1) [**15 points**] Consider discrete-time LTI system depicted below.

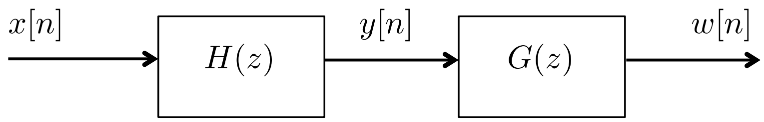

Figure 1: Discrete-time LTI system.

Suppose that $H(z) = (1 - 2z^{-1})(1 - \frac{1}{2}z^{-1})$ with ROC $|z| > 0$. Find the impulse response $g[n]$ of a *stable* system G such that $w[n] = x[n-2]$.

Hint: Recall the following z-Transform relations, which you may find useful:

$$\begin{aligned} x[n] = \alpha^n u[n] \longleftrightarrow X(z) &= \frac{1}{1 - az^{-1}}, \quad \textit{ROC: } |z| > |a| \\ x[n] = -\alpha^n u[-n-1] \longleftrightarrow X(z) &= \frac{1}{1 - az^{-1}}, \quad \textit{ROC: } |z| < |a|. \end{aligned}$$

2) **[10 points]** Recall that the discrete-time convolution operation satisfies several standard arithmetic properties. Namely, for all functions $f[n], g[n]$, and $h[n]$, we have:

- Commutativity: $f[n] * g[n] = g[n] * f[n]$
- Associativity: $f[n] * (g[n] * h[n]) = (f[n] * g[n]) * h[n]$
- Distributivity: $f[n] * (g[n] + h[n]) = (f[n] + g[n]) + (f[n] + h[n])$

Note, however, that for arbitrary functions $g[n], h[n]$, the relationship

$$f[n]\,(g[n] * h[n]) = (f[n]g[n]) * (f[n]h[n]) \qquad (1)$$

does *not* hold for all $f[n]$, though it does hold for the trivial cases $f[n] = 1$ and $f[n] = 0$.

Besides the trivial examples given above, find another function $f[n]$ for which (1) holds for all $g[n], h[n]$.

3) Consider the system shown below.

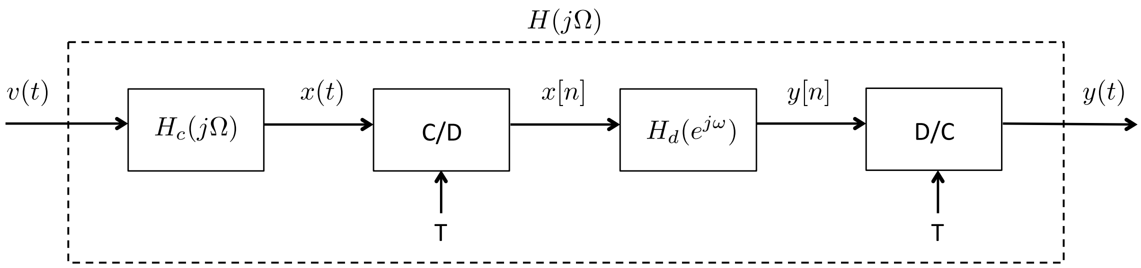

Figure 2: System for digital processing of analog signals.

The anti-aliasing filter $H_c(j\Omega)$ is an ideal low-pass continuous-time filter with unit gain and cutoff frequency π/T. The frequency response of the LTI discrete time system between the converters is given by:

$$H_d(e^{j\omega}) = j\omega e^{-j\omega/3}, \quad |\omega| < \pi.$$

Assume that the C/D and D/C converters are ideal.

a) [**5 points**] Find the impulse response $h_d[n]$ of the discrete-time LTI system.

b) [**5 points**] What is the effective continuous-time frequency response of the overall system, $H(j\Omega)$?

c) [**5 points**] Choose the most accurate statement:

(i) $y(t) = x(t - 1/3)$

(ii) $y(t) = \frac{d}{dt}x(3t)$

(iii) $y(t) = T\frac{d}{dt}x(t - 1/3)$

(iv) $y(t) = T\frac{d}{dt}x(t - T/3)$

(v) $y(t) = \frac{d}{dt}x(t - 1/3)$

(vi) $y(t) = \frac{d}{dt}x(t - T/3)$