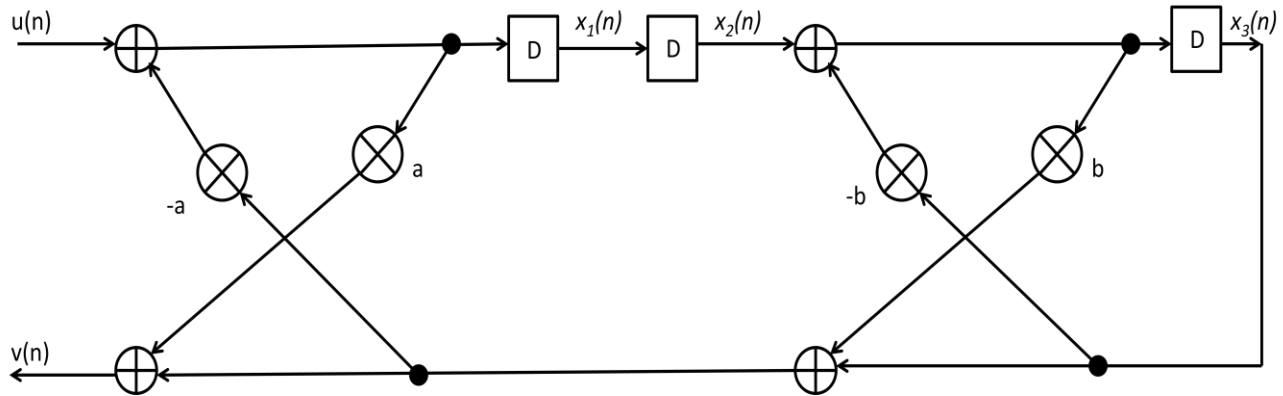


[DSP] [6+7+3+7+3+6+5+3=40 points]

In this problem, we will derive the transfer function of the 3rd order IIR lattice filter shown below in two ways. First, we will use the state-space description method, and then the polynomial reduction method. We will also study other aspects of this filter.



Assume $|a| < 1$ and $|b| < 1$.

(a) Describe the above filter by state-variable description.

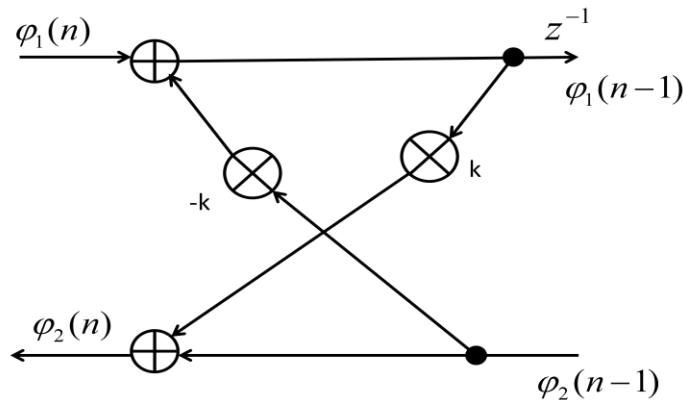
$$\mathbf{x}(n + 1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b}u(n)$$

$$y(n) = \mathbf{c}^T \mathbf{x}(n) + du(n)$$

Where, $\mathbf{x}(n + 1) = \begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \end{bmatrix}$

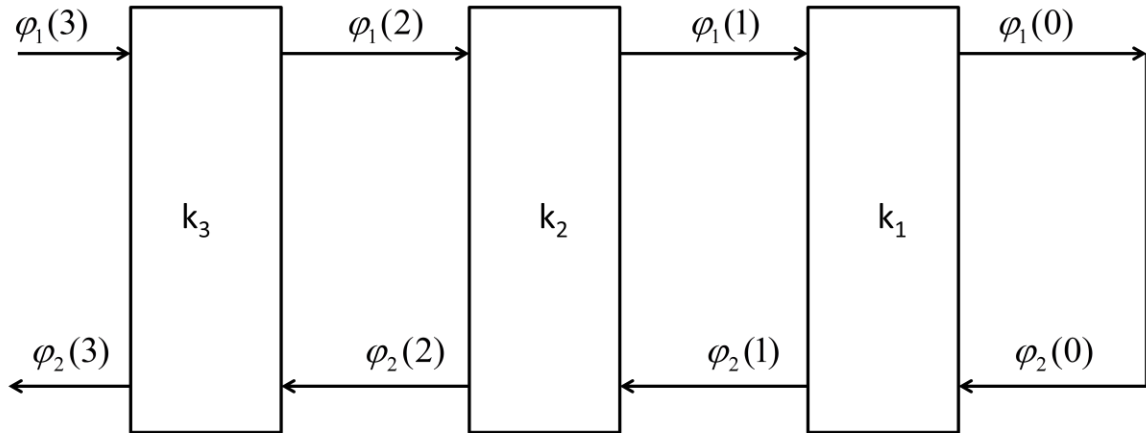
(b) Compute the transfer function $\frac{V(z)}{U(z)}$ from the state-space description in (a).

(c) Now we will compute the transfer function using modular description of each stage of the lattice. Consider a typical stage described below. The input and output polynomials represent polynomials in z . The term n represents the order of the module and not time. The modular degree reduction reduces the degree of the top and bottom polynomials from left to right. Each block is characterized by a k coefficient.



Compute $\varphi_1(n)$ and $\varphi_2(n)$ in terms of $\varphi_1(n - 1)$ and $\varphi_2(n - 1)$. Note the z^{-1} (delay element) on top right signal.

(d) Consider the filter in a modular form shown below, where each module is described by the block diagram in (c).



What are k_1, k_2 and k_3 in terms of a and b ?

Assume $\varphi_1(0) = \varphi_2(0) = 1$

Using the iterative computation in (c), compute $\varphi_1(3)$ and $\varphi_2(3)$ as polynomials in z . You should first use $\varphi_1(0)$ and $\varphi_2(0)$ to compute $\varphi_1(1)$ and $\varphi_2(1)$.

Compute $H(z) = \frac{V(z)}{U(z)} = \frac{\varphi_2(3)}{\varphi_1(3)}$. It should be same as the result you in (b).

(e) Can the poles of this filter be ever outside the unit circle if $|a| < 1$ and $|b| < 1$?

Assume $a \neq 0, b \neq 0$.

(f) What type of filter is $H(z)$? Justify. What can you say about the relationship between poles and zeros of $H(z)$?

(g) Find $\sum_{n=-\infty}^{\infty} h^2(n)$. [Hint: Use Parseval's Relation].

(h) If $u(n) = (-1)^n$, find $v(n)$.