Find conditions for charge neutrality: *(0.5 points for recognizing that they need to solve charge neutrality equation in order to find E_F).*

$$\rho = q(N_d^+ + p) - q(N_a^- + n) = 0$$

$$(N_d^+ + p) = (N_a^- + n)$$

$$(n - p) = (N_d^+ - N_a^-)$$

How to determine if the dopants are fully ionized? *(0.5 points for determining that the dopants must be fully ionized. They could guess this as well, but then if they guess and get the answer wrong, they don't get this piece of partial credit.)*

n-type semiconductor with $N_V > N_C$, so the Fermi level is going to have be to above mid-gap, putting it over 5kT from E_A. Therefore the acceptors are fully ionized: $N_a^- \approx N_a$.

We know that the maximum value of $(N_d^+ - N_a^-)$ is 2 x 10^{15} cm^{-3}. Therefore, let's check the value of $(n - p)$ when $E_F = E_D - 3kT$.

$$(n - p) = N_C e^{(E_F - E_C)/kT} - N_V e^{(E_V - E_F)/kT} = N_C e^{(E_D - E_C - 3kT)/kT} - N_V e^{(E_V - E_D + 3kT)/kT}$$

$$(n - p) = (8.3\times10^{16}\text{cm}^{-3})e^{(-4-78)/26} - (1.7\times10^{19}\text{cm}^{-3})e^{(-346+78)/26}$$

$$(n - p) = (3.54\times10^{15}\text{cm}^{-3}) - (5.68\times10^{14}\text{cm}^{-3}) = 2.97\times10^{15}\text{cm}^{-3}$$

Since $(n - p) > (N_d^+ - N_a^-)_{\text{MAX}}$, then we know that $E_D - E_F > 3kT$ and that the donors must also be fully ionized: $N_d^+ \approx N_d$.

Now that we know this, we can solve the charge neutrality equation:

$$(n - p) = (N_d - N_a)$$

$$(N_d - N_a) = N_C e^{(E_F - E_C)/kT} - N_V e^{(E_V - E_F)/kT} = N_C e^{(E_F - E_V - E_G)/kT} - N_V e^{(E_V - E_F)/kT}$$

$$(N_d - N_a) = N_C e^{(E_F - E_V)/kT} e^{-E_G/kT} - N_V e^{-(E_F - E_V)/kT}$$

(0.5 points for using the correct relation(s) for n and p.)

(0.5 points for recognizing that the hole concentration is not insignificant.)

Let's define $x \equiv e^{(E_F - E_V)/kT}$, then:

$$\left((N_d - N_a)\cdot e^{E_G/kT}\right)\cdot x = N_C x^2 - N_V e^{E_G/kT}$$

$$N_C x^2 - \left((N_d - N_a)\cdot e^{E_G/kT}\right)\cdot x - N_V e^{E_G/kT} = 0$$

Solve quadratic equation:

$$x = \frac{(N_d - N_a)\cdot e^{E_G/kT} \pm \sqrt{(N_d - N_a)^2 \cdot e^{2E_G/kT} + 4N_C N_V e^{E_G/kT}}}{2N_C}$$

$$x = \frac{(2\times10^{15})\cdot e^{0.35/0.026} \pm \sqrt{(2\times10^{15})^2 \cdot e^{0.70/0.026} + 4(8.3\times10^{16})(1.7\times10^{19})e^{0.35/0.026}}}{2(8.3\times10^{16})}$$

$$x = \frac{(2\times10^{15})\cdot(7.02\times10^{5}) \pm \sqrt{(4\times10^{30})\cdot(4.93\times10^{11}) + (5.64\times10^{36})(7.02\times10^{5})}}{1.66\times10^{17}}$$

$$x = \frac{(1.40\times10^{21}) \pm \sqrt{(5.93\times10^{42})}}{1.66\times10^{17}} = 2.31\times10^{4}$$

$$e^{(E_F - E_V)/kT} = 9.90\times10^{3}$$

$E_F - E_V = 0.261\,\text{eV}$ or $E_C - E_F = 0.089\,\text{eV}$ *(1.0 points for getting the correct numerical answer).*

(b) At what energy is the Fermi level at $T = 0$?

First determine ionization state of donors vs. E_F:

$$N_a^+ = 0, \quad E_F > E_D$$
$$N_d^+ = N_d, \quad E_F < E_D$$

$$N_a^- = 0, \quad E_F < E_A$$
$$N_a^- = N_a, \quad E_F > E_A$$

Now, we can determine ρ vs. E_F: *(0.5 points for understanding that you need to find the energy at which the net charge goes from positive to negative).*

$$\rho = -qN_a^-, \qquad E_D < E_F < E_C$$
$$\rho = +q(N_d^+ - N_a^-), \qquad E_A < E_F < E_D$$
$$\rho = +qN_d^+, \qquad E_V < E_F < E_A$$

$$\rho = -q(8\times10^{15}\,\mathrm{cm}^{-3}), \qquad E_D < E_F < E_C$$
$$\rho = +q(2\times10^{15}\,\mathrm{cm}^{-3}), \qquad E_A < E_F < E_D$$
$$\rho = +q(1\times10^{16}\,\mathrm{cm}^{-3}), \qquad E_V < E_F < E_A$$

The only place where we could have charge neutrality would be at $E_F = E_D$, since this is where ρ changes sign. Therefore:

$E_C - E_F = 0.004\,\mathrm{eV}$ (Don't know $E_g(T)$, so can't express answer in terms of E_F-E_V).

(0.5 points for getting the correct numerical answer).