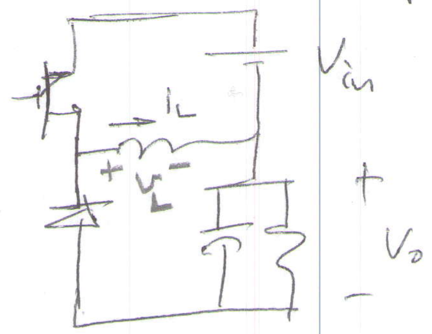
Solution 9A Spring 2013



$V_{in} = 30\,V$, $V_o = 60\,V$, $f_s = 200\,kHz$

$P_{o\,max} = 180\,W$. $T_s = \frac{1}{200} = 5\,\mu s$

(a) at the border of Cont./Discont. mode –

$$P_o = \frac{1}{3} P_{o,max} = \frac{180}{3} = 60\,W.$$

$$\frac{V_o}{V_{in}} = \frac{D}{1-D} = \frac{60}{30} = 2$$

$\therefore$ $2 - 2D = D$

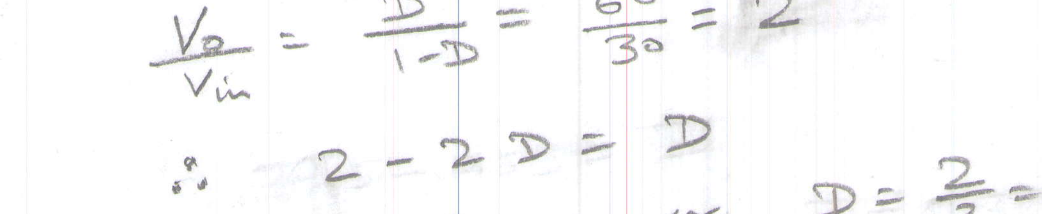
or $3D = 2$ or $D = \frac{2}{3} = 0.667$

$$I_{in} = \frac{P_o\,(=P_{in})}{V_{in}} = \frac{60}{30} = 2\,A$$

$$I_o = \frac{P_o}{V_o} = \frac{60}{60} = 1\,A$$

$\therefore$ $I_L = I_{in} + I_o = 3\,A$

$\hat{I}_L = 2 \times I_L = 6\,A$



During $0 < t < DT_s$ $V_L = V_{in} = 30\,V$

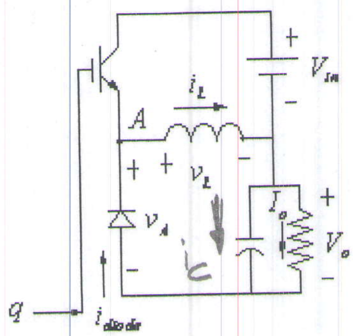
$\therefore$ $L_{crit} \frac{\Delta I}{\Delta T} = V_{in}$ $\Rightarrow$ $L_{crit} = \frac{30\,V \times 0.667 \times 5\,\mu s}{6\,A}$

$= 16.667\,\mu H$

Solution 2/2

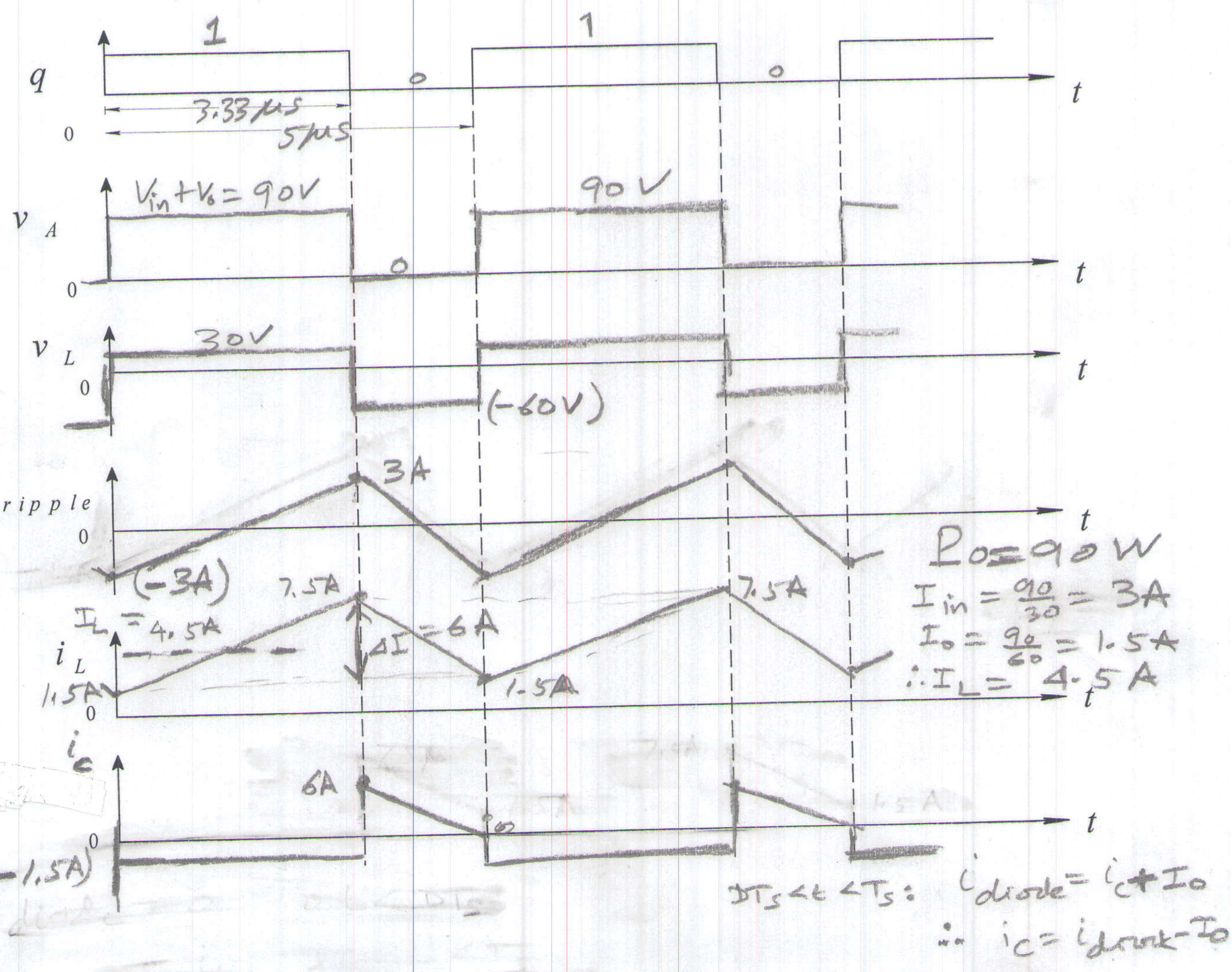
Problem 9A) Power Electronics (2 pts)

A Buck-Boost converter shown below is designed to operate with $V_{in} = 30V$, $V_o = 60V$ and $f_s = 200\,kHz$. The maximum output power level of this converter is $P_{o,\max} = 180W$. Assume ideal components.



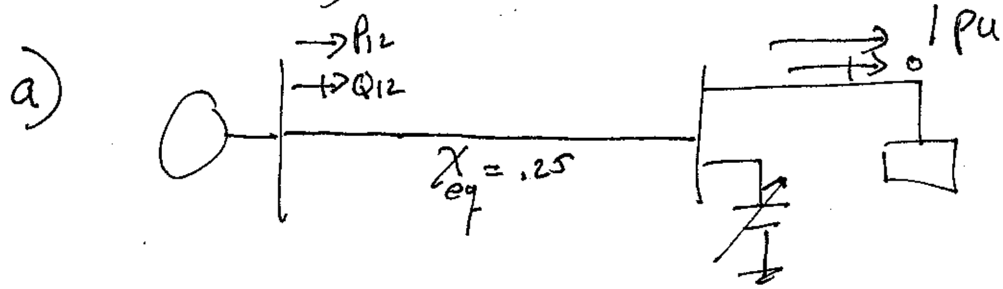
(a) (1 pt) It is to be designed such that if the output power is equal to or higher than one-third of $P_{o,\max}$, it remains in the continuous-current conduction mode; below this output power level, it goes into the discontinuous-conduction mode. Calculate the value of the inductor L to satisfy this design condition.

(b) (1 pt) This converter is operating at $P_o = 90W$. Calculate and draw the waveforms of the following variables as a function of time in the figure below, labeling both axes as appropriate: $v_A, v_L, i_{L,ripple}, i_L$ and i_{diode}. (Note: $i_{L,ripple} = i_L - I_L$).



Problem 9B) Power Systems

a)



$$P_{12} + jQ_{12} = V_1\angle\theta_1 \left[I_{12} \right]^* = V_1\angle\theta_1 \left[\frac{V_1\angle\theta_1 - V_2\angle\theta_2}{.25j} \right]^*$$

$$= 1 \left[\frac{1 - (1\cos\theta_2 + j\,1\sin\theta_2)}{.25j} \right]^*$$

$$= \left[\frac{(1 - 1\cos\theta_2) - j\,1\sin\theta_2}{.25j} \right]^*$$

$$= \frac{(1 - \cos\theta_2) + j\sin\theta_2}{-.25j}$$

$$= \frac{(4 - 4\cos\theta_2) + j\,4\sin\theta_2}{-j}$$

$$= j(4 - 4\cos\theta_2) - 4\sin\theta_2$$

$P_{12} = 1 = -4\sin\theta_2$, $\sin\theta_2 = -1/4$, $\theta_2 = -14.4775^\circ$

Part b) $Q_{12} = 4 - 4\cos\theta_2 = 4 - 4(.9682) = .1270$ pu mvars

Q supplied by SVC:

$$P_{21} + jQ_{21} = V_2\angle\theta_2 \left[I_{21} \right]^*$$

$$= V_2\angle\theta_2 \left[\frac{V_2\angle\theta_2 - V_1\angle\theta_1}{.25j} \right]^*$$

$$= V_2\angle\theta_2 \left[\frac{V_2\angle-\theta_2 - 1}{-.25j} \right]$$

$$= \frac{|V_2|^2 - |V_2|\angle\theta_2}{-.25j}$$

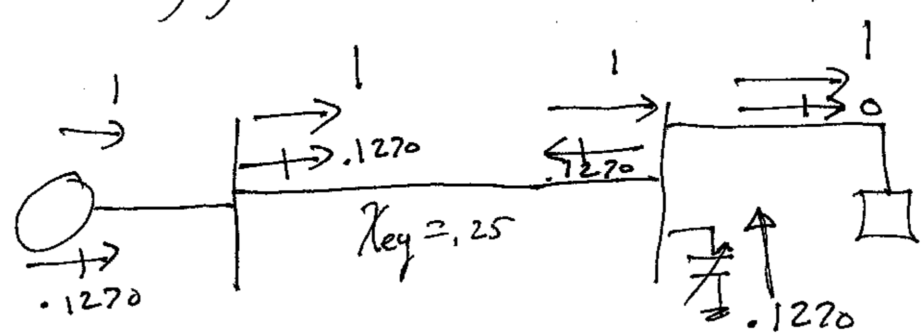
$$= \frac{1 - (\cos\theta_2 + j\sin\theta_2)}{-.25j} = \frac{(4 - 4\cos\theta_2) - j4\sin\theta_2}{-j}$$

$$= j(4 - 4\cos\theta_2) + 4\sin\theta_2$$

$$P_{21} = 4(\sin(-14.725^\circ)) = -1$$

$$Q_{21} = +.1270$$

Resulting Power Flow



Part c)

One Line opened $x_{eq} = .5$

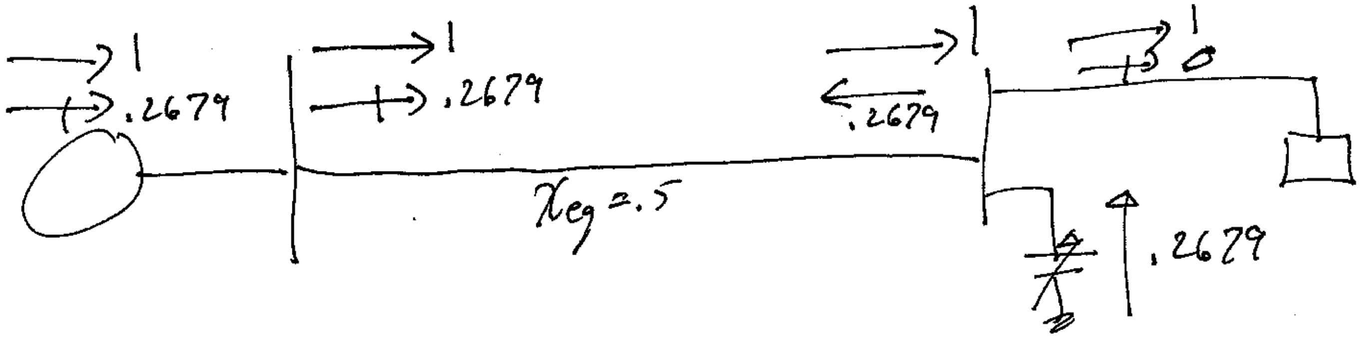
$$P_{12} + jQ_{12} = \frac{(1-\cos\theta_2) + j\sin\theta_2}{-.5j}$$

$$1 + jQ_{12} = j(2 - 2\cos\theta_2) - 2\sin\theta_2$$

$$1 = -2\sin\theta_2 \qquad \sin\theta_2 = -.5$$

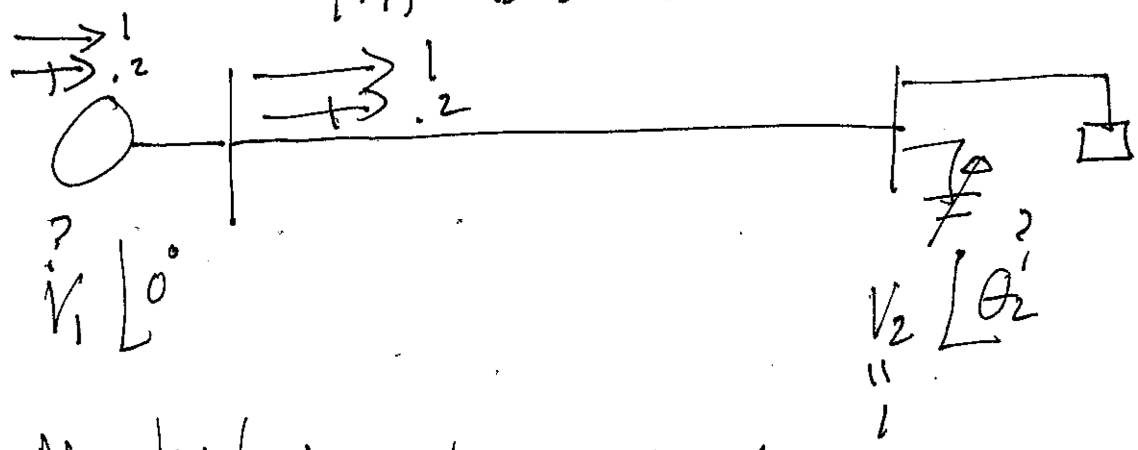
$$\theta_2 = -30^\circ$$

$$Q_{12} = 2 - 2\cos\theta_2 = 2 - 2(.866) = .2679$$



Note Q gen is > Q^{max} for generator

d) We want to bring Q_{12} down to 20 mVAR or .2 pu. The only way is to reduce $|V_1|$ below 1.0



Now $|V_1|$ is unknown, θ_2 is unknown.
assume θ_1 is still zero, V_2 is still 1.0

$$1 + .2j = V_1 \angle \theta_1 \left[\frac{V_1 \angle \theta_1 - V_2 \angle \theta_2}{.5j} \right]^*$$

$$= |V_1| \left[\frac{|V_1| - (1 \cos\theta_2 + j \sin\theta_2)}{.5j} \right]^*$$

$$= |V_1| \left[\frac{(|V_1| - \cos\theta_2) - j \sin\theta_2}{.5j} \right]^*$$

$$= |V_1| \left[\frac{(|V_1| - \cos\theta_2) + j \sin\theta_2}{-.5j} \right]$$

$$= \frac{(|V_1|^2 - |V_1|\cos\theta_2) + j |V_1| \sin\theta_2}{-.5j}$$

$$1 + .2j = 2j\left(V_1^2 - V_1\cos\theta_2\right) - 2V_1\sin\theta_2$$

$$1 = -2V_1\sin\theta_2 \Rightarrow V_1\sin\theta_2 = -.5$$

$$.2 = 2\left(V_1^2 - V_1\cos\theta_2\right) \Rightarrow V_1^2 - V_1\cos\theta_2 = .1$$

$$V_1^2\sin^2\theta_2 = .25 \qquad V_1^2\cos^2\theta_2 = \left(V_1^2 - .1\right)^2$$

$$V_1^2\left(\sin^2\theta_2 + \cos^2\theta_2\right) = \left(V_1^2 - .1\right)^2 + .25$$

$$V_1^2 = V_1^4 - 0.2V_1^2 + .01 + .25$$

let $x = V_1^2$ $\quad x^2 - 1.2x + .26 = 0$

Using quadratic Formula $x = .9162$, $V_1 = .9572$

$$V_1\sin\theta_2 = -.5$$

$$\sin\theta_2 = -.5/.9572 = -.5223$$

$$\theta_2 = -31.49^\circ$$

Voltage at gen must be dropped to .9572 pu to get gen MVAR to 20 MVAR output.