a) Density of states

Using a wavefunction with periodic boundary conditions, one needs $k_x L_x = 2\pi n_x$, where $n_x$ is an integer and $L_x$ is a crystal dimension in the x-direction. Then any quantity needs to be summed over the states as

$$\sum_{n_x, n_y}$$

which can be changed to an integral as

$$\frac{L_x L_y}{(2\pi)^2} \int dk_x dk_y$$

since a change in $\Delta n_x$ corresponds to a change $(L_x/2\pi)\Delta k_x$. Then changing to polar coordinates $dk_x dk_y = kdkd\phi$. Integrating over angle this is

$$\frac{A}{2\pi} \int kdk$$

If one wanted to be able to integrate over energy, use $E = \hbar v_F k$ to change $k$ and $dk$ to get

$$\frac{2\pi A}{(hv_F)^2} \int EdE$$

so that the density of states per unit area is

$$D(E) = \frac{2\pi}{(hv_F)^2} E$$

This needs to be multiplied by 2 since there is a degeneracy of two conduction and valence bands and 2 since there are two spin states for each, giving

$$D(E) = \frac{8\pi}{(hv_F)^2} E$$

b) The charge (number) density per area for the capacitor is

$$n = \frac{K_o \epsilon_o}{d} \cdot V/q$$

In [125]:
```
q=1.6E-19
n=3.9*8.85E-14*20/500E-7/q
print 'n=',n,'electrons/cm2' #per cm2
```

```
n= 8.62875e+11 electrons/cm2
```

The density of states is

$$D(E) = \frac{2 \cdot 2}{2\pi} \frac{E}{(\hbar v_F)^2}$$

and the density of electrons added is

$$n = \int_0^{E_F} dE\, f(E) D(E) \ .$$

where the Fermi factor is unity at T=0 up to $E_F$ (and nearly the same at RT since it will be found that $k_B T \ll E_F$) so that

$$E_F = \frac{h\sqrt{n\pi}}{2\pi} v_F$$

In [126]:
```
h=6.63E-34
vF=1E8
EF=h*sqrt(n*pi)*vF/2/pi
print EF/q, 'V'
```

```
0.108583180137 V
```

c) To calculate the density of atoms in the lattice, examine the cell defined by $\vec{a}_1, \vec{a}_2$. There are 4 corner atoms, each shared by 4 similar cells. In addition there is one basis atom per cell. The area of this cell is $A = 2 \times (1/2)a\sqrt{3}a/2$ .

In [127]:
```
a=2.47E-8
A=2*2*(1/2)*a*sqrt(3)*a/2
natoms=2/A
print natoms,'atoms per cm2'
```

```
1.89267245551e+15 atoms per cm2
```

In [128]:
```
ratio = n/natoms
print 'number of electrons per atom =','%.3g' %ratio
```

```
number of electrons per atom = 0.000456
```