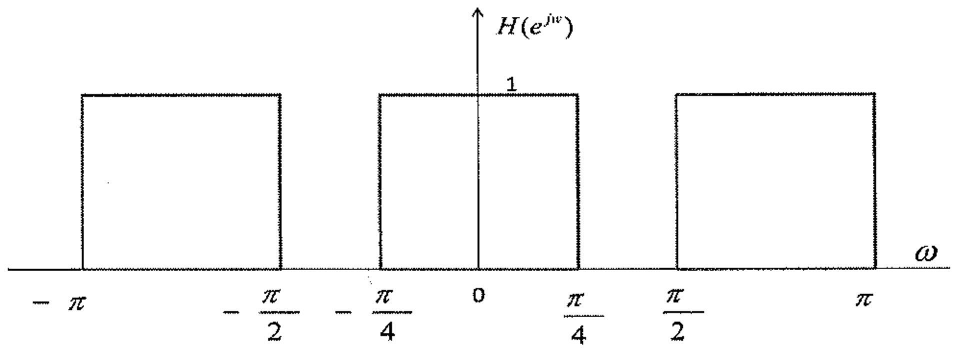
DSP Solution, WPE, Fall-2012

[DSP]

This problem consists of two independent parts: PART-I and PART-II.

PART-I: [6+2+6+6+5=25 points]

Consider h(n) whose DTFT $H(e^{jw})$ is shown below:



Let $g(n) = h(2n)$, we are interested in evaluating $G(e^{jw})$ in 2 ways.

a. Find $h(n) = IDTFT[H(e^{jw})]$
b. Find $g(n) = h(2n)$. Substitute *n* by *2n* in $h(n)$.
c. Compute and sketch $G(e^{jw})$, the DTFT of $g(n)$.
d. Now observe that $g(n) = h(2n)$.

   Thus $G(e^{jw}) = \frac{1}{2}[H\left(e^{\frac{jw}{2}}\right) + H\left(e^{j(\frac{w}{2}-\pi)}\right)]$

   Use $H(e^{jw})$ shown above to sketch (i) $H\left(e^{\frac{jw}{2}}\right)$ (ii) $H\left(e^{j(\frac{w}{2}-\pi)}\right)$ and (iii) $G(e^{jw})$.

   (Note: There will be aliasing involved.) You should see that the answers of part (c) and (d) are the same.

$H(e^{jw})$

e. An input signal $x(n) = \frac{\sin(\frac{n\pi}{2})}{n\pi}$ is filtered by the above filter. What is the output y(n)?

a) $h(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/2} e^{jwn}\,dw + \int_{-\frac{\pi}{4}}^{\pi/4} e^{jwn}\,d\omega + \int_{\frac{\pi}{2}}^{\pi} e^{jwn}\,dw\right]$

$= \frac{1}{2\pi jn}\left[e^{-j\frac{\pi}{2}n} - e^{-j\pi n} + e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n} + e^{j\pi n} - e^{j\frac{\pi}{2}n}\right]$

$= \frac{1}{2\pi jn}\left[-2j\sin\frac{n\pi}{2} + 2j\sin n\pi + 2j\sin\frac{n\pi}{4}\right]$

$= \frac{\sin n\pi + \sin\frac{n\pi}{4} - \sin\frac{n\pi}{2}}{n\pi}$

[Extra Page]

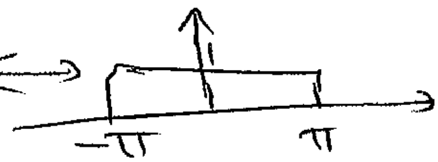
b) $$g(n) = \frac{\sin 2n\pi + \sin \frac{n\pi}{2} - \sin n\pi}{2n\pi}$$

$$= \frac{\sin 2n\pi}{2n\pi} - \frac{\sin n\pi}{2n\pi} + \frac{\sin n\pi/2}{2n\pi}$$

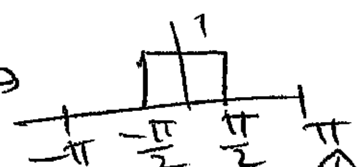
$$= \frac{1}{2}\delta(n) + \frac{\sin n\pi/2}{2n\pi}$$

$$\boxed{\begin{array}{l} \frac{\sin 2n\pi}{2n\pi} = \delta(n) \\ \frac{\sin n\pi}{2n\pi} = \frac{1}{2}\delta(n) \end{array}}$$

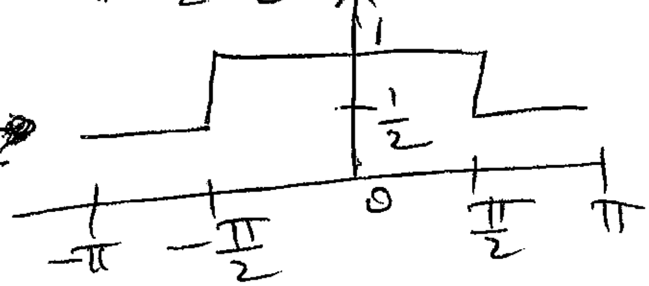
c) $\delta(n) \longleftrightarrow$



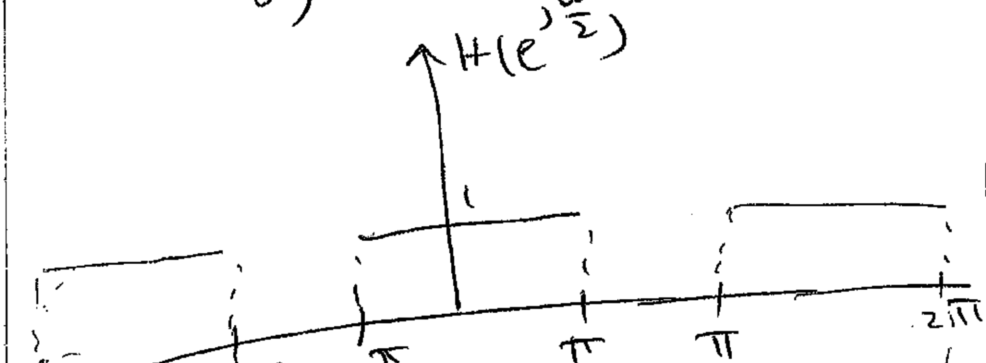
$\frac{\sin n\pi/2}{n\pi} \longleftrightarrow$



$g(n) \longleftrightarrow G(e^{j\omega}) \Longrightarrow$

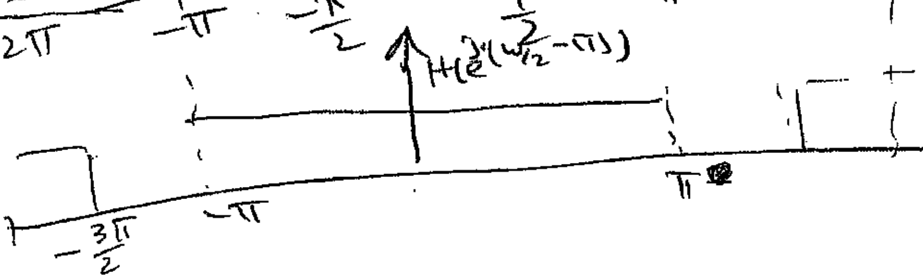


d)



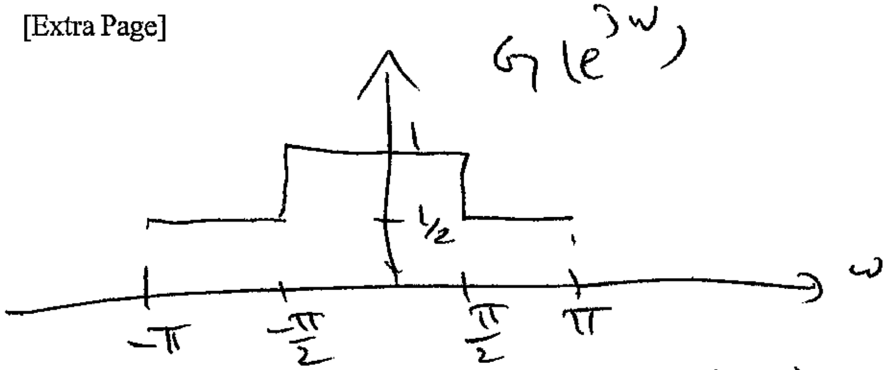
Periodic in 4π

(stretch by factor 2)



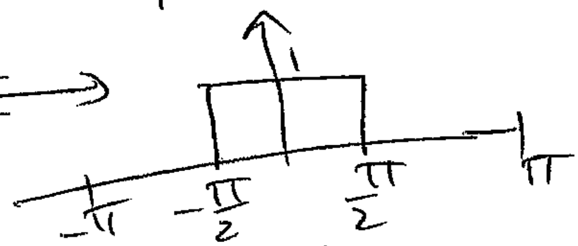
① shift $H(e^{j\omega})$ by π to right
② keep only parts for $-\frac{\pi}{2} < \omega < \frac{\pi}{2}$
③ stretch by factor 2

[Extra Page]



same as part (c)

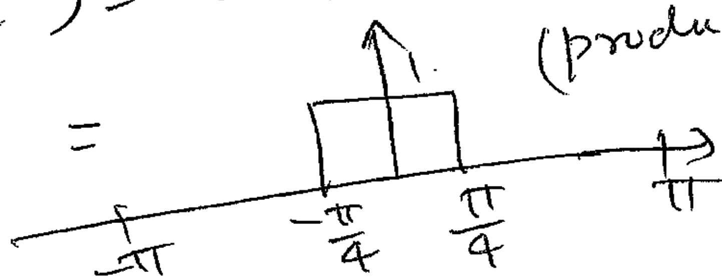
e) $X(e^{jw}) \Longleftrightarrow$



$$Y(e^{jw}) = X(e^{jw})\, H(e^{jw})$$

(product)

=



$$y(n) = \frac{\sin n\pi/4}{n\pi}$$

(a low-pass signal limited to $\pi/4$ rad/s)

PART-II: [15 points]

Let the DFT of a 6-point complex sequence (a,b,c,d,e,f) be another complex sequence (A,B,C,D,E,F), i.e., (a,b,c,d,e,f) $\leftrightarrow$ (A,B,C,D,E,F)

Complete the DFT/IDFT table below by finding the unspecified *x(n)* or *X(k)*. The first 9 DFT/IDFT pairs correspond to DFT size 6, and the last 6 pairs correspond to DFT size 12. You do not need to show any of your calculations. Return this page with your answers.

	x(n)	*X(k)*	
	(A,B,C,D,E,F)	(6a, 6f, 6e, 6d, 6c, 6b)	$N x((-n))_N$
	(A, F, E, D, C, B)	(6a, 6b, 6c, 6d, 6e, 6f)	
	(A*, B*, C*, D*, E*, F*)	(6a*, 6b*, 6c*, 6d*, 6e*, 6f*)	
	(A, F, E, D, C, B)	(6a, 6b, 6c, 6d, 6e, 6f)	
	(a*,b*,c*,d*,e*,f*)	(A*, F*, E*, D*, C*, B*)	
	(a, f, e, d, c, b)	(A, F, E, D, C, B)	
	(a*, f*, e*, d*, c*, b*)	(A*,B*,C*,D*,E*,F*)	
$(-1)^n x(n) = e^{j\frac{2\pi}{6}(3)n} x(n)$	(a,-b, c,-d, e,-f)	(D, E, F, A, B, C)	
	(a,-f, e, -d, c, -b)	(D, C, B, A, F, E)	
	(a, 0, b, 0, c, 0, d, 0, e, 0, f, 0)	(A, B, C, D, E, F, A, B, C, D, E, F)	
	(A, B, C, D, E, F, A, B, C, D, E, F)	(12a, 0, 12f, 0, 12e, 0, 12d, 0, 12c, 0, 12b, 0)	
	(A, 0, B, 0, C, 0, D, 0, E, 0, F, 0)	(6a, 6f, 6e, 6d, 6c, 6b, 6a, 6f, 6e, 6d, 6c, 6b)	
	($\frac{A}{6}$, 0, $\frac{F}{6}$, 0, $\frac{E}{6}$, 0, $\frac{D}{6}$, 0, $\frac{C}{6}$, 0, $\frac{B}{6}$, 0)	(a, b, c, d, e, f, a, b, c, d, e, f)	
	($\frac{A}{12}$, $\frac{F}{12}$, $\frac{E}{12}$, $\frac{D}{12}$, $\frac{C}{12}$, $\frac{B}{12}$, $\frac{A}{12}$, $\frac{F}{12}$, D/12, C/12, B/12)	(a, 0, b, 0, c, 0, d, 0, e, 0, f, 0)	
	(D, 0, E, 0, F, 0, A, 0, B, 0, C, 0)	(6a, −6f, 6e, −6d, 6c, −6b, 6a, −6f, 6e, −6d, 6c, −6b)	