Note that e_0=8.85*10^{-12} F/m and m_0=4p*10^{-7} H/m.

1. (1 point) Find the 10 percent and 60 percent depth of penetration of an electromagnetic wave of amplitude

$$E_y = E_0 e^{-\frac{x}{\delta}}$$

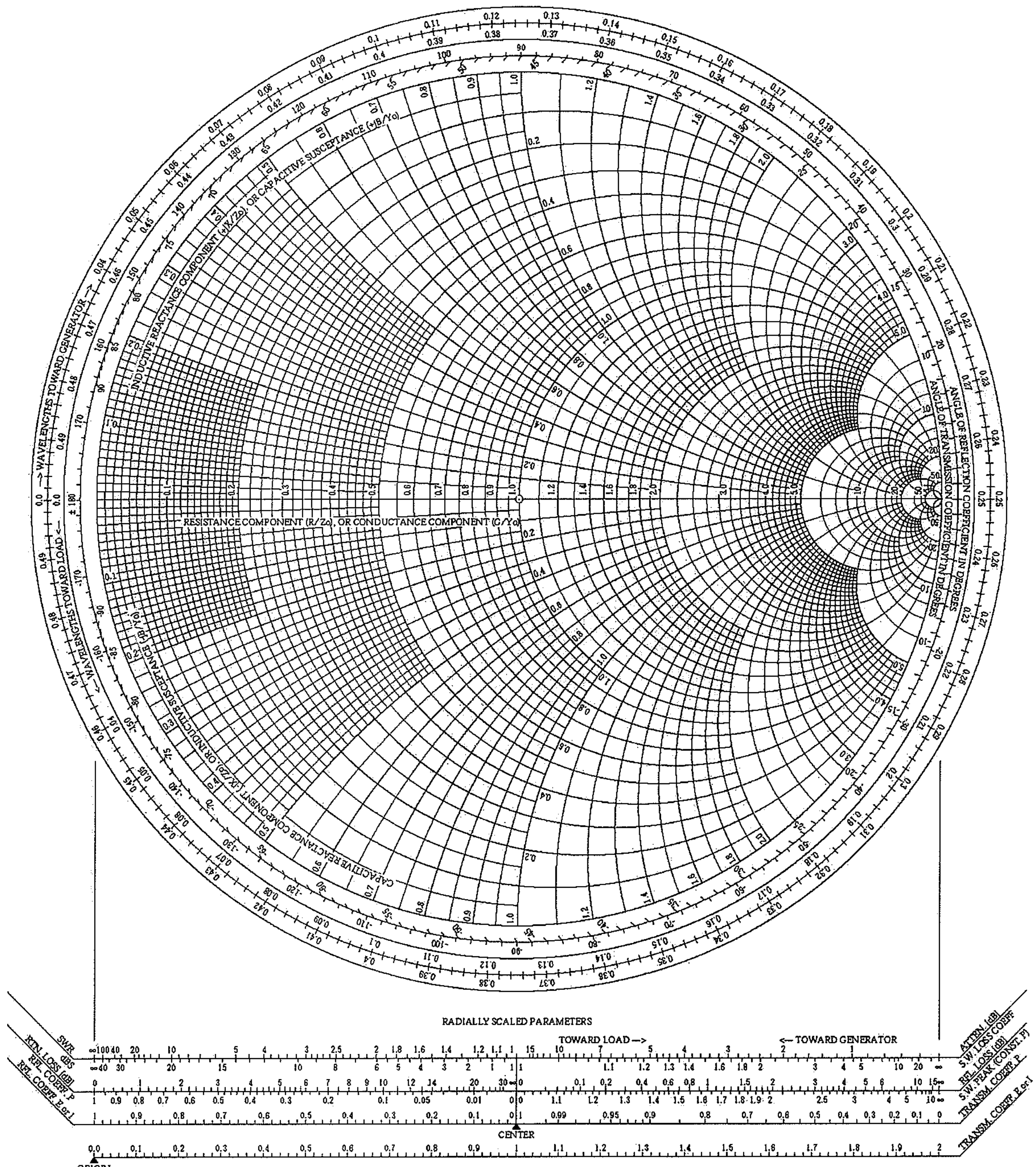
   E_o is the initial amplitude traveling in the x-direction.

2. (1 point) A plane wave having a frequency of 1600 MHz is traveling in a medium where the relative dielectric constants are 1 and the conductivity is 0.1 S/m. If the rms electric field intensity of the wave is 10 V/m, what is the (a) conduction current, (b) displacement current density and (c) total current density.

3. (2 point) Design a circuit that includes a load (Z_L=0.1+j0.1, ohms) connected to shortest length transmission line that makes the input the same as the load. The input to the line is connected to a shunt 10nF capacitor. This parallel combination is then connected to a $\lambda/4$ section of line. Assume all line characteristic impedances are 50 ohms.
   a. What is the input impedance to the quarter-wave line and specific line lengths in meters of the transmission line sections of all lines if the operating frequency is 20 GHz and the dielectric constant of the transmission line is 4?
   b. Design an equivalent transmission line in wavelengths to realize a 10nF capacitor if the line is terminated in a short.

The Complete Smith Chart

Black Magic Design



Problem 1 Solution

$$E_y = E_o e^{-x/\delta}$$

a) 10% depth of penetration

$$E_o e^{-x/\delta} = 0.1 E_o$$

$$\ln(e^{-x/\delta} = 0.1)$$

$$\frac{-x}{\delta} = -2.3$$

$$x_{10\%} = 2.3\,\delta.$$

b) 60% depth of penetration

$$e^{-x/\delta} = 0.6$$

$$x_{60\%} = 0.51\,\delta$$

Problem 2 Solution

If $\mathcal{E}_0 = 10\,V/m$, $E(t) = 10\sqrt{2}\sin\omega t$

a) $J_c = \sigma\mathcal{E} = 0.1 \times 10 = 1\ A/m^2$

b) $\frac{d}{dt}E = 10\sqrt{2}\,\omega\cos\omega t$

$= 10\sqrt{2}\,\omega\sin(\omega t + \pi/2)$

$J_d = \frac{\partial D}{\partial t} = \varepsilon\frac{d\mathcal{E}}{dt}$

$= \omega\varepsilon E_0 \sin(\omega t + \pi/2)$; $f = 1600 \times 10^6$

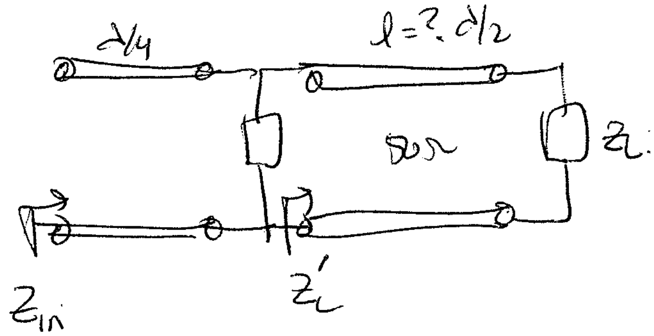
$= 2\pi \cdot 1.6 \times 10^9 \times 10 \times \frac{10^{-9}}{36\pi}\sin(\omega t + \pi/2)$

$= 0.883\sin(\omega t + \pi/2)$

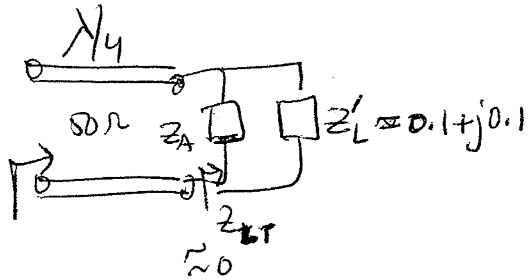
c) Current density in phasor form

$J_t = J_c + J_d = 1 + j0.88\ (A/m^2)$

Problem 3 Solution



a)

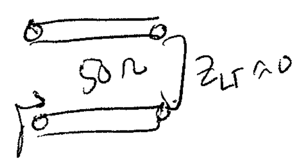


$$l = \lambda/2 = \frac{c}{f\sqrt{4}} \quad \frac{3\cdot10^8}{20\cdot10^9\cdot2} = \frac{1.5\cdot10^{-1}}{20} = 0.0909$$

9.09 cm

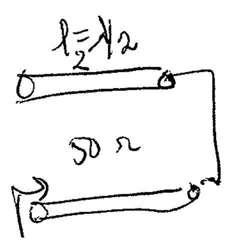
$$Z_A = \frac{-j}{\omega C} = \frac{-j}{2\pi\cdot20\cdot10^9\cdot10\cdot10^{-9}} = \frac{-j}{400\pi}$$

$$Z_A = -j\,7.95\times10^{-4} \simeq 0$$



$Z_{in} \approx \infty$

b) $Z_A = \frac{-j}{400\pi}$



$Z_{in} \approx 0 = Z_A$