a cross-over frequency at $\omega_0 \approxeq 10^7 rad/dec$. At ω_0 the phase margin is very small (less than 5^o). This result in a closed loop system with a lightly damped resonant mode around ω_0.

   Thus, although the unit step response converges to 10 in steady state, the transient has a large overshoot and high frequency lightly damped oscillations.

4. By adding a capacitor in parallel to R_1, the compensator becomes dynamic with the following transfer function:

$$K(s) = \frac{R_2}{R_1 || \frac{1}{sC} + R_2} = \frac{R_2}{\frac{R_1}{1+sCR_1} + R_2} = \frac{R_2(1 + sCR_1)}{R_1 + R_2 + sCR_1R_2} = \frac{R_2}{R_1 + R_2} \frac{1 + sCR_1}{1 + sC\frac{R_1R_2}{R_1+R_2}}$$

   The compensator zero is at $z = -\frac{1}{CR_1}$ and the pole is at $p = -\frac{1}{CR_1} - \frac{1}{CR_2}$. With $C = 2pF = 2\,10^{-12}F$, we obtain

$$z \approxeq 5.55\,10^6\,rad/sec, \qquad p \approxeq 5.55\,10^7\,rad/sec$$

   These correspond to

$$K(s) = \frac{1.8\,10^{-8}s + 0.1}{1.8\,10^{-8}s + 1}$$

   The compensated open loop Bode plot can be sketched easily using asymptotes since z and p are a decade apart for each other. The new cross-over frequency is about $1.1 \cdot 10^7\,rad/sec$ which correspond to a phase margin of about 40^o degree.

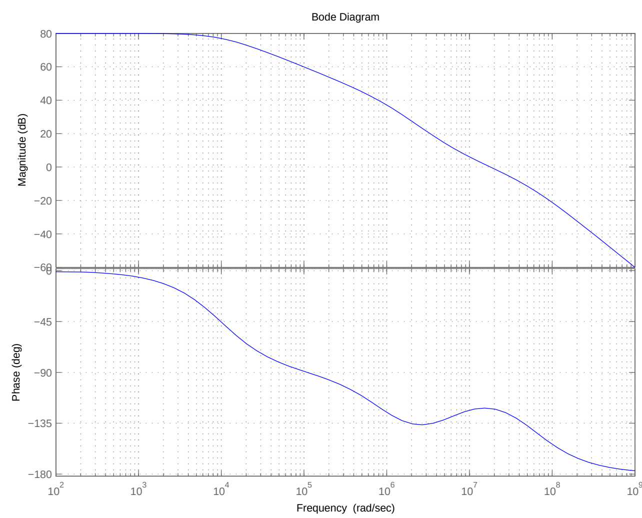

Figure 3: Lead compensated open loop Bode plot.