

Instructions:

1. Answer all problems.
2. There are 2 problems with a total of 40 points (15 points for Problem 1 and 25 points for Problem 2).
3. Upsampling and downsampling operations are defined mathematically in the sequence domain (see below). You do not have to derive the relationships in the frequency domain. For example, if you know the relationship between $X_u(e^{j\omega})$ and $X(e^{j\omega})$, then simply use it if it is pertinent to your answer.
4. Show work for partial credit.

Upsampling/Downsampling:



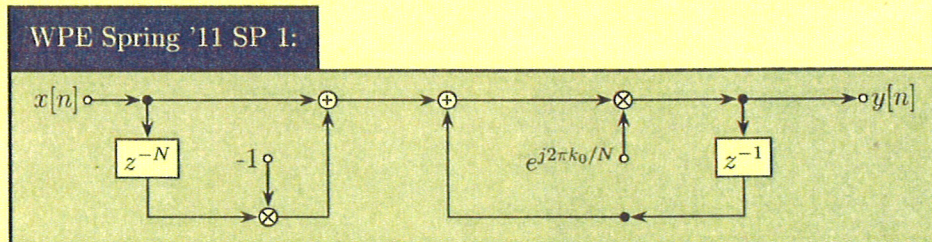
$$x_u[n] = \begin{cases} x[n/L] & \text{for } n = 0, \pm L, \pm 2L, \dots, \\ 0 & \text{otherwise.} \end{cases}$$



$$\begin{aligned} x_d[n] &= x[Mn] \\ &= p[Mn]x[Mn]. \end{aligned}$$

Where $p[n]$ is a periodic unit pulse with period M .

1. Consider the following system:

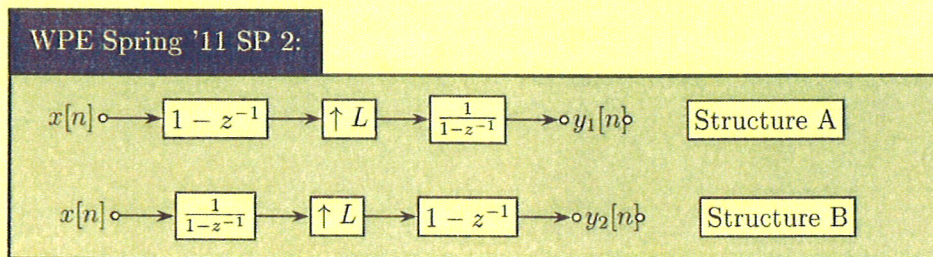


Assume the input to be a finite duration sequence of length $L \leq N$ and $0 \leq k_0 < N$ (assume integer, for simplicity). Answer the following questions:

- (a) Derive the difference equation for the output, $y[n]$. Does this equation represent an FIR or IIR filter? Explain your answer and comment on the stability of the filter. [6 points]
- (b) Derive the relationship between $Y(e^{j\omega})$ and $X(e^{j\omega})$ and sketch the magnitude response of the filter. Assume $N = 8$ and $k_0 = 2$. [6 points]
- (c) This filter is sometimes referred to as a sliding DFT. Explain this statement based on your derivations above. [3 points]

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2. John Smith is currently taking a DSP class, where he is learning introductory material on multirate systems. He learned that an ideal interpolator can be implemented by an upsampler followed by an ideal LPF. While surfing the Internet, he learned that multiplier-free DSP structure (labeled Structure A in the figure below) can be used as a basic block in a practical interpolator. John did not understand how this structure works and came to you for help:



- (a) Derive expression for $Y_1(e^{j\omega})$ and explain how Structure A approximates an interpolator. [10 points]
- (b) Thinking about cascade relationships from system theory, John wondered if Structure B is equivalent to Structure A. Explain how the two structures are different and why Structure B does not function as an interpolator. [10 points]
- (c) John thought that, if there is an interpolation filter implemented with one or both of the structures in the figure, there exists a similar structure for implementing a downsampler. What do you think of this assertion? If you agree, sketch the appropriate structure. [5 points]
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