

Solution

(1) (2 points) A uniform plane wave in air with $E_i(z) = \mathbf{a}_x E_0 \exp(-j\beta_0 z)$ impinges normally onto the surface at $z=0$ of a highly conducting medium having constitutive parameters ϵ_0 , μ_0 , and σ ($\sigma/\omega\epsilon_0 \gg 1$).

- Define the reflection coefficient. (0.5 points)
- Derive the expression for the fraction of the incident power absorbed by the conducting medium. (1.0 points)
- Obtain the fraction of the power absorbed at 1 MHz if the medium is iron ($\epsilon_r=1$, $\mu_r=4000$, and $\sigma=1e10^7$). (0.5 points)

Solution:

$$a) \Gamma = \frac{E_r}{E_i} = \frac{\eta_c - \eta_0}{\eta_c + \eta_0}; \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \text{ (or } 377 \Omega)$$

$$\eta_c = \sqrt{j\omega\mu/\sigma}$$

$$|\eta_c| \ll \eta_0.$$

$$b) |\Gamma|^2 = \left| \frac{\eta_c - \eta_0}{\eta_c + \eta_0} \right|^2 = \left| \frac{1 - \eta_c/\eta_0}{1 + \eta_c/\eta_0} \right|^2 \approx |1 - 2\eta_c/\eta_0|^2$$

$$= (1 - 2\eta_c/\eta_0)(1 - 2\eta_c^*/\eta_0)$$

$$\approx 1 - 4 \operatorname{Re}(\eta_c)/\eta_0.$$

$$\text{Fraction of power absorbed, } F = 1 - |\Gamma|^2$$

$$= \frac{4 \operatorname{Re}(\eta_c)}{\eta_0}$$

$$= \frac{4}{\eta_0} \operatorname{Re} \sqrt{\frac{j\omega\mu}{\sigma}} = \frac{4}{\eta_0} \sqrt{\frac{\omega\mu}{2\sigma}}$$

$$c) f = 1 \text{ MHz} \Rightarrow \omega = 2\pi \times 10^6 \text{ Hz}$$

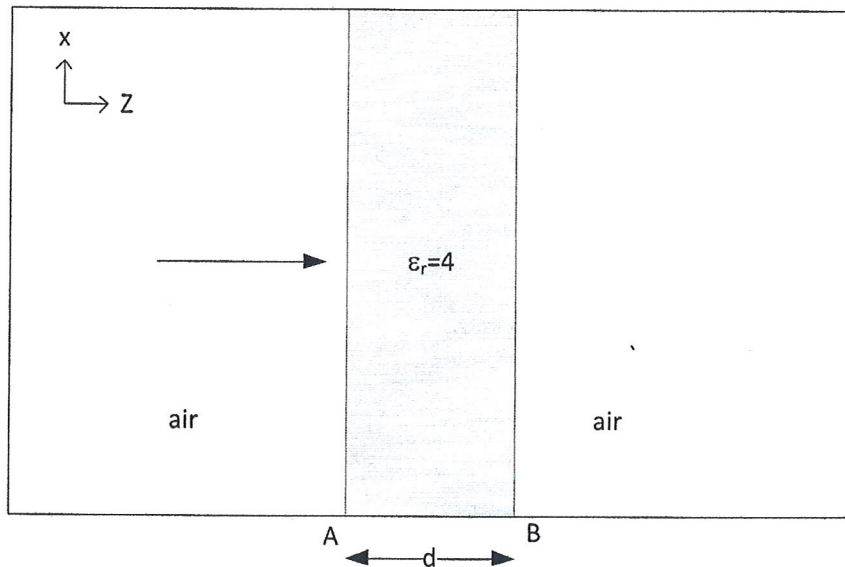
$$\mu = \mu_r \mu_0 = 4000 \cdot 4\pi \times 10^{-7} \text{ H/m}$$

$$\sigma = 10^7 \text{ (S/m)}$$

$$\therefore F = 4.21 \times 10^{-4} \text{ or } 0.42\%$$

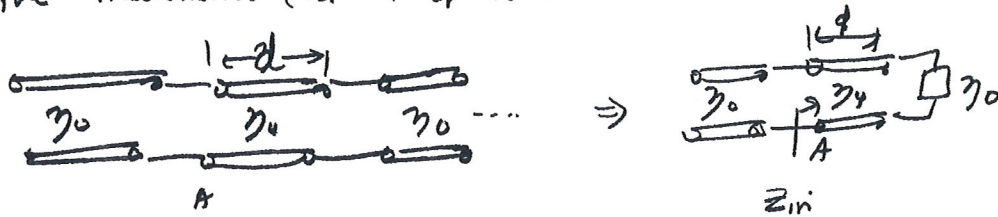
Solution

(2) (1 point) If a plane wave traveling in air in the +z direction passes through a uniform lossless medium with relative dielectric constant of 4 with normal incidence, derive an expression and compute how thick ($z = d$) the medium would need to be in wavelengths so the signal passes through to the other side with no reflection at Point A. Assume the $\epsilon_r=4$ medium extends to infinity in the x and y directions and that the air regions extends into infinity in all direction except where the $\epsilon_r=4$ medium exist.



Solution:

View the medium (air + $\epsilon_r=4$) as a line with impedance (η_0, η_4)



Find Z_{in} + set

$$Z_{in} = Z_0 \cdot \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l}$$

$$= \eta_4 \cdot \frac{\eta_0 + j \eta_4 \tan \beta l}{\eta_4 + j \eta_0 \tan \beta l}$$

where $Z_0 = \eta_4$ + $Z_L = \eta_0$

if $\tan \beta l = 0 \Rightarrow Z_{in} = \eta_0$

$$\therefore d (= d) = \lambda / 2$$

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- (3) (1 point) If a 1.5 pF capacitance is needed to tune a circuit using a transmission line configuration. What two stub designs can be used to realize this capacitor value in a coaxial cable configuration operating 10 GHz and coax dielectric constant of 4? Sketch and label each design with the line lengths (in wavelengths) and impedances. Assume the system impedance is 50 ohms.

Solution:

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3)

$$Z_{in} = -j \frac{1}{\omega c} = -j \frac{1}{2\pi(10 \times 10^9)(1.5 \times 10^{-12})} = -j \frac{1}{2\pi(0.015)} = -j \frac{1}{0.03\pi}$$
$$= -j 10.6$$

$$\bar{Z}_{in} = \frac{-j 10.6}{Z_0 (= 50)} = -j 0.2122$$

$$Z_{in, o.c. load} = -j Z_0 \cot \beta l$$

$$\left(\bar{Z}_{in, o.c. load} = -j \cot \beta l \right) \frac{\sin \beta l}{\cos \beta l}$$

$$\tan \beta l = \frac{-j 1}{Z_{in, o.c. load}}$$

$$= \frac{-j 1}{-j 0.2122}$$

$$\tan \beta l = +4.7125$$

$$\beta_{o.c.l.} = \tan^{-1}(4.71) = 78.0^\circ \quad \left(\beta = \frac{2\pi}{\lambda} \right)$$

$$l_{o.c.l.} = \frac{\lambda}{2\pi} \times 78^\circ$$

$$= 0.216 \lambda$$

$$\boxed{l_{o.c.l.} = 3.24 \text{ mm}}$$

$$\lambda = \frac{c}{f \sqrt{\epsilon_r}} = \frac{3 \cdot 10^8}{(10 \cdot 10^9) 2} = 0.015$$

April 16, 2011 Problems T-line Quiz:

Solution

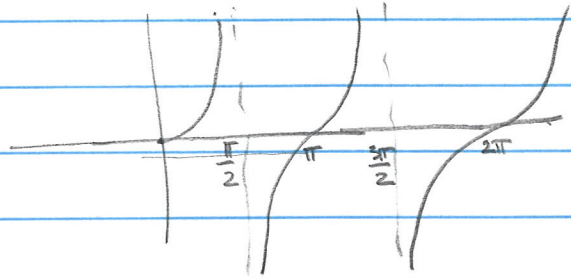
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3) continued

$$Z_{in, s.c. load} = +j Z_0 \tan \beta l$$

$$-j 0.2122 = j \tan \beta l$$

$$\beta l_{s.c.} = \tan^{-1}(-0.2122)$$



$$\beta l_{s.c.} = -11.9^\circ + n\pi$$

$$l_{s.c.} = \frac{\lambda}{2\pi} \times (11.9^\circ + 180^\circ)$$

$$= \frac{\lambda}{2\pi} \times 168^\circ \quad \text{since } -11.9^\circ \text{ is non-physical}$$

$$l_{s.c.} = 0.466 \lambda$$

$$\lambda = \frac{c}{f \sqrt{\epsilon_r}} = \frac{3 \cdot 10^8}{(10 \cdot 10^9)(\sqrt{4})} = 0.015$$

$$l_{s.c.} = 0.466(0.015)$$

$$\boxed{l_{s.c.} = 6.99 \text{ mm}}$$