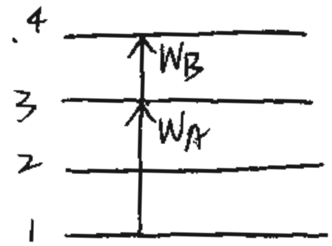

Rate equations:

$$\frac{dN_4}{dt} = W_B N_3 - \gamma_4 N_4$$

$$\frac{dN_3}{dt} = W_A N_1 - W_B N_3 + \gamma_{43} N_4 - \gamma_3 N_3$$

$$\frac{dN_2}{dt} = \gamma_{42} N_4 + \gamma_{32} N_3 - \gamma_2 N_2$$

Steady state: $N_2 = \frac{1}{\gamma_2}\left(\gamma_{42} N_4 + \gamma_{32} N_3 \right)$

$$\begin{cases} W_B N_3 - \gamma_4 N_4 = 0 \quad \Rightarrow \quad N_3 = \frac{\gamma_4}{W_B} N_4 \\ W_A N_1 - W_B N_3 + \gamma_{43} N_4 - \gamma_3 N_3 = 0 \end{cases}$$

$$\Rightarrow \quad N_4 = \frac{W_A}{\gamma_4 - \gamma_{43} + \gamma_3 \gamma_4 / W_B} \cdot N_1$$

$$\Delta N_{42} = N_4 - N_2$$

$$= N_4 - \frac{\gamma_{42} + \frac{\gamma_{32}}{W_B}\gamma_4}{\gamma_2} N_4$$

$$= \frac{\gamma_2 - \gamma_{42} - \frac{\gamma_{32}}{W_B}\gamma_4}{\gamma_2} \cdot \frac{W_A N_1}{\gamma_4 - \gamma_{43} + \gamma_3 \gamma_4 / W_B}$$

To achieve inversion: $\Delta N_{42} > 0$

Since: $\gamma_4 > \gamma_{43}$, so $\gamma_4 - \gamma_{43} + \gamma_3 \gamma_4 / W_B > 0$

Then require: $\gamma_2 - \gamma_{42} - \dfrac{\gamma_{32}}{W_B} \gamma_4 > 0$

so: $$W_B > \frac{\gamma_{32} \gamma_4}{\gamma_2 - \gamma_{42}}$$

assuming $\gamma_2 > \gamma_{42}$.

This should hold for a good laser system where lower level "2" should be short-lived.