There are two Parts and they are assigned 20 points each (for a total of 40/40)

Part #1 (20 points total):
Consider an inertial system modeled as a lumped scalar linear system

$$\frac{d^2x(t)}{dt^2} = u(t).$$

Here $u(t)$ represents a force applied to it by a controller, while $x(t)$ represents its position at time $t \in [0, \infty)$. The controller input is taken as the difference $v(t) = r(t) - x(t)$, between a reference signal $r(t)$ and the position $x(t)$. The controller output is the applied force $u(t)$.

Address the following three independent sub-parts to Part 1:
1. **(5 points)** Consider the control law $u(t) = Kv(t)$ and determine whether the feedback system is asymptotically stable for any choice of the gain K. (Recall that a system is asymptotically stable if all of its poles have negative real part.)

2. **(5 points)** Consider a dynamic control law where

$$u(t) = v(t) - \int_0^t v(\tau)e^{-2(t-\tau)}d\tau.$$

Determine whether the feedback system is asymptotically stable for this choice of control law. (Prove that it is, or prove that it is not.)

3. **(10 points)** Design a control law which stabilizes the feedback system and has the additional property that, at steady state, the position follows exactly a sinusoidal reference signal $r(t) = \sin(t)$.

Part #2 (20 points total):
Consider a dynamical system, described by the delay-differential equation

$$\frac{d}{dt}x(t) = u(t) - Kx(t-1) - Kx(t-2),$$

where $x(t)$ represents the state and $u(t)$ the input. Determine the maximal value of K for which the system is stable. (Hint: You may use Nyquist's stability criterion.)