
Problem 1 (15 points)

You are given the signal

$$s(t) = 2 \sin \pi t.$$

The signal is to be transmitted using four-bit PCM.

(a) What is the Nyquist sampling rate for this signal? [5 points]

(b) Suppose the sampling period is $T_s = 0.9$ secs. Find the PCM waveform for the first 5 sec. [5 points]

(c) Suppose $T_s = 0.9$ secs. Find the mean square quantization noise power. [5 points]

Problem 2 (25 points)

A binary communication system has 2 symbols 0 and 1. The probabilities of these symbols occurring are given by

$$\begin{cases} P(0) = \frac{1}{5} \\ P(1) = \frac{4}{5} \end{cases}$$

These symbols are transmitted by signals $-s(t)$ and $s(t)$ respectively, where $s(t)$ is time-limited to the interval $[0, T_b]$, i.e.,

$$s(t) = 0, \quad \forall t \notin [0, T_b] \quad \text{and} \quad \int_0^{T_b} s^2(t)dt = E_s.$$

An integrator is used to receive the binary signal which has been transmitted through a channel disturbed by white Gaussian noise having a power spectral density

$$\mathcal{S}_n(\omega) = \eta/2$$

where η is a constant. If v is the output of the integrator at $t = T_b$, $E_s = 1$ and $\eta = 2 \times 10^{-3}$.

(a) Determine the noise power at the output of the integrator at $t = T_b$. [7 points]

(b) Sketch the probability density functions of the possible signals at the output of the integrator. [7 points]

(c) Determine the optimal threshold of decision. [6 points]

(d) Calculate the overall bit error rate (expressed in terms of the Gaussian error function $\Phi_c(z) = \frac{2}{\pi} \int_z^{\infty} e^{-t^2} dt$). [5 points]