Part a:

The phase of the reflected beam can be found by:

$$\left|\frac{E_{\mathrm{t}}}{E_{\mathrm{i}}}\right| e^{-i\Psi} = \frac{(1-R)(1+R\,\mathrm{e}^{i\delta})}{(1-R\,\mathrm{e}^{-i\delta})(1+R\,\mathrm{e}^{i\delta})} e^{-i\delta/2}$$

$$= \frac{(1-R)+R(1-R)e^{i\delta}}{1+R^2-2R\cos\delta} e^{-i\delta/2} = \frac{(1-R)+R(1-R)(\cos\delta+i\sin\delta)}{1+R^2-2R\cos\delta} e^{-i\delta/2}$$

So

$$\Psi = \frac{\delta}{2} - \arctan\left\{\frac{R(1-R)\sin\delta}{(1-R)+R(1-R)\cos\delta}\right\} = \frac{\delta}{2} - \arctan\left\{\frac{R\sin\delta}{1+R\cos\delta}\right\}$$

Part b:

1) Group delay:

$$\tau = \frac{d\Psi}{d\omega} = \frac{d\Psi}{d\delta}\frac{d\delta}{d\omega}$$

First, $\frac{d\delta}{d\omega} = \frac{d}{d\omega}\left[\frac{2n(\omega)\omega l}{c}\right] = \frac{2l}{c}\left[n(\omega)+\omega\frac{dn(\omega)}{d\omega}\right] = \frac{2n_g l}{c}$

and

$$\frac{d\Psi}{d\delta} = \frac{1}{2} - \frac{d}{d\delta}\arctan\left(\frac{R\sin\delta}{1+R\cos\delta}\right)$$

$$= \frac{1}{2} - \frac{1}{1+\left(\frac{R\sin\delta}{1+R\cos\delta}\right)^2}\left[\frac{R\cos\delta}{1+R\cos\delta} + \frac{R^2\sin^2\delta}{(1+R\cos\delta)^2}\right]$$

$$= \frac{1}{2} - \frac{R\cos\delta + R^2}{1+R^2+2R\cos\delta}$$

So group delay is:

$$\tau = \frac{d\Psi}{d\omega} = \frac{2n_g l}{c}\left[\frac{1}{2} - \frac{R\cos\delta+R^2}{1+R^2+2R\cos\delta}\right]$$

2) To find its extreme values, let $d\Psi/d\omega = 0$, which leads to $\sin\delta = 0$, so $\delta = m\pi$, where *m* is an integer.

When *m* is even, $\cos\delta = 1$, group delay $\tau = {n_g l}/{c}\left(1-\frac{2R}{1+R}\right) = \frac{n_g l}{c}\frac{1-R}{1+R}$ is the minimum.

When m is odd, $\cos\delta = -1$, group delay is $\tau = {n_g l}/{c}\left(1-\frac{2R}{1+R}\right) = \frac{n_g l}{c}\frac{1+R}{1-R}$ is the maximum.