Problem 1.

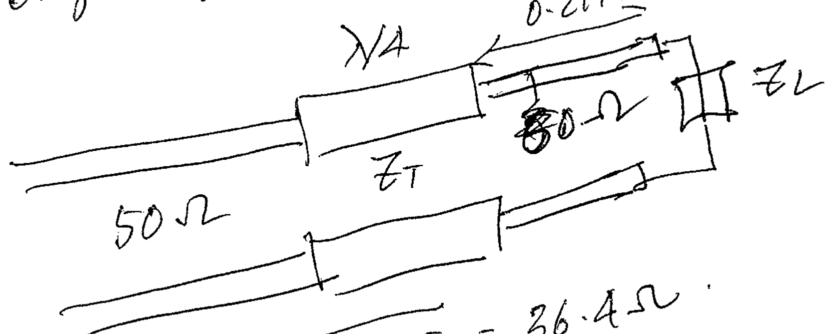
Smith Chart
mark $0.3\angle -25^\circ$, $z_L = 1.55 - j0.45$.
Rotate towards generator $(0.5 - 0.2825)\lambda$
$= 0.2175\lambda$.

$f\lambda_g = v_p$ ie $\lambda_g = \frac{v_p}{f} = \frac{1\times10^8}{6\times10^9} = 1.67\text{cm}$

(a) Thus $0.2175\lambda = 3.625\text{mm}$

(b) $z_{min} = 0.53$ $\quad Z_{min} = 0.53\times 50 = 26.5\Omega$

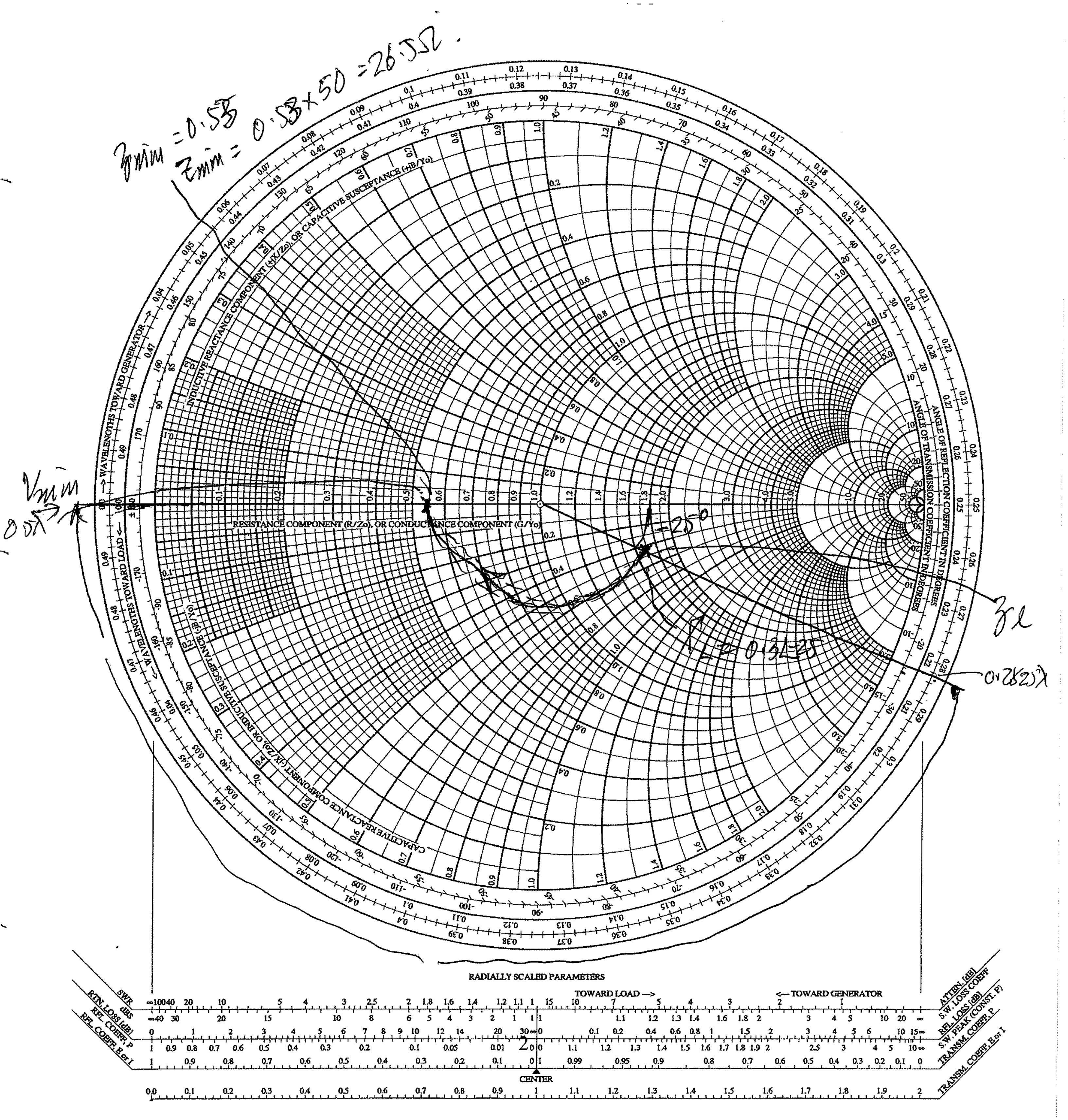
(c) Design of Matching $\lambda/4$ circuit



$Z_T = \sqrt{50\times 26.5} = 36.4\Omega$.

$\lambda/4 = \frac{1.65}{4} = 0.4125\text{ mm}$

Smith Chart



zmin = 0.53
Zmin = 0.53×50 = 26.5Ω
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
Vmin
0.0λ
25°
zL
ΓL = 0.3∠-25
0.282λ
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER

2. $\nabla \times \bar{H} = \bar{J} + j\omega\epsilon\bar{E} = \sigma\bar{E} + j\omega\epsilon\bar{E}$

$$= j\omega\left(\epsilon + \frac{\sigma}{j\omega}\right)\bar{E}$$

$$\epsilon_c = \left(\epsilon - j\frac{\sigma}{\omega}\right), \quad \tan\delta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon}$$

$$\gamma^2 = -\omega^2\mu\epsilon_c$$

$$\gamma = j\omega\sqrt{\mu\left(\epsilon - j\frac{\sigma}{\omega}\right)} = \alpha + j\beta.$$

$$\mu = \mu_0$$

$$\mu_0\left(25.5 \times 8.85\times10^{-12} - j\frac{0.5}{2\pi\times915\times10^{6}}\right)$$

$$\epsilon_c = \epsilon_0\left(25.5 - j\frac{0.5}{2\pi\times6\times10^{9}}\right)$$

$$= 8.854\times10^{-12}\left(25.5 - j\frac{0.5}{2\pi\times6\times10^{9}}\right) = 8.854\times10^{-12}(25.5 - j1.326\times10^{-11})$$

$$= 2.256\times10^{-10} - j\,1.173\times10^{-22}$$

Note that $\tan\delta = \frac{\epsilon'}{\epsilon''} = \frac{1.326\times10^{-11}}{25.5} \ll 1$

$$k_c^2 = \omega^2\mu_0\epsilon_c = (2\pi\times6\times10^{9})^2\times4\pi\times10^{-7}\times8.85\times10^{-12}\times(25.5 - j1.326\times10^{-11})$$

$$k_c = \sqrt{\omega^2\mu_0\epsilon_c} = 6.35\times10^{2} - j\,1.65\times10^{-1}$$

$\beta = 6.35\times10^{2}$ rad/m, $\alpha = 1.65\times10^{-1}$ Np/m.

$$\delta = \frac{1}{\alpha} = \frac{1}{1.65\times10^{-1}} = \underline{6.06\,m}.$$

Attenuation $\alpha = 1.65 \times 10^{-1}$ Np/m $\equiv 8.68 \times 1.65 \times 10^{-1}$ dB/m

$= 1.43$ dB/m

$$\eta = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854\times10^{-12}\times(25 - j\,1.326\times10^{-11})}}$$

$$= 377\sqrt{\frac{1}{(25-j\,1.326\times10^{-11})}}$$

$$= 74.65 + j\,1.94\times10^{-11}$$

$$\simeq 74.65\ \Omega.$$

At the borehole the power density is

$$2.75\ \text{kW/m}^2 \equiv \frac{|E_i|^2}{2\eta}$$

$$|E_i| = \sqrt{2.75\times10^{3}\times 2\times 75}$$

$$= 642.3\ \text{V/m}.$$

This decays as $e^{-\alpha z}$

Hence $642.3\, e^{-0.165 z}$ V/m $= \frac{10\times10^{-6}\,\text{V}}{.01} = 10^{-5}\times10^{2} = 10^{-3}$ V/m

$$\text{ie}\quad e^{-0.165 z} = \frac{10^{-3}}{642.3}$$

$$-0.165 z = \ln\frac{10^{-3}}{642.3} = \ln 1.557\times10^{-6}$$

$$z = \frac{+13.37}{+0.165} = \underline{81\text{m}}.$$