Cascade pumping of a four-level laser system

Suppose a four-level system is "cascade pumped" with two separate pumping transition probabilities $W_{13} = W_A$ and $W_{34} = W_B$. The optical approximation $\hbar \omega >> kT$ applies for all transitions, and there are significant downward relaxation rate γ_{ij} between all levels

or a total decay rate for each energy level $\gamma_i = \sum_{j < i} \gamma_{ij}$.

a) Solve for the steady-state population difference $\Delta N_{42} \equiv N_4 - N_2$ in this system as a function of the two pumping rates W_A and W_B . (7 points)

b) Find what steady-state conditions are needed for an inversion on the $4 \rightarrow 2$ transition and how this inversion depends on the two pumping rates. (3 points)

