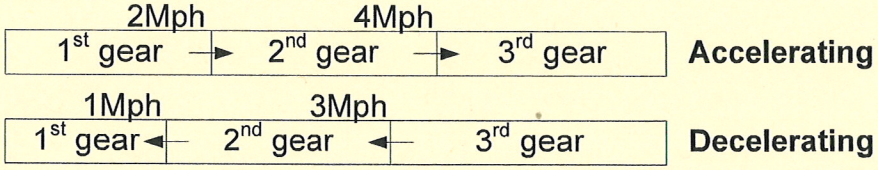
You are asked to design a hardware platform to control the automatic gear changes in a go kart racer. To keep matters tractable we shall assume the following for the car:

- The car has three gears
- The accelerator has two positions, either off or on
- The car's speed varies from 0-7Mph
- When the accelerator position is "off" the car does not move if it was not moving previously. If it was moving previously it slows down.
- When the accelerator is on the car continues to accelerate and the gear change, i.e., going from 1→2 and 2→3 at the speeds shown below.



- However, to avoid hunting, i.e., rapid changes in gears, the speeds at which they are changed down (when the accelerator position is off), i.e., going from 3→2 and from 2→1 are at 1Mph lower as shown below, i.e., the system has hysteresis.

a) Identify the number of bits required for representing the accelerator position ($A_0..A_n$), for the number of gears ($G_0...G_k$) and for the speed ($S_0...S_m$). Then draw up a truth table for acceleration and deceleration that maps the gear used for a particular speed. Specify, what is the total length of this truth table? [0.5 point]
b) Write the canonical sum-of-products expression for the gear used. [0.5 points]
c) Next, using a Karnaugh map identify the minterms to realize the simplest circuit using a sum-of-products form for this function. [1 point]
d) Draw the circuit using only "AND", "OR" and "NOT" gates. [0.5 points]
e) You are now told that an additional safety feature needs to be added to the system. You are told that an additional input to your circuit from the brake needs to be added. When the brake is pushed **on** (B0=1) the car should be slowed down using the prior gears used for deceleration. Using the minimum of costs, how would you modify your previous circuit to incorporate this change? Here cost corresponds to the number of gates. [0.5 points]
f) A new manager is brought into the company and he instructs you to identify if the car is upshifting (gears going from 1→2, or 2→3) or downshifting by turning on a light (L0=1 corresponds to upshifting). Assume a don't care for values where the previous gear ($PG_0...PG_k$) is the same as the present one ($G_0...G_k$). Draw the truth table for this new system. [0.5 points]
g) Draw the Karnaugh map and identify the minterms for this new system. What is the minimum number of D flip-flops needed to realize this new system. [0.5 points]