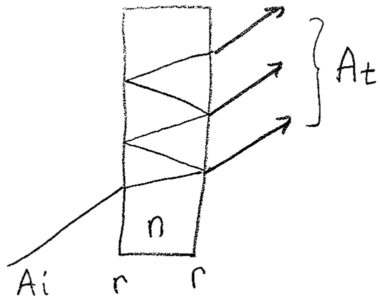
Optics Problem (Solution) Spring 2011

Part 1:



r = amp. refl. from air to glass

t = amp. trans. from air to glass

r' = amp. refl. from glass to air

t' = amp. trans. from glass to air

a) $$A_t = A_i\, t t' e^{i\delta/2}\left(1 + r'^2 e^{i\delta} + r'^4 e^{2i\delta} + \dots\right)$$

$$= \frac{A_i\, t t' e^{i\delta/2}}{1 - r'^2 e^{i\delta}} = \frac{A_i\, e^{i\delta/2}(1-R)}{1 - Re^{i\delta}}$$

where $R = r'^2$ = int. refl.

$T = tt'$ = int. trans.

and $R + T = 1$

The phase is given by

$$\angle\left\{\frac{A_t}{A_i}\right\} = \delta/2 + \arctan\left\{\frac{\mathrm{imag}\left[(1-R)/(1-Re^{i\delta})\right]}{\mathrm{real}\left[(1-R)/(1-Re^{i\delta})\right]}\right\}$$

Note: $$\frac{1-R}{1-Re^{i\delta}} = \frac{1-R}{1-Re^{i\delta}} \cdot \frac{1-Re^{-i\delta}}{1-Re^{-i\delta}} = \frac{(1-R)(1-Re^{-i\delta})}{1+R^2-2R\cos(\delta)}$$

So: $$\mathrm{Im}\left\{\frac{1-R}{1-Re^{i\delta}}\right\} = \frac{-(1-R)R\sin(-\delta)}{1+R^2-2R\cos(\delta)} = \frac{(1-R)R\sin(\delta)}{1+R^2-2R\cos(\delta)}$$

$$\mathrm{Re}\left\{\frac{1-R}{1-Re^{i\delta}}\right\} = \frac{(1-R)(1-R\cos(-\delta))}{1+R^2-2R\cos(\delta)} = \frac{(1-R)(1-R\cos(\delta))}{1+R^2-2R\cos(\delta)}$$

$$\therefore\ \angle\left\{\frac{A_t}{A_i}\right\} = \delta/2 + \mathrm{atan}\left\{\frac{(1-R)R\sin(\delta)}{(1-R)(1-R\cos(\delta))}\right\}$$

$$= \frac{\delta}{2} + \mathrm{atan}\left\{\frac{R\sin(\delta)}{1-R\cos(\delta)}\right\}$$

b) If $\delta \ll 1$, $\sin\delta \approx \delta$
$\cos\delta \approx 1$
$\mathrm{atan}(x) \approx x$

and $\angle\left\{\frac{A_t}{A_i}\right\} = \frac{\delta}{2} + \frac{R\delta}{1-R}$,

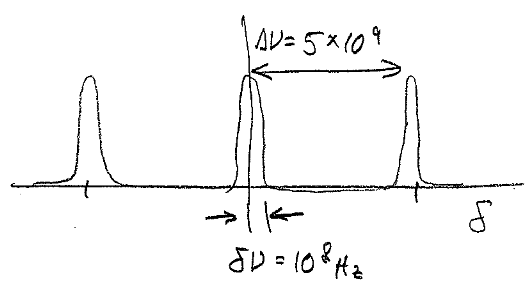
The relationship is linear with proportionality constant $\frac{R}{1-R}$

Part 2: Etalon properties
$n = 1.5$
$l = 2$ cm

$$\therefore \quad \Delta\nu = \frac{c}{2nl} = \frac{3\times10^{8}\,\mathrm{m/s}}{(2)(1.5)\,2\times10^{-2}\,\mathrm{m}} = 5\times10^{9}\,\mathrm{Hz}$$

$$\delta\nu = 10^{8}\,\mathrm{Hz}$$

$$\therefore \quad \text{Finesse} = F = \frac{5\times10^{9}}{10^{8}} = 50$$



To scan across the entire free spectral range ($\delta = 0 \to 2\pi$), the round-trip cavity length increases by λ_{eff} (or λ/n), corresponding to a mirror shift of $\frac{\lambda}{2n}$. Since the spectral bin (10^{8} Hz) is $1/50$ of the free spectral range, the mirror positional tolerance is:

$$\Delta l = \left(\frac{\lambda}{2n}\right)\left(\frac{1}{50}\right) = \left(\frac{1\,\mu m}{3}\right)\left(\frac{1}{50}\right) = \frac{1}{150}\,\mu m = 6.7\ \mathrm{nm}$$